

## A Rational Approach for Fundamental Period of Low and Medium Rise Steel Building Frames

Cinitha. A<sup>1</sup>, Umesha. P. K<sup>2</sup>, Nagesh. R. Iyer<sup>2</sup>

<sup>1</sup> Scientist, CSIR-Structural Engineering research centre, India,

<sup>2</sup> Chief Scientist, CSIR-Structural Engineering Research Centre, India

<sup>3</sup> Director, CSIR-Structural Engineering Research Centre, India

**ABSTRACT:** The fundamental periods of buildings are essential to calculate the design base shear and lateral forces. Most seismic codes specify empirical formulae to estimate the fundamental vibration period of buildings. These empirical formulae are used for both low- and medium- rise buildings. These formulae depend on the building materials (steel, reinforced concrete etc), building types (frame, shear wall etc) and overall dimensions of the buildings. In this paper, for finding the fundamental natural period of steel moment resisting frames, numerical studies are carried out, and by regression analysis, empirical formulae are derived for low- and medium- rise buildings. The numerical studies carried out includes, the influence of the plan and bay dimensions, normalized stiffness and height of storeys and various cross sections as columns and beams on fundamental period of the moment resisting frames.

**Keywords:** Dynamic Characteristics, Fundamental Period, Regression Analysis, Normalized stiffness

### I. INTRODUCTION

The design of structures to natural hazards such as earthquakes and cyclones demands safety of structures which is governed by the fundamental natural period and the amount of damping in each mode of vibration. Fundamental period of a building and its damping has a remarkable effect on the magnitude of its response. The ability to predict these characteristics, at the design stage, would enable to design safe structures. The fundamental periods of buildings are calculated using the equations specified in building codes for calculating the design base shear and lateral forces. Building codes provide empirical formula that depends on the building material (steel, reinforced concrete etc), building type (frame, shear wall etc) and overall dimensions. Lagomarsino (1993), Tamura (1996), Goel and Chopra (1997) and Tremblay and Rogers (2005) have conducted several field studies on the dynamic characteristics of moment resisting frames. The empirical formula suggested by Goel and Chopra (1997) is adopted in most of the seismic design codes not incorporates, the effect of parameters such as the plan area and symmetry of the building. In this paper, for finding the fundamental natural periods of steel moment resisting frames numerical studies are carried out, and by regression analysis, expressions are derived for low- and medium - rise buildings. The numerical studies carried out includes, the influence of the plan and bay dimensions, normalized stiffness and height of storeys and various cross sections as columns and beams on fundamental period of the moment resisting frames.

### II. NUMERICAL MODELING

The numerical studies are carried out with a developed finite element program. The elements used in the program are 3D beam elements with 12 degrees of freedom. The consistent mass matrix is used for analysis. The idealization employs the structure with only one element for each member, which reduces the number of degrees of freedom involved and the computational time. It is assumed that there is no damping in the structure and no time varying forces acts on it. Having defined generalized mass and stiffness matrices, the approximate mode shapes and frequencies of the structure are determined by solving the homogenous equations of the undamped system. The transformation of the mode shapes which result from the solution of the eigenvalue problem in the structural coordinate system to real coordinate system is also accounted. The Cholesky's factorization is used to decompose the stiffness matrix while conducting eigen analysis. Simultaneous iteration method is used to evaluate eigen values and eigen vectors from structural stiffness and mass matrices. The numerical model is validated (Cinitha et al 2008) with experiments conducted on small scale model, Krawinkler et al (1985).

### III ESTIMATION OF FUNDAMENTAL PERIODS

The fundamental periods of building frames are highly scattered. The Empirical formulae suggested in building codes are used to calculate the design base shear and lateral forces. Based on their studies Goel and Chopra (1997) suggested an expression as given in equ (1). The fundamental periods found from these expressions are highly conservative. Hence in this study an attempt is made to improve the estimation of periods of steel moment resisting frames without infill. The study is focused on buildings with height varying from 9 to 30m. Studies are carried out with different frame configurations as given in Table.1. For all cases the dimensions of the bay-width and storey-height are assumed to be same. The fundamental natural period is found as per the codal provisions of IS: 1893 (Part 1) 2002 by equ.(2)

$$T = 0.035H^{0.75} \quad (1)$$

$$T_a = 0.085 h^{0.75}, \text{for steel frame building} \quad (2)$$

Where H is the height of the building measured in feet (ft) and h is the height measured in meters.

### 3.1 NUMERICAL STUDIES AND RESULTS

Numerical studies are carried out to study the effect of the plan and bay dimensions, normalized stiffness and height of storeys of multi bay multi storey frames and various cross section as beams and columns on the fundamental frequency. The ratios of length to width of plan dimensions are taken as plan ratio in the present study.

Fig.1 shows the variation of fundamental frequency with height of the building for various plan dimensions of the building. As reported in empirical formula given in IS:1893-2002, it is found that irrespective of the plan dimensions of the building fundamental frequency decreases with increase in height of the building. The storey stiffness of the building frames are calculated based on the expressions developed by Schultz (1992) and they are normalized according to the maximum structural stiffness observed in each case. Fig.2 shows the fundamental frequency versus normalized stiffness behaviour for buildings with 3x3, 3x4, 3x5 and 3x6 bays. Frames with smaller bays have high fundamental frequency as compared to frames with larger bays. The variation shows a non-linear trend. Fig.3 shows the variation in fundamental frequency with increase in storey height and bay width. A decreasing trend in frequencies is observed with increase in storey height and bay dimensions. As a specific case, frames with different plan ratio are also studied. The fundamental frequency is found for these frames by varying the bay-width from 3m to 6m (3m, 4m, 5m and 6m) and the storey height has been kept constant as 3m. The height of the frame for all cases is assumed as 24m. It is observed that the variation in fundamental frequency is marginal with increase in plan ratio as shown in Fig.4. The buildings with larger bay-width show higher frequency.

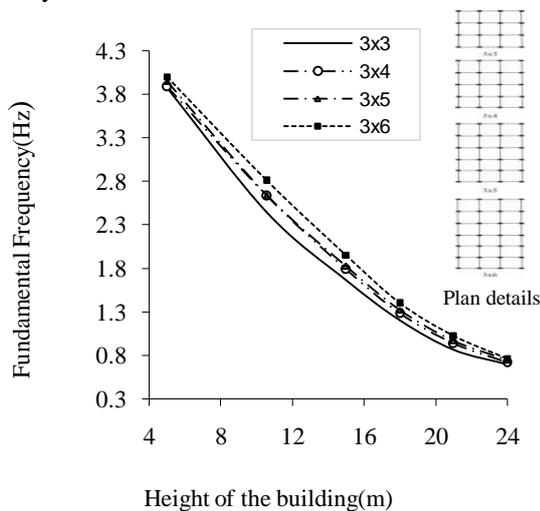


Fig. 1 Variation of fundamental frequency with height of the building

Cases	Parameters varied	
Number of storeys	3, 4, 5, 6 and 7	
Height of Storey (m)	3, 4 and 5	
Spacing of Columns (m)	3, 4, 5, 6 and 9	
Cross Section Details (open sections), (Wide flange sections as per AISC standards)	Beams	Columns
	W14 x 30 , W24 x 62	W16 x 26 , W 18 x 50
Cross Section Details (closed section)	Beams	columns
Case-1	200x200x12	200x200x12
Case-2	300x300x16	300x300x16
Case-3	400x400x20	400x400x20
Case-4	500x500x25	500x500x25

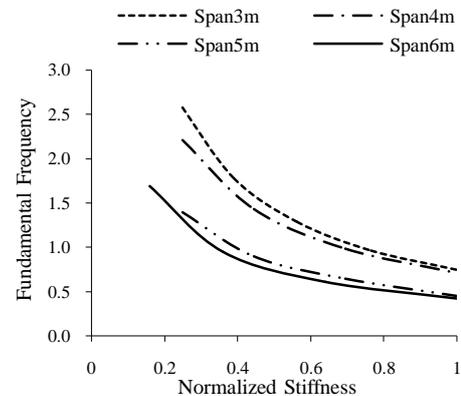


Fig. 2 Variation of fundamental frequency with normalized Stiffness

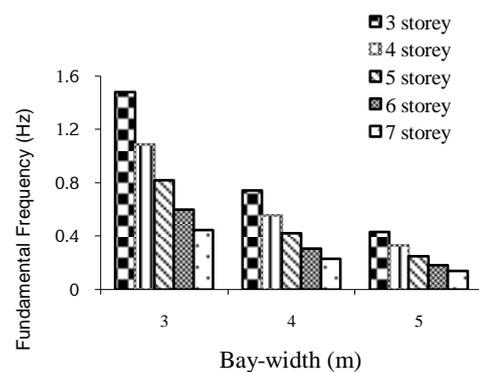


Fig. 3 Fundamental Frequency with Increase in storey height and bay-width

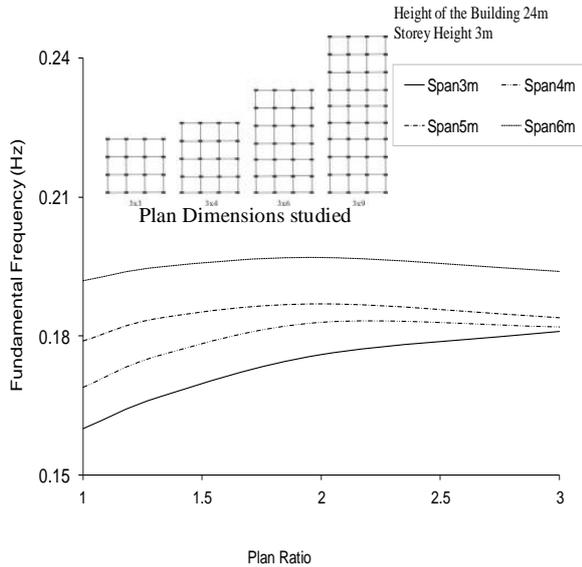


Fig. 4 Fundamental Frequency vs Plan Ratio

**III. REGRESSION ANALYSIS**

The regular steel moment resisting frames without infill are studied. Based on the height and plan area, parametric studies on 75 regular steel framed structures a regression analysis is carried out for finding their fundamental period. The fundamental periods of the buildings with plan dimensions of 3x3, 3x4, 3x5 and 3x6 (BxD) with heights varied as 3, 4, 5 and 6m, best fit curves are plotted and are shown in Fig.5. In the present study, buildings with a height less than 10m is considered as low-rise buildings and those with 10-30m is considered as medium rise buildings. Among the numerical data, 15 cases with a height of 9m are considered for the prediction of an expression for low rise buildings (0-10m height). The remaining 60 cases are with height of the building as 12, 15, 24 and 30m are considered for the prediction of an expression for medium rise buildings (10-30m, height).

**4.1 LOW RISE BUILDINGS**

For low rise buildings (i.e., buildings with height less than 10m), from the curve fit the fundamental natural period is derived as a power relation as given in equ.(3) and shown in Fig.6

$$T = 0.056 (BD)^{0.3289} \tag{3}$$

Where, B and D are the length and width of the building plan respectively in 'm'. The present estimation of periods for low-rise buildings is having about 0-13% error as shown in Table.2. Where as the period estimated with the equation suggested in IS: 1893-2002 and Goel and Chopra (1997), the error is found to be 0-77%.

**4.2 MEDIUM RISE BUILDINGS**

For medium-rise buildings (heights about 10-30m), natural period vs. plan area relationships are derived as power relationships as shown in Fig.7. The variation of the constants with height of the building in power relationships are again fitted to an exponential relationship as shown in the Figs. 8 and 9 respectively. The generalized power relationship in terms of constants  $C_0$  and  $\alpha$  is as given in equ.(4). The expressions for  $C_0$  and  $\alpha$  value are given

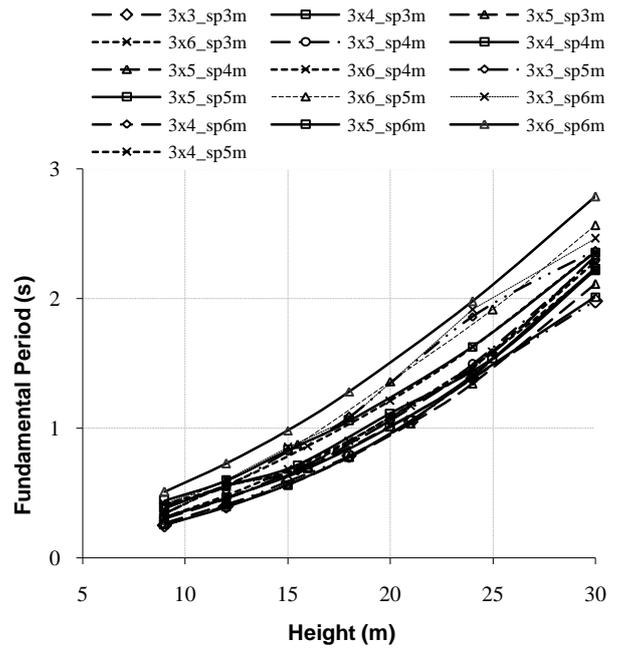


Fig.5 The fundamental Period vs. Height of regular frames studied

in equ. (5) and (6) respectively.

The expression is as follows

$$T = C_0 (BD)^{0.3289\alpha} \tag{4}$$

$$C_0 = 0.0247e^{0.1305 H} \tag{5}$$

$$\alpha = 0.4773e^{-0.0441 H} \tag{6}$$

Where, H is height, Length (B) and width (D) of the building respectively in 'm'. The present estimation of periods for medium – rise buildings is having 0-23% error as shown in Table 3. Where as the period estimated with the equation suggested in IS: 1893: 2002 and Goel and Chopra (1997), the error is found to be about 0-61% as shown in Table 4.

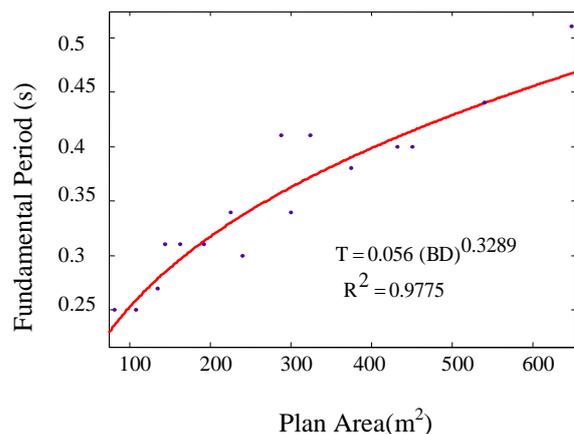


Fig.6 Fundamental Period vs. Plan Area for Low-Rise Buildings

**Table 2. Time period (s) for Low- Rise buildings < 10m height, Equ.(3)**

Numerical study	Present study (equ.3)	%error	IS:1893 (equ.2)	% error
0.25	0.24	-5.63	0.44	76.67
0.25	0.26	3.74	0.44	76.67
0.27	0.28	3.37	0.44	63.58
0.31	0.30	-4.41	0.44	42.48
0.31	0.29	-8.04	0.44	42.48
0.31	0.31	1.09	0.44	42.48
0.30	0.34	12.41	0.44	47.22
0.41	0.36	-12.67	0.44	7.73
0.34	0.33	-2.90	0.44	29.90
0.34	0.36	6.74	0.44	29.90
0.38	0.39	2.78	0.44	16.23
0.40	0.41	3.67	0.44	10.42
0.41	0.37	-9.22	0.44	7.73
0.40	0.41	2.29	0.44	10.42
0.44	0.44	0.07	0.44	0.38
0.51	0.47	-8.33	0.44	-13.40

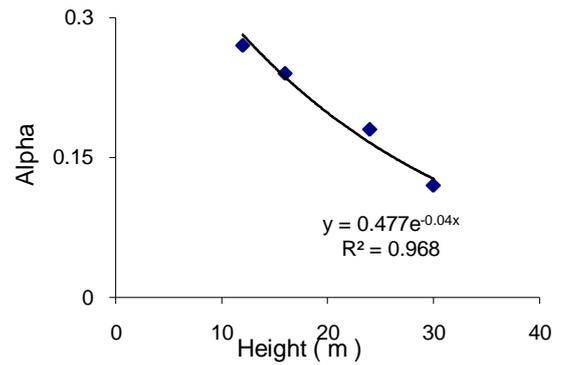


Fig. 8 Alpha vs Height

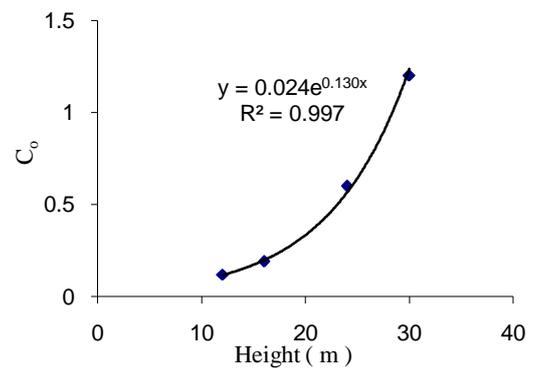


Fig.9 C<sub>o</sub> vs. Height

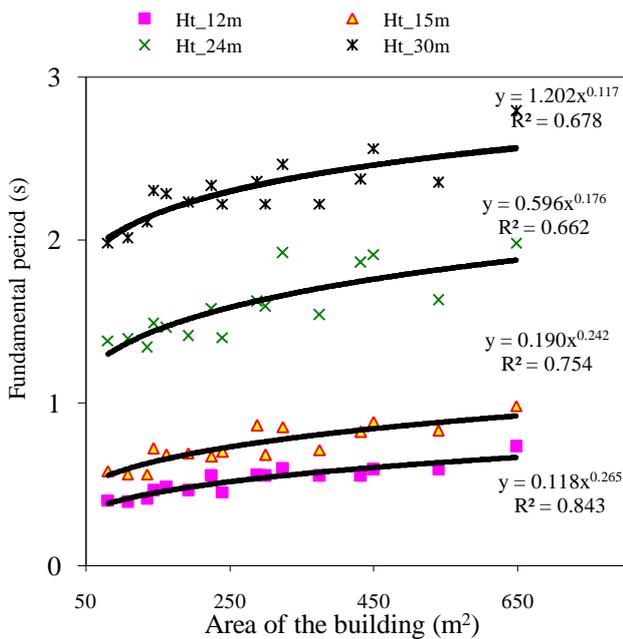


Fig.7 Fundamental Period vs. Plan Area for Medium-Rise Buildings

Area/Height (m)	Numerical Study				Present Study				% error			
	12	15	24	30	12	15	24	30	12	15	24	30
81	0.400	0.581	1.381	1.98	0.41	0.52	1.17	2.17	1.705	10.98	15.06	9.37
108	0.390	0.561	1.392	2.01	0.44	0.55	1.23	2.25	13.1	1.03	11.55	11.75
135	0.410	0.561	1.342	2.11	0.47	0.59	1.28	2.31	14.55	-4.57	4.80	9.52
162	0.480	0.681	1.462	2.28	0.49	0.61	1.31	2.37	2.991	9.93	9.94	3.73
144	0.460	0.721	1.492	2.30	0.48	0.59	1.29	2.33	3.969	17.37	13.46	1.30
192	0.460	0.691	1.412	2.23	0.52	0.64	1.35	2.42	12.73	7.44	4.09	8.37
240	0.450	0.701	1.402	2.22	0.55	0.67	1.40	2.49	22.69	3.61	-0.23	11.99
288	0.560	0.861	1.622	2.36	0.58	0.71	1.45	2.54	3.78	17.94	10.72	7.82
225	0.550	0.671	1.582	2.33	0.54	0.66	1.39	2.47	-1.42	0.88	12.13	5.83
300	0.550	0.681	1.592	2.22	0.59	0.71	1.46	2.56	6.886	-4.83	8.42	15.21
375	0.550	0.711	1.542	2.22	0.63	0.75	1.51	2.63	13.81	-6.08	1.89	18.53
450	0.590	0.881	1.912	2.56	0.66	0.79	1.56	2.69	11.67	10.49	18.47	5.20
324	0.600	0.851	1.922	2.46	0.6	0.73	1.47	2.58	0.122	14.53	23.19	5.00
432	0.550	0.821	1.862	2.37	0.65	0.78	1.55	2.68	18.43	4.90	16.84	13.04
540	0.590	0.831	1.632	2.35	0.69	0.82	1.60	2.76	17.55	0.73	1.54	17.28
648	0.730	0.981	1.982	2.79	0.73	0.86	1.65	2.82	-0	12.07	16.46	1.10

Table 4. Time period in 's' for Medium- Rise buildings with IS:1893 Equ.(2)

Area/ Height (m)	Numerical Study				IS:1893as given in equ2				% error			
	12	15	24	30	12	15	24	30	12	15	24	30
81	0.400	0.581	1.381	1.98	0.55	0.65	0.92	1.09	37.01	11.70	33.21	44.97
108	0.390	0.561	1.392	2.01	0.55	0.65	0.92	1.09	40.52	15.69	33.69	45.79
135	0.410	0.561	1.342	2.11	0.55	0.65	0.92	1.09	33.67	15.69	31.22	48.36
162	0.480	0.681	1.462	2.28	0.55	0.65	0.92	1.09	14.17	-4.73	36.87	52.21
144	0.460	0.721	1.492	2.30	0.55	0.65	0.92	1.09	19.14	-10.0	38.14	52.63
192	0.460	0.691	1.412	2.23	0.55	0.65	0.92	1.09	19.14	-6.11	34.63	51.14
240	0.450	0.701	1.402	2.22	0.55	0.65	0.92	1.09	21.78	-7.45	34.17	50.92
288	0.560	0.861	1.622	2.36	0.55	0.65	0.92	1.09	-2.14	-24.7	43.11	53.83
225	0.550	0.671	1.582	2.33	0.55	0.65	0.92	1.09	0.00	-3.30	41.67	53.24
300	0.550	0.681	1.592	2.22	0.55	0.65	0.92	1.09	0.00	-4.73	42.03	50.92
375	0.550	0.711	1.542	2.22	0.55	0.65	0.92	1.09	0.00	-8.75	40.15	50.92
450	0.590	0.881	1.912	2.56	0.55	0.65	0.92	1.09	-7.11	-26.4	51.74	57.44
324	0.600	0.851	1.922	2.46	0.55	0.65	0.92	1.09	-8.66	-23.8	52.00	55.71
432	0.550	0.821	1.862	2.37	0.55	0.65	0.92	1.09	0.00	-21.0	50.45	54.03
540	0.590	0.831	1.632	2.35	0.55	0.65	0.92	1.09	-7.11	-21.9	43.46	53.63
648	0.730	0.981	1.982	2.79	0.55	0.65	0.92	1.09	-24.9	-33.9	53.45	60.95

Fig 10 and Fig.11 show the fundamental period versus height of the building with closed and open sections respectively, for various plan dimensions of the building. Irrespective of the plan dimensions of the building and irrespective of cross sections fundamental period increases with increase in height of the building. This increase is steep for lighter sections and shallow for heavier sections. It was observed that buildings with heavy sections have

lower fundamental period as compared to buildings with lighter sections. A similar behaviour was observed with frames with open sections.

Figs 12 (a) to (d) show the comparison of fundamental period for buildings with different plan dimensions for various cross sections of beams and columns. For buildings with lower height there is no significant change in fundamental period irrespective of the cross section of beams and column, whereas for buildings with height more than 20m, fundamental period decreases for buildings rectangular in plan. This decrease in

fundamental period is not very significant for buildings with heavier cross section.

#### IV. RESULTS AND DISCUSSIONS

Based on the numerical studies an expression for finding the fundamental natural periods of low- and medium-rise building is derived. The expression suggested for low-rise buildings are found to be closer to numerical studies than the equation suggested in IS: 1893-2002. The fundamental period predicted with the suggested expression is found to be 0-13% error for low rise building frames compared to IS: 1893-2002 provisions, where the error is 0-77%. But for medium-rise buildings the error is 23% and this can be further improved with analysis of large sampling data.

In general considerable amount of uncertainties exist in the estimation of frequency of the structures by different methods. This depends on the two parameters mass and stiffness, whereas the mass can be predicted fairly accurately. The stiffness in real structure becomes unpredictable because of modeling accuracy of the boundary conditions, materials used in construction and its characterizations etc. Hence, detailed numerical studies were conducted to study the effect of height of the storeys, normalized stiffness of the building and plan and bay dimensions and various cross sections as columns and beams on the fundamental period of moment resisting steel space frames. It is found that the fundamental natural frequency decreases with increase in height of the building irrespective of plan dimensions of the building. The fundamental natural frequencies of frames are decreasing with increase in normalized stiffness and bay-width. It is also found that the variation in fundamental natural frequency is not much significant with increase in plan ratio, but the buildings with larger bay-width have higher frequency. Irrespective of the plan dimensions of the building and irrespective of cross sections fundamental period increases with increase in height of the building. This increase is steep for lighter sections and shallow for heavier sections. It was observed that buildings with heavy sections have lower fundamental period as

compared to buildings with lighter sections. A similar behaviour was observed with frames with open sections.

#### V. CONCLUSIONS

In this paper based on numerical studies separate expressions are suggested for fundamental natural period of low- and medium- rise buildings. It is found that the fundamental natural frequency decreases with increase in height and normalized stiffness of the building irrespective of plan dimensions of the building. But with increase in plan area the fundamental frequency of the buildings is found to be increasing. It was also observed that buildings with heavy sections as beams and columns have lower fundamental period as compared to buildings with lighter sections.

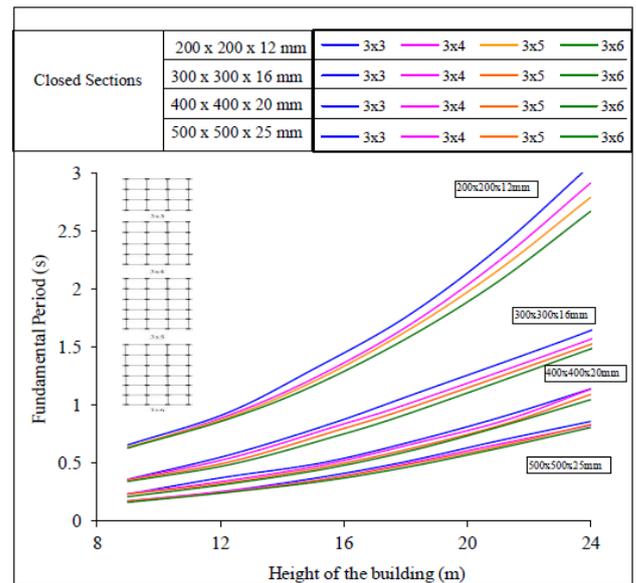


Fig.10 Fundamental period versus Height of the building with closed sections

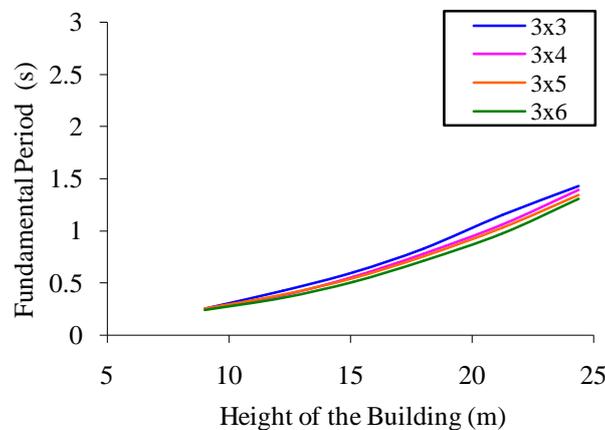


Fig.11 Fundamental period versus Height of the building with Open sections

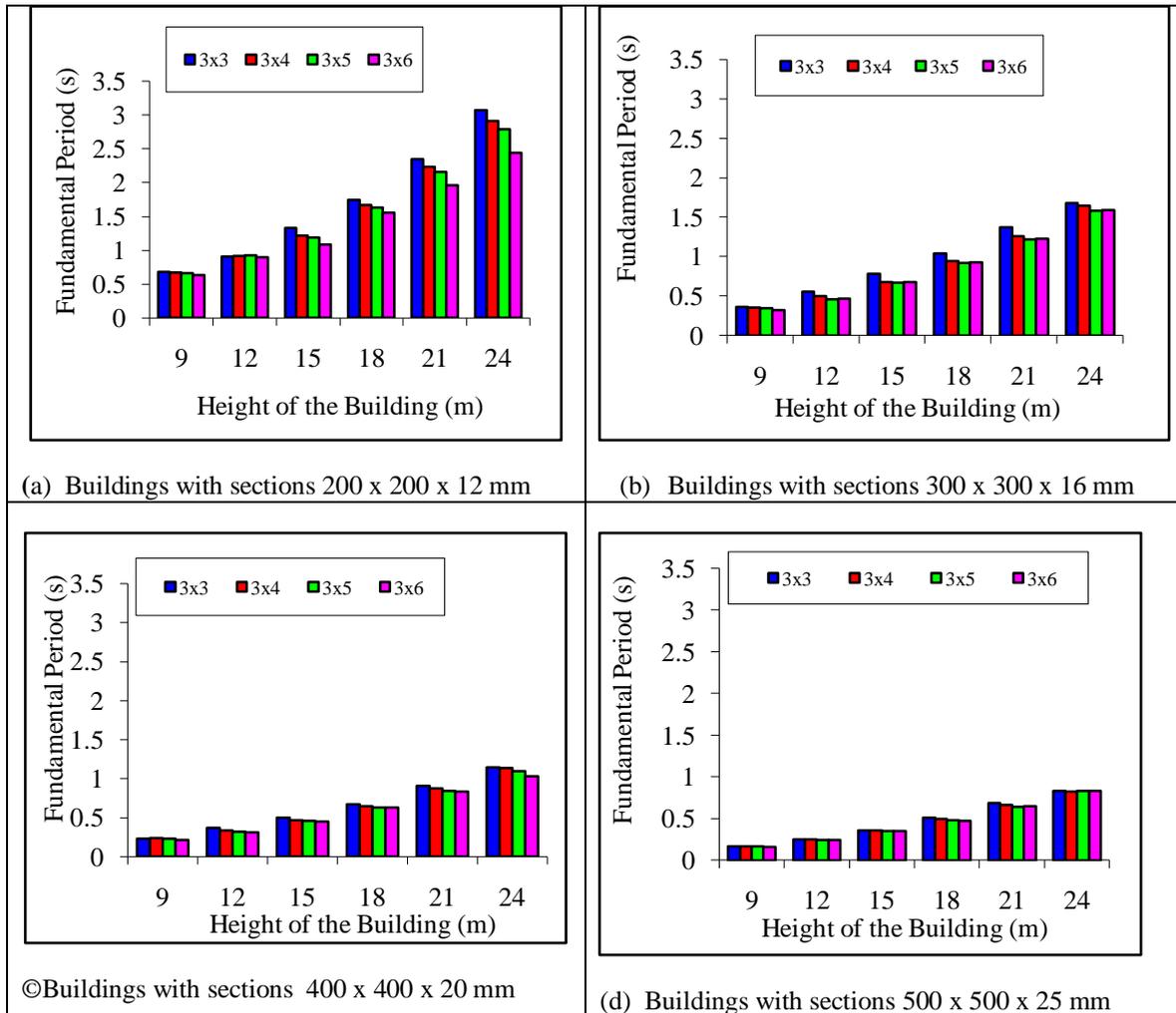


Fig. 12 Comparison of fundamental period for different plan dimensions of the buildings

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