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Fuzzy Hypothesis Testing Of Anova Model with Fuzzy Data

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Abstract: *A new statistical technique for testing the fuzzy hypotheses of one factor ANOVA model using their samples having fuzzy data is proposed without using h-level concept. In the proposed technique, the decision rules that are used to accept or reject the null and alternative hypotheses are provided in which the notions of pessimistic degree and optimistic degree are not used. The idea of the proposed procedure has been clarified by illustrative numerical examples.*
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I. Introduction

Analysis of variance (ANOVA) is a statistical technique that will enable us to test the null hypothesis that the three or more populations means are equal against the alternative hypothesis that they are not equal by using their samples information. The ANOVA technique was originally used in the analysis of agricultural research data. Due to the strength and versatility of the technique, the ANOVA technique is now used in all most all research areas especially in Social Science Research and Managerial Decision Making. In traditional statistical testing [7], the observations of sample are crisp and a statistical test leads to the binary decision. However, in the real life, the data sometimes cannot be recorded or collected precisely. The statistical hypotheses testing under fuzzy environments has been studied by many authors using the fuzzy set theory concepts introduced by Zadeh [18]. The application by using fuzzy sets theory to statistics has been widely studied in Manton et al. [8] and Buckley [3] and Viertl [13]. Arnold [2] proposed the fuzzification of usual statistical hypotheses and considered the testing hypotheses under fuzzy constraints on the type I and II errors. Saade [10], Saade and Schwarzlander [11] considered the binary hypotheses testing and discussed the fuzzy likelihood functions in the decision making process by applying a fuzzified version of the Bayes criterion. Grzegorzewski [6] and Watanabe and Imaizumi [14] proposed the fuzzy test for testing hypotheses with vague data, and the fuzzy test gave the acceptability of the null and alternative hypotheses. The statistical hypotheses testing for fuzzy data by proposing the notions of degrees of optimism and pessimism was proposed by Wu [15]. Viertl [12] investigated some methods to construct confidence intervals and statistical tests for fuzzy data. Wu [17] proposed some approaches to construct fuzzy confidence intervals for the unknown fuzzy parameter. Arefi and Taheri [1] developed an approach to test fuzzy hypotheses upon fuzzy test statistic for vague data. A new approach to the problem of testing statistical hypotheses for fuzzy data using the relationship between confidence intervals and testing hypotheses is introduced by Chachi et al.[4]. Mikihiko Konishi et al. [9] proposed the method of ANOVA for the fuzzy interval data by using the concept of fuzzy sets. Hypothesis testing of one factor ANOVA model for fuzzy data was proposed by Wu [16] using the h-level set and the notions of pessimistic degree and optimistic degree by solving optimization problems.

In this paper, we propose a new statistical fuzzy hypothesis testing of ANOVA model for finding the significance among more than two population means when the data of their samples are fuzzy data. We provide the decision rules which are used to accept or reject the fuzzy null and alternative hypotheses. In the proposed technique, we convert the given fuzzy hypothesis testing of one factor ANOVA model with fuzzy data into two hypothesis testing of one factor ANOVA models with crisp data namely, upper level model and lower level model; then, we test the hypothesis of each of the one factor ANOVA models with crisp data and obtain the results and then, we obtain a decision about the population means on the basis of the proposed decision rules using the results obtained. In the decision rules of the proposed testing technique, we are not using degrees of optimism and pessimism and h-level set which are used in Wu [16]. For better understanding, the proposed fuzzy hypothesis testing technique of ANOVA model for fuzzy data is illustrated with numerical examples.

II. Preliminaries

We need the following results which can be found in [5, 7].

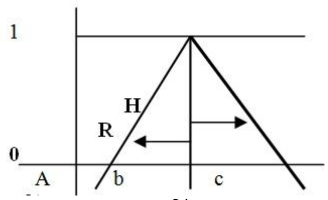
Let $D=\{[a,b],\ a\le b$ and a and b are in R$\}$ the set of all closed bounded intervals on the real line R.

Result 2.1: Let $A=[a,b]$ and $B=[c,d]$ be in D. Then, $A=B$ if $a=c$ and $b=d$.

Result 2,2 : If s^2 is the variance of a sample of size n drawn from the population with variance σ^2, then $E\left(\frac{ns^2}{n-1}\right)=\sigma^2$, that is, $\frac{ns^2}{n-1}$ is an unbiased estimator of σ^2.

A triangular fuzzy number $\tilde{a}=(a,b,c)$ can be represented as an interval number form as follows.

$$[\tilde{a}]=[b-(b-a)r, b+(c-b)h];\quad 0\le h,r\le 1. \qquad (1)$$



Fuzzy number $\tilde{a}=(a,b,c)$

Note that r is the level of pessimistic and h is the level of optimistic of the fuzzy number $\tilde{a} = (a,b,c)$.

III. One -Factor ANOVA Model

The basic principle of ANOVA technique [7] is to test the difference among the means of populations by studying the amount of variation within each of the samples relative to the amount of variation between the samples. Samples under consideration in ANOVA model are assumed to be drawn from normal populations of equal variances. A one-factor between-subjects ANOVA is used when the analysis involves only one factor with more than two levels and different subjects in each of the experimental conditions.

Consider a sample of size N of a given random variable X drawn from a normal population with variance σ^2 which is subdivided into s classes according to some factor of classification. The main objective is to test the null hypothesis that the factor of classification has no effect on the variables (or) there is no difference between various classes (or) the classes are homogeneous.

Now, let μ_i be the mean of ith population class. The testing hypotheses are given below:

Null hypothesis, $H_0 : \mu_1 = \mu_2 = ... = \mu_s$ against

Alternative hypothesis, $H_A : \text{not all } \mu_i\text{'s are equal}$

Let x_{ij} be the value of the jth member of the ith class which contains n_i values, $\bar{x}_i$ be the mean value of ith class and $\bar{x}$ be the general mean of all the N $(= \sum_i n_i)$ values .

Now, the total variation, $Q = \sum_i \sum_j (x_{ij} - \bar{x})^2 =$

$$\sum_i \sum_j (x_{ij} - \bar{x}_i)^2 + \sum_i n_i(\bar{x}_i - \bar{x})^2$$

$$= Q_2 + Q_1$$

where $Q_1 = \sum_i n_i(\bar{x}_i - \bar{x})^2$ is the variation between classes and $Q_2 = \sum_i \sum_j (x_{ij} - \bar{x}_i)^2$ is the variation within classes (or) the residual variation.

Now, since ith class is a sample of size n_i from the population with the variance σ^2, we can conclude that $M_2 = \frac{Q_2}{N-s}$ is an unbiased estimator of σ^2 with degrees of freedom $N-s$ and since the entire group is a sample of size N from the population with variance σ^2, we can conclude that $M_1 = \frac{Q_1}{s-1}$ is an unbiased estimator of σ^2 with degrees of freedom $s-1$.

Now, since the `population is normal and the estimates M_1 and M_2 are independent, the ratio $\frac{M_1}{M_2}$ or $\frac{M_2}{M_1}$, whichever is greater than or equal to one, follows a F-distribution with $(s-1, N-s)$ degrees of freedom in the first case or $(N-s, s-1)$ degrees of freedom in the later case. Then, we employ the $F-\text{test}$ for testing the hypotheses.

The above results are displayed in the form a table, known as the one-factor ANOVA table as given below:

Source of Variation	Sum of Squares	Degrees of freedom	Mean Square	F-value
Between Classes	Q_1	$s-1$	$M_1 = \frac{Q_1}{s-1}$	$F = \frac{M_1}{M_2}$ (or) $F = \frac{M_2}{M_1}$
Within Classes	Q_2	$N-s$	$M_2 = \frac{Q_2}{N-}$	

The decision rules in the F-test to accept or reject the null and alternative hypotheses are given below:

(i) If $M_1 < M_2$ and $F = \frac{M_1}{M_2} < F_0$ where F_0 is the table of F for $(s-1, N-s)$ degrees of freedom at α level, the null hypothesis is accepted, Otherwise, alternative hypothesis is accepted.

(ii) If $M_2 < M_1$ and $F = \frac{M_2}{M_1} < F_0$ where F_0 is the table of F for $(N-s, s-1)$ degrees of freedom at α level, the null hypothesis is accepted, Otherwise, alternative hypothesis is accepted.

Note : For easy computing the values Q, Q_1 and Q_2, we use the following formulae:

$$Q = \sum_i \sum_j x_{ij}^2 - \frac{T^2}{N} \quad \text{where} \quad T = \sum_i \sum_j x_{ij};$$

$$Q_1 = \sum_i \left(\frac{T_i^2}{n_i} \right) - \frac{T^2}{N} \quad \text{where } T_i = \sum_j x_{ij}$$

and $Q_2 = Q - Q_1$.

IV. Fuzzy One-Factor Anova Model

Suppose that we test the fuzzy hypotheses of one-factor ANOVA model with the data of the given samples are triangular fuzzy numbers. Using the relation (1), we convert the given fuzzy ANOVA model into interval ANOVA model. Construct two crisp ANOVA models from the interval ANOVA model namely, upper level model which is formed using the upper limits of each of the interval values and lower level model which is formed using the lower limit of each of the interval values. Then, we analyze lower level model and upper level model using the crisp one-factor ANOVA technique.

Let α be the level of significance.

Now, the null hypothesis, $\tilde{H}_0 : \tilde{\mu}_1 = \tilde{\mu}_2 = \ldots = \tilde{\mu}_s$ against

thealternative hypothesis, $\tilde{H}_A :$ not all $\tilde{\mu}_i$'s are equal.

Thisimpliesthatthenullhypothesis,

$[\tilde{H}_0]:[\tilde{\mu}_1]=[\tilde{\mu}_2]=\ldots=[\tilde{\mu}_s]$ against

thealternativehypothesis,

$[\tilde{H}_A]:$ not all $[\tilde{\mu}_i]$'s are equal

Thatis,thenullhypothesis,

$[H_0^L, H_0^U]:[\mu_1^L, \mu_1^U]=[\mu_2^L, \mu_2^U]=\ldots=[\mu_s^L, \mu_s^U]$

againstthealternativehypothesis,

$[H_A^L, H_A^U]:$ not all $[\mu_i^L, \mu_i^U]$'s are equal.

This implies the following two sets of hypotheses:

(1) the null hypothesis, $H_0^L : \mu_1^L = \mu_2^L = \ldots = \mu_s^L$

againstthealternativehypothesis,

$H_A^L :$ not all μ_i^L's are equal.

and

(2) thenull hypothesis, $H_0^U : \mu_1^U = \mu_2^U = \ldots = \mu_s^U$

againstthealternativehypothesis,

$H_A^U :$ not all μ_i^U's are equal.

Note that (1) is the hypotheses of the lower level model and (2) is the hypotheses of the upper level model.

Suppose that if at α level of significance, the null hypothesis of the lower level model is accepted for $0 \le h \le h_0$ where $0 \le h_0 \le 1$ and the null hypothesis of the upper level model is accepted for $0 \le r \le r_0$ where $0 \le r_0 \le 1$, then, the fuzzy null hypothesis of the fuzzy

Now, we consider the lower level model. The hypotheses are given below:

Package design (i)	Store j (Observation j)		
	1	2	3
1	11+2h	16+2h	
2	15+4h	15+5h	13+2h
3	18+3h	17+3h	20+3h
4	19+4h	24+3h	

ANOVA model is accepted for $0 \le h \le h_0$ and $0 \le r \le r_0$ at α level of significance. Otherwise, the fuzzy alternative hypothesis of the fuzzy ANOVA model is accepted at α level of significance.

Remark 4.1. If when $h=0$ or $r=0$, that is, center level, the fuzzy null hypothesis of the fuzzy ANOVA model is accepted at α level of significance. Then, only we study about the acceptance levels of null hypothesis in the directions of pessimistic and optimistic.

The proposed statistical technique to test the fuzzy hypothesis of ANOVA model with fuzzy data is illustrated with the following numerical examples.

Example 4.1. A food company wished to test four different package designs for a new product. Ten stores, with approximately equal sales volumes, are selected as the experimental units. Package designs 1 and 4 are assigned to three stores each and package designs 2 and 3 are assigned to two stores each. We cannot record the exact sales volume in a store due to some unexpected situations, but we have the fuzzy data for sales volumes. The fuzzy data are given below:

Package design (i)	Store j (Observation j)		
	1	2	3
1	(9,11,13)	(14,16,18)	
2	(11,15,19)	(10,15,20)	(11,13,15)
3	(15,18,21)	(14,17,20)	(17,20,23)
4	(15,19,23)	(21,24,27)	

We wish to test whether or not the (fuzzy) mean sales are the same for the four designs.

Let $\tilde{\mu}_i$ be the mean sales for the ith design.

Now, the null hypothesis, $\tilde{H}_0 : \tilde{\mu}_1 = \tilde{\mu}_2 = \tilde{\mu}_3 = \tilde{\mu}_4$

against the alternative hypothesis,

$\tilde{H}_A :$ not all $\tilde{\mu}_i$'s are equal.

Now, the interval ANOVA model for the given fuzzy ANOVA model is given below:

Package Design (i)	Store j (Observation j)		
	1	2	3
1	[11-2r,11+2h]	[16-2r,16+2h]	
2	[15-4r,15+4h]	[15-5r,15+5h]	[13-2r, 13+2h]
3	[18-3r,18+3h]	[17-3r,17+3h]	[20-3r, 20+3h]
4	[19-4r,19+4h]	[24-3r, 24+3h]	

Now, the upper level model and lower level model for the above interval ANOVA model are given below:

Upper level model: Lower level model:

Package design (i)	Store j (Observation j)		
	1	2	3
1	11-2r	16-2r	
2	15-4r	15-5r	13-2r
3	18-3r	17-3r	20-3r
4	19-4r	24-3r	

the null hypothesis, $H_0^L : \mu_1^L = \mu_2^L = \mu_3^L = \mu_4^L$ and the alternative hypothesis,

$H_A^L :$ not all μ_i^L's are equal.

Now, the ANOVA table is given below:

Source of Variation	Sum of Squares	Degree of freedom	Mean square	F-ratio
Between Samples	Q_1^L	3	$M_1^L = \frac{Q_1^L}{3}$	$F_r^L = \frac{M_1^L}{M_2^L}$
Within Samples	Q_2^L	6	$M_2^L = \frac{Q_2^L}{6}$	

where $Q_1^L = 3.73r^2 - 12.73r + 91.26$ and $Q_2^L = 5.17r^2 - 1.6r + 32.34$.

Now, $F_r^L > T$, for all r ; $0 \le r \le 0.87$ where T (= 4.76) is the table value of F at 5% level of significance with (3,6) degrees of freedom. Therefore, the null hypothesis H_0^L of the lower level model is rejected for all r ; $0 \le r \le 0.87$.

Now, we consider the upper level model. The hypotheses are given below:

the null hypothesis, $H_0^U : \mu_1^U = \mu_2^U = \mu_3^U = \mu_4^U$ and the alternative hypothesis, H_A^U : not all μ_i^U's are equal.

Now, the ANOVA table is given below:

Source of Variation	Sum of Squares	Degree of freedom	Mean square	F-ratio
Between Samples	Q_1^U	3	$M_1^U = \frac{Q_1^U}{3}$	$F_h^U = \frac{M_1^U}{M_2^U}$
Within Samples	Q_2^U	6	$M_2^U = \frac{Q_2^U}{6}$	

where $Q_1^U = 3.73h^2 + 12.73h + 91.26$ and $Q_2^U = 5.17h^2 + 1.6h + 32.34$.

Now, $F_h^U > T$, for all h ; $0 \le h \le 1$ where T (= 4.76) is the table value of F at 5% level of significance with (3,6) degrees of freedom. Therefore, the null hypothesis H_0^U of the upper level model is rejected for all h ; $0 \le h \le 1$.

Thus, since the null hypotheses H_0^L and H_0^U of the lower level data and upper level data are rejected for all r ; $0 \le r \le 0.87$ and h ; $0 \le h \le 1$ (note that null hypotheses are not rejected at $r = 0$ and $h = 0$, that is, the centre level), the fuzzy null hypothesis $\tilde{H}_0$ of the fuzzy ANOVA model is rejected. Therefore, the alternative hypothesis $\tilde{H}_A$ I of the fuzzy ANOVA model is accepted. It says that the factor level fuzzy means $\tilde{\mu}_i$ are not equal. Thus, we conclude that there is a relation between package designs and sales volumes.

Remark 4.2. The decision obtained by the proposed fuzzy hypothesis testing technique for the Example 4.1. is same as in Wu [16].

Example 4.2. Four different machines are used to produce milk pouches of 100 ml each by a city diary . Before these pouches are dispatched for local distribution, the quality assurance manager selects two samples of pouches from machine1 and machine 4 and three samples of pouches from the machine 2 and the machine3 and determines the number of pouches that does not meet the specifications under the Weights and Measures Act. We cannot record the exact number of pouches in a sample due to some unexpected situations, but we have the fuzzy data for number of pouches. The fuzzy data are given below:

Machine (i)	Sample j		
	1	2	3
1	(8,9,14)	(11,14,18)	
2	(7,11,15)	(8,10,14)	(8,11,14)
3	(12,15,18)	(12,14,19)	(14,17,23)
4	(12,15,19)	(19,21,24)	

Test that there is a significant difference in the performance of the machines.

Let $\tilde{\mu}_i$ be the mean number of non-specifications pouches for the ith machine.

Now, the null hypothesis, $\tilde{H}_0 : \tilde{\mu}_1 = \tilde{\mu}_2 = \tilde{\mu}_3 = \tilde{\mu}_4$ and

the alternative hypothesis, $\tilde{H}_A$: not all $\tilde{\mu}_i$'s are equal.

Now, the interval ANOVA model for the given fuzzy ANOVA model is given below:

Machine (i)	Sample j		
	1	2	3
1	[9-r,9+5h]	[14-3r,14+4h]	
2	[11-4r,11+4h]	[10-2r,10+4h]	[11-3r,11+3h]
3	[15-3r,15+3h]	[14-2r,14+5h]	[17-3r,17+6h]
4	[15-3r,15+4h]	[21-2r,21+3h]	

Now, the upper level model and lower level model for the above interval ANOVA model are given below:

Upper level model: Lower level model:

Machine (i)	Sample j		
	1	2	3
1	9-r	14-3r	
2	11-4r	10-2r	11-3r
3	15-3r	14-2r	17-3r
4	15-3r	21-2r	

Now, we consider the lower level model. The hypotheses are given below:

Machine (i)	Sample j		
	1	2	3
1	9+5h	14+4h	
2	11+4h	10+4h	11+3h
3	15+3h	14+5h	17+6h
4	15+4h	21+3h	

the null hypothesis, $H_0^L : \mu_1^L = \mu_2^L = \mu_3^L = \mu_4^L$ and the alternative hypothesis, H_A^L : not all μ_i^L's are equal.

Now, the ANOVA table is given below:

Source of Variation	Sum of Squares	Degree of freedom	Mean square	F-ratio
Between Samples	Q_1^L	3	$M_1^L = \frac{Q_1^L}{3}$	$F_r^L = \frac{M_1^L}{M_2^L}$
Within Samples	Q_2^L	6	$M_2^L = \frac{Q_2^L}{6}$	

Where $Q_1^L = 1.23r^2 + 3.07r + 82.26$ and $Q_2^L = 5.17r^2 - 8.6r + 35.84$.

Now, $F_r^L < T$, for all r ; $0 \le r \le 0.14$ where T (= 4.76) is the table value of F at 5% level of significance with (3,6) degrees of freedom. Therefore, the null hypothesis H_0^L of the lower level model is accepted for all r ; $0 \le r \le 0.14$.

Now, we consider the upper level model. The hypotheses are given below:

The null hypothesis, $H_0^U : \mu_1^U = \mu_2^U = \mu_3^U = \mu_4^U$ and

The alternative hypothesis, H_A^U : not all μ_i^U's are equal.

Now, the ANOVA table is given below:

Source of Variation	Sum of Squares	Degree of freedom	Mean square	F-ratio
Between Samples	Q_1^U	3	$M_1^U = \frac{Q_1^U}{3}$	$F_h^U = \frac{M_1^U}{M_2^U}$
Within Samples	Q_2^U	6	$M_2^U = \frac{Q_2^U}{6}$	

where $Q_1^U = 2.56h^2 - 0.4h + 82.26$ and $Q_2^U = 6.34h^2 - 7h + 35.84$.

Now, $F_h^U < T$, for all h ; $0 \le h \le 0.23$ where T (= 4.76) is the table value of F at 5% level of significance with (3,6) degrees of freedom. Therefore, the null hypothesis H_0^U of the upper level model is accepted for all h ; $0 \le h \le 0.23$.

Thus, since the null hypotheses H_0^L and H_0^U of the lower level model and upper level model are accepted for all r ; $0 \le r \le 0.14$ and h ; $0 \le h \le 0.23$ (note that null hypotheses are accepted at $r = 0$ and $h = 0$, that is, the centre level), the fuzzy null hypothesis $\widetilde{H}_0$ of the fuzzy ANOVA model is accepted for all r ; $0 \le r \le 0.14$ and h ; $0 \le h \le 0.23$. Thus, we conclude that four machines are equal only if r ; $0 \le r \le 0.14$ and h ; $0 \le h \le 0.23$, that is, the maximum level of pessimistic is 0.14 and the maximum level of optimistic is 0.23.

Remark 4.3. From the Example 4.2., we observe that the acceptance of the fuzzy null hypothesis for not all r and h always, but for some specific levels of r and h , that is, r ; $0 \le r \le 0.14$ and h ; $0 \le h \le 0.23$.

V. Conclusion

In this paper, we propose a new statistical fuzzy hypothesis testing of single-factor ANOVA model with the fuzzy data. In the proposed technique, we transfer the fuzzy ANOVA model into two crisp ANOVA models. Based on the decisions of hypotheses of two crisp ANOVA models, we take a decision on the fuzzy hypothesis of the fuzzy ANOVA model. Since the proposed technique in this paper is mainly based only on the crisp models, the proposed technique can be extended to multi-factor fuzzy ANOVA model and the experimental design analysis having fuzzy data.

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