**The Interval Valued Fuzzy Graph Associated with the Graphical Structure of the Cyclic Group**

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**Abstract**

In this paper, we have constructed the interval - valued fuzzy graph (IVFG) corresponding to crisp graph based on the inverse of the elements of the Cyclic group and analyze various properties. The edges of the IVFG corresponding to the crisp graph can be determined if the Inverse of the element in a group is identified. We proved few theorems and some results, which are carrying over to construct IVFG.

**Keywords:** Crisp graphs, Interval valued fuzzy graphs, Cyclic groups

Subject Classification: MSC 2010 20M17, 20N15, MSC 2020

**1. Introduction**

Graphs are used to symbolize relation between the objects. But uncertainty and vagueness happen to be significant elements in relations. To address this, Rosenfeld [9] developed the theory of fuzzy graphs giving membership degree to nodes and arcs of crisp graphs. But there may be incidences in which one do not have an extra information about the choice of the membership degrees. In these situations, it is better “to represent the membership degree” by means of an interval. “Replacing the membership” degree of nodes and arcs “in fuzzy graphs by interval valued fuzzy sets such that they satisfy some particular condition, interval-valued fuzzy graphs were defined”. Thus interval valued fuzzy graph (IVFG) provides a better explanation “of vagueness and uncertainty within the” particular interval than fuzzy graphs. “Hongmei and Lianhua [4]” introduced IVFGs generalizing the fuzzy graphs. Several new works on the fuzzy graphs are appearing in literature regularly. Recently, AL-Hawary and Hourani [7] introduce “direct product, semi-strong product and strong product of the intuitionistic fuzzy graphs” [3]. We can associate any number of IVFGs with a particular crisp graph. This paper is an earnest effort to construct a particular type of IVFG from a crisp graph based on the degrees of the nodes of crisp graph. Throughout our work, we consider only simple, connected, undirected graphs, IVFG [13] associate inverse of the element of the group. “N. Naga Maruthi Kumari and R. Chandra Sekhar” introduced [1] “Coloring of Interval Valued Fuzzy Graphs and Traffic Light Problem”. Mini Gopala Krishnan, et.al., 2019 have introduced [2] generator graphs for cyclic groups. This topic is more interesting to study “the finite groups are represented in the form of graphs”. Kumari, M.N.N., Sekhar, C.R. [5] explained well about coloring of interval valued fuzzy graph using alpha cut.

A. Nagoorgani, K. Radha [6] introduced Isomorphism on fuzzy graphs. Now we are continuing to work on crisp graphs to interval valued fuzzy graphs [8].

Here we are working on graphs, which are formed by the cyclic groups.

* Explaining the relationship between nodes of the graphs and the elements of the group
* The edges of the graphs, which are formed by using the generator of the cyclic groups [10]
* Explaining the relation between the inverse of an element in a group , and the degree of in the corresponding graph’ [12].
* IVFG associated with graphs corresponding to groups based on the inverse of every element of the group [11].
* The IVFG associated with a cyclic group of order is regular IVFG. If G is cyclic group, is a graph generated by ‘a’ then is equal to number of generators of the group G. The Interpretation of cyclic groups as graphs was explained in [14]

**2. Preliminaries**

**2.1. Definition**

A graph is an ordered pair, where is non-empty set and is a set of unordered pairs of elements of. The set is called the vertex set of and the set is called edge set of the . The order of a graph is the number of vertices in V and it is denoted by ‘A’ and size of the graph is the number of edges in E and is denotes ‘B’. -graph is a graph having ‘A’ vertices and ‘B’ edges. A graph is simple if it has no loop or multiple edges.

**Example.** Let and, is having 4 vertices and 3 edges then the graph was drawn in the figure 1.1



Fig 1.1

**2.2. Definition**

Let G be a non-empty subset with respect to the binary operation \* is said to be a group. It is denoted by A group is satisfying the following axioms

The binary operation \* is associative.

If there exist an identity element , such that

For every , there exist, such that

**2.3. Definition**

The graph , if the order of pair ), where is an interval valued fuzzy set on V and “is an interval-valued fuzzy relation on E”.

**Example.** A graph such that . “Let A be an interval valued fuzzy set of V and B is an interval valued fuzzy set of. The IVFG is drawn in the following figure 1.2



Fig 1.2

**2.4. Definition**

IVFG associated with the inverse of the elements of the group.

**2.5. Definition**

IVFG associated with identity and inverse graphs corresponding to groups based on the inverse of every element of the group.

**2.6. Definition**

Let be a simple connected graph with . The IVFG defined as the pair I= of IVFG, where an IVFS is a set on V and is an IVFS is a set on E, such that

 Then is an IVFG.

**2.7. Definition**

Let be the graph corresponding to the cyclic group generatedby ‘g’ i.e. . In the graph , the element ‘a’ is adjacent to the vertex ‘b’ iff

**Example.**

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**2.8. Definition**

An IVFG is said to be simple connected IVFG. If and .

**2.9. Definition**

An IVFG **‘**I’ “is said to be connected if any two nodes are joined by a path”.

**2.10. Definition**

An IVFG **‘**I’ “is said to be regular if every node of ‘I’ has the same degree”.

**2.11. Definition**

Two arcs are said to be comparable if their membership degrees are such that either and or and. They are said to be equal if their membership degree is equal.

**2.12. Definition**

A connected IVFG ‘I ‘is said to be strongly connected if there exists a strong path between every two nodes of ‘I’.

**3. Results**

**3.1. Theorem**

Let be a simple connected graph with by using the following membership, and ,

I is interval valued fuzzy graph.

**Proof:** Let be a simple connected graph with

Consider and. We prove that A is an IVFS on V and B is an IVFS on E satisfying the requirement of an IVFG.

To show that A is an IVFS on V

Since the member ship of all the nodes and the edges are in the interval [0, 1],

From the construction define, we have,

,

Since is connected,

 And so clearly.

Clearly, since

Again, since

Thus we have, . Hence A is an IVFS on V.

To show that B is an IVFS on E

We have,

Since and,

And since,

Also, implies that and

Thus we have, Hence B is an IVFS on E.

Now we Show A and B satisfies the requirement of an IVFG. We know that if ‘x’ and ’y’ are any two real numbers in [0, 1], then

And.

Thus) constructed as above is an IVFG.

**Example.** Figure 1.3 shows as crisp graph and its associated IVFG ‘I’. Using ‘I’, we can verify the procedure

,

,

Now

∴

,

,

.

Hence G is an IVFG



 Fig 1.3

**3.2. Theorem**

If G is a cyclic group, generated by then the correspondence graph . Intervalvalued fuzzy graph

**Proof:** Given is the IVFG Corresponding to the cyclic group G

Hence, if G is a group, then in IVFG I of



Fig 1.4

**3.3. Theorem**

If be the IVFG associated with the graph G\* with ‘n’ vertices and ‘m’ edges, corresponding to the cyclic group G. Then

Where,

**Proof:** Let be the IVFG associated with the graph with ‘n’ vertices and ‘m’ edges, corresponding to the cyclic group G.

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Similarly, we can solve that

 =

**4. CONCLUSION**

In this paper, we established the results in particular type of IVFG from the simple connected graph, I is called the IVFG based on the inverse of the element in G. We proved that the theorems and its results are carrying over to construct IVFG. The work can be continued in future by constructing IVFG using some other parameters based on groups. We look forward to establish few results in the particular direction towards applications in Neural Networking.

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