Radiation Properties of the Array Pattern Synthesis Using Fibonacci and Normalized Modified Binomial (Nmb) Polynomials

Abdolkhalegh Mohammadi and Mohammad Pourbagher, Javad Nourinia
Microelectronics Research Laboratory, Urmia university, IRAN

Abstract: *This paper presents radiation property of the array antenna pattern synthesis using new polynomial. Fibonacci and Smooth Normalized Modified Binomial (SNMB) are two polynomials which the property of their pattern are investigated. Fibonacci have a good side lobe level but the lobes are smooth and no tapering is observed. SNMB array type can be accomplished to have tapered minor lobe which is suitable for Radar and low-noise systems.*

Keywords- *polynomials, array, minor lobe, first null, side lobe level (SLL)*

I. Introduction:

Antenna array radiation pattern synthesis has taken major interest since the beginning of the array antenna era. The problem is generally to synthesize the optimum complex excitation coefficients for a given array geometry that will yield an array factor which is – in some sense– close to a desired array factor. Especially when it comes to the subject of shaped pattern synthesis, optimization techniques with iterative procedures are used. Genetic algorithms (GA) [1] are the most widely used methods in pattern synthesis, which can almost deal with all the synthesizing problems. Particle swarm optimization (PSO) [2] and simulated annealing (SA) [3] have already been used in array synthesis for different requirements too. Immune algorithm (IA) [4] is also a new heuristic optimization algorithm which has powerful function of global search. Although some studies like [5] focus on the problem with a different point of view, i.e., attempting to find the optimum geometry under the existence of prescribed excitation coefficients, most of the literature deals with the problem of obtaining the optimum coefficients for one dimensional linear arrays, possibly due to its practical use. Various methods have been applied for the solution of the problem. Among these are non-iterative methods such as Fourier Transform Method, Woodward-Lawson Method, and Taylor Line Source Synthesis Method.

Currently, development of wireless application such as radar and communication with low noise becomes rapidly. The antenna plays an important role as the key device in transmitting and receiving the signal. Generally it is desired to antenna to provide maximum directivity, narrow beamwidth and low side lobe level especially far out minor lobes in order to reduce the noise entering through those minor lobes. The antenna array is one of the most suitable candidates that can fulfill these requirements. In most cases, the elements of an array are identical. The total field of the array is determined by the vector addition of the fields radiated by the individual elements. This is usually not the case depends on the separation between the elements. To provide very directive patterns, it is necessary that the field from the elements of the array interfere constructively (add) in the desired directions and interfere destructively (cancel each other) in the remaining space. In an array with identical elements, there are at least five parameters that can control the shape of the overall pattern of the antenna [6]. Excitation amplitude of the individual elements is one of the features that help us to control the pattern of the array antenna. In N-element linear array with uniform spacing and nonuniform amplitude there are three famous distributions: uniform, binomial and Tschebyscheff. An uniform amplitude array yields the smallest half-power beamwidth. It is followed, in order, by the Dolph-Tschebyscheff and binomial arrays. In contrast, binomial arrays usually possess the smallest side lobes followed, in order, by the Dolph-Tschebyscheff and uniform arrays. As a matter of fact, binomial arrays with element spacing equal or less than $\lambda/2$ have no side lobes. It is apparent that the designer must compromise between side lobe level and beamwidth. a criterion that can be used to judge the relative beamwidth and side lobe level of one design to another is the amplitude distribution (tapering) along the source. It has been shown analytically that for a given side lobe level the Dolph-Tschebyscheff array produces the smallest beamwidth between the first nulls. Conversely, for a given beamwidth between the first nulls, the Dolph-Tschebyscheff design leads to the smallest possible side lobe level. Uniform arrays usually possess the largest directivity.

For some applications, such as radar and low-noise systems, it is desirable to sacrifice some beamwidth and low inner minor lobes to have all the minor lobes decay as the angle increases on either side of the main beam [7]. In these applications the side lobes should be tapered. In this paper radiation properties of array pattern synthesis using some new special polynomials is discussed

II. Array pattern synthesis using Fibonacci polynomials

In this part the design procedures of the array pattern synthesis using modified Fibonacci polynomials will be illustrated. The issue refers to compute the radiation pattern and half power beamwidth (HPBW) and directivity of N-element linear array with uniform spacing and nonuniform amplitude which are excited by Fibonacci polynomials. In the following The Fibonacci polynomials will be introduced. The formula of Fibonacci polynomials is

$$F_n = F_{n-1} + F_{n-2} \quad (1)$$
$$F_0 = 0$$
$$F_1 = 1$$

The generated polynomial will be
0,1,1,2,3,5,8,13,21,34,55,89,...

To use this polynomial for excitation of N-elements we write the Fibonacci polynomial with n=N+1 term, after that we again write these n terms in a degradation form, now if N be even, the same terms will be repeated in a degradation form without any change. Also if N be odd, the maximum term will be omitted from the degradation form terms. Let N=4, the Fibonacci polynomial will be

0,1,1,2,3,5

N is even so we write this polynomial in a degradation form directly after initial terms

0,1,1,2,3,5,5,3,2,1,1,0

Now we omit the additional terms from both sides until the number of terms be equal to number of elements (4 elements).

~~0~~,~~1~~,~~1~~,~~2~~,3,5,5,3,~~2~~,~~1~~,~~1~~,~~0~~

So the amplitude of excitation coefficients for 4-elements array will be

3,5,5,3

Again for a 5-elements array the Fibonacci polynomial will be

0,1,1,2,3,5,8

Now these terms will come with degradation form and the maximum term will be omitted.

0,1,1,2,3,5,8,5,3,2,1,1,0

Now we omit the additional terms from both sides until the number of terms be equal to number of elements (five elements). The answer is: 3,5,8,5,3

The polynomials of Fibonacci for different N are brought here:

N=1 1
N=2 2 2
N=3 2 3 2
N=4 3 5 5 3
N=5 3 5 8 5 3
N=6 5 8 13 13 8 5
N=7 5 8 13 21 13 8 5
N=8 8 13 21 34 34 21 13 8

Now if we use some changes to these polynomials the features of radiation will be better. For example we can add the difference between the smallest and the greatest excitation coefficient to all elements, so the directivity will be better. Now we compare the radiation pattern, HPBW and Directivity of an array with 10 elements which excited by Fibonacci, uniform, binomial and Tschebyscheff. In Fig. 1 We observe that the pattern of fibonacci is between the Uniform and Tschebycheff. It shows that the directivity of fibonacci is better than Tschebycheff but not as well as Uniform. Tschebycheff pattern has the smallest first minor lobe. We do not observe any good tapering from Tschebycheff and fibonacci patterns. But Uniform array has tapered.

III. Normalized Modified Binomial (NMB) :

Using the binomial method for excitation of arrays has some problems. For example the differences between excitation coefficients are very much and it makes some practical problems. In modified binomial polynomial this problem has been solved by keeping other features. In this method we omit two sides of pascal's triangle for several times according to number of element. After that we use the new triangle and by the knowledge of number of element the appropriate row of triangle is chosen. For example for a three element array we delete the sides for three times. Then the third row of new triangle is chosen.

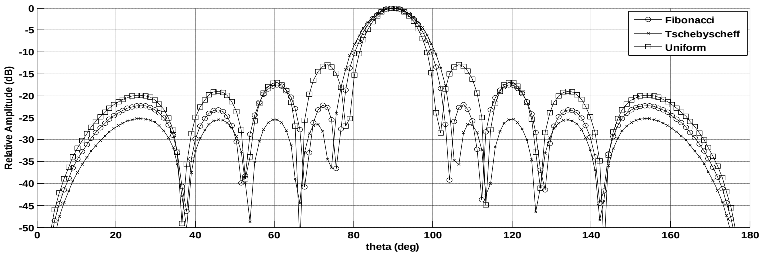

Fig. 1. Comparison of Fibonacci, Uniform and Tschebyscheff

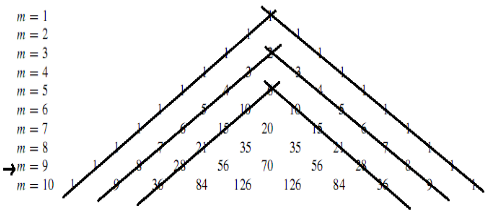

Fig. 2. Deleted sides of pascal's triangle

After that the excitation coefficients are normalized to the big one. In fig. 2 this method is shown completely.

For formulizing the above steps we start with binomial expansion:

$$(1+x)^{N-1} = \binom{N-1}{0} + \binom{N-1}{1}x + \binom{N-1}{2}x^2 + \ldots + \binom{N-1}{N-1}x^{N-1} \quad (2)$$

N is the number of elements. So the excitation coefficients are:

$$\binom{N-1}{0}, \binom{N-1}{1}, \binom{N-1}{2}, \ldots, \binom{N-1}{N-1} \quad (3)$$

From Posteriori reasoning we will find out that the Nth row of new triangle is the 3*Nth of the Pascal's triangle which some coefficient are omitted. For finding the excitation coefficient by new triangle we can trace the following:

$$\binom{N^{old}-1}{N^{new}}, \binom{N^{old}-1}{N^{new}+1}, \ldots, \binom{N^{old}-1}{N^{old}-1-N^{new}} \quad (4)$$

For example for a five-element array the results is:

N^{new} =5

N^{old} =3*5=15

$\binom{15-1}{5}, \binom{15-1}{6}, \binom{15-1}{7}, \binom{15-1}{8}, \binom{15-1}{9}$

The coefficients are:

2002,3003,3432,3003,2002

Now we normalize coefficients to the bigger one.

0.58, 0.87, 1, 0.87, 0.58

The new triangle for excitation coefficient is:

N=1 1
N=2 1 1
N=3 0.8 1 0.8

N=4 0.71 1 1 0.71
. 0.58 0.87 1 0.87 0.58
. 0.5 0.8 1 1 0.8 0.5
0.41 0.68 0.9 1 0.9 0.68 0.41
0.36 0.68 0.84 1 1 0.84 0.68 0.36

Now if we use some changes to this triangle the features of radiation will be better.

For example if the difference between the smallest and the greatest excitation coefficient be added to all elements, the directivity will be better. The excitation coefficient will be:

N=1 1
N=2 1 1
N=3 1 1.2 1
N=4 1 1.29 1.29 1
N=5 1 1.29 1.42 1.29 1
N=6 1 1.3 1.5 1.5 1.3 1
N=7 1 1.27 1.49 1.59 1.49 1.27 1
N=8 1 1.24 1.48 1.64 1.64 1.48 1.24 1

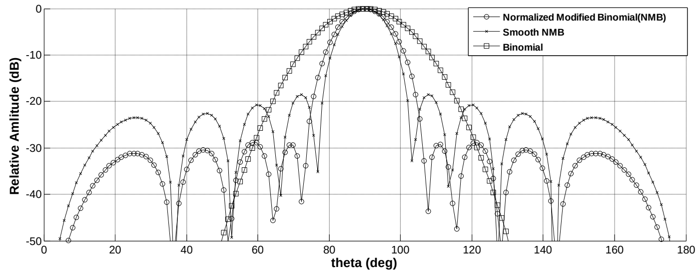

Fig. 3. Comparison between Normalized Modified Binomials, smooth NMB and Binomial

We call this polynomial Smooth Normalized Modified Binomials (SNMB). Now we compare this new polynomial with binomial and other special polynomials. At first in fig. 3 We observe the effect of changes to polynomials. By adding the difference of smallest and largest coefficient to all excitation coefficients, the amplitudes will be more smooth and it help us to have a directivity better than NMB.

The most important part of Fig. 3 is the tapering of smooth NMB polynomial pattern. As in this figure is observed, the NMB and Binomial (which has no side lobe) has no tapering.

In fig. 4 the pattern of Smooth NMB and Tschebycheff are compared. The directivity of smooth NMB is better than Tschebycheff, although the side lobe level is worse. Tschebycheff has no tapering in minor lobes but the tapering in Smooth NMB is evident. As mentioned above this property is suitable for some application like Radar and low-noise system.

There are other polynomials which have this property (tapering). Hermite polynomials and A continuous line-source distribution that yields decaying minor lobes and, in addition, controls the amplitude of the sidelobe is that introduced by Taylor [8] in an unpublished classic memorandum. It is referred to as the Taylor (one-parameter) design.

Both polynomials have some disadvantages, for example Hermite in spite of having tapered minor lobe is sacrifices directivity. Totally the disadvantage of designing an array with decaying minor lobes as compared to a design with equal minor lobe level (Dolph-Tschebyscheff), it yields about 12 to 15% greater half-power beamwidth. Also Taylor designing methods are more applicable for large arrays [3].

As it is observed in fig. 4 the Smooth NMB has a better directivity than Tschebycheff. It has tapered minor lobe without sacrificing the directivity.

The advantage of Tschebycheff and Taylor designing methods is their capability in controlling the side lobe level, in other words for a given side lobe level, the smallest possible first-null beamwidth (or the smallest possible sidelobe level for a given first-null beamwidth)is achieved by Dolph-Tschebyscheff array design. If this property of Dolph-Tschebyscheff design be combined by Smooth NMB tapering property the best answer will occur. Then a trade-off between side lobe level and tapering ratio could be made.

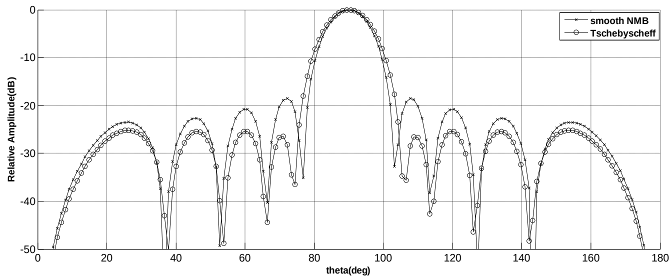

Fig. 4. Comparison between Tschebycheff and smooth NMB

IV. Conclusion:

In this paper radiation properties of array pattern synthesis using some special polynomials is discussed. In Radar and low noise systems it is desirable to sacrifice some beamwidth and low inner minor lobes to have all the minor lobes decay as the angle increases on either side of the main beam. In radar applications this is preferable because interfering or spurious signals would be reduced further when they try to enter through the decaying minor lobes. Thus any significant contributions from interfering signals would be through the pattern in the vicinity of the major lobe. The best polynomial that gives this property is Hermite polynomial. In spite of giving this tapered minor lobes Hermite polynomials reduces directivity. In this paper the end was to excite the amplitude by some new polynomials to give us a tapered lobe by keeping Directivity constant. Smooth NMB do this, by keeping directivity it has a good tapered minor lobes.

REFERENCES

[1] K. Yan and Y. Lu, "Side lobe reduction in array pattern synthesis using genetic algorithm," IEEE Trans. Antennas Propag., vol. 45, no. 7, July 1997, pp. 1117-1122.

[2] M. M. Khodier and C. G. Christodoulou, "Linear array geometry synthesis with minimum sidelobe level and null control using particle swarm optimization," IEEE Trans. Antennas Propag., vol, 53, no. 8, Aug. 2005, pp. 2674-2679.

[3] V. Murino, A. Trucco, and C. Regazzoni, "Synthesis of unequally spaced arrays by simulated annealing," IEEE Trans. Signal Process., vol. 44, no. 1, Jan 1996, pp. 119-123.

[4] De Castro L N.Von Zuben F J. "The Clonal Selection Algorithm with Engineering Applications". Workshop Proceedings of GECC'00, Workshop on Artificial Immune Systems and Their Applications. Orlando:IEEE Tran. 2000, pp. 36~37.

[5] Kelley, D.F.; Stutzman, W.L.; , "Array antenna pattern modeling methods that include mutual coupling effects," Antennas and Propagation, IEEE Transactions on , vol.41, no.12, pp.162531632, Dec 1993.

[6] Constantine A. Balanis, "ANTENNA THEORY" , Wiley,New Jersey,2005,p.283

[7] Constantine A. Balanis, "ANTENNA THEORY" , Wiley,New Jersey,2005,p.410

[8] T. T. Taylor, "One Parameter Family of Line-Sources Producing Modified Sin(πu)/πu Patterns," Hughes Aircraft Co. Tech. Mem. 324, Culver City, Calif., Contract AF 19(604)-262-F-14, 1953