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Inventory Model: Deteriorating Items with Price and Time Dependent Demand Rate

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ABSTRACT: *This study presents a deterministic inventory model for deteriorating products under the condition of instantaneous replenishment. The rate of deterioration is assumed to be a constant fraction of on hand inventory and demand is a function of selling price and decreases exponentially with time. It is shown that the developed model can be related to Cohen Model and Standard model without deterioration. A numerical example demonstrates the effectiveness of the developed model.*

Keywords: *Demand; deterioration; inventory; optimal; shortage.*

I. INTRODUCTION

It is of considerable interest and important to analysis the inventory models for deteriorating items. In many inventory systems, the effect of deterioration is an important factor and cannot be ignored. Deterioration may be defined as decay or damage or spoilage, so that the item can not be used as in its original state. Thus for example, blood, certain food items, photographic films, fruits, chemicals, radio active substances are some examples of items in which deterioration plays a major role.

Ghare and Schrader (Ghare and Schrader, 1963) developed a model for exponentially decaying inventory considering constant demand. Emmons (1968) also developed a model for exponential decaying products, where the product decayed at one rate into a new product, which decayed at a second rate.

Cohen (1977) developed a model for joint pricing and ordering policy for exponentially decaying inventory with known demand and constant decay rate. Mukherjee (1987) extended it by considering time varying decay rate. Kumar and Sharma (2009) has extended Mukherjee's (1999) model by considering shortages.

In this study, we consider demand as a function of time and price both and decreases exponentially. Deterioration is assumed to be constant fraction of on hand inventory. More ever the developed model reduces Cohen's model (1977) and standard model without deterioration.

II. PROPOSED ASSUMTIONS & NOTATIONS

The model is developed under following assumptions and notations.

1 Deterioration rate λ is a constant fraction of on hand inventory.

2 Demand rate $D(t,p)$ is known and decreases exponentially, i.e. at time $t,\ t \geq 0$

$$D(t,p) = \frac{e^{-\theta\, t}}{d(p)}$$

θ is constant governing the decreasing rate of demand. p is the selling price per unit and $d(p)$ is the function of p.

3 Lead time is zero, shortages are not allowed.

4 The replenishment rate is infinite and T is the cycle time.

5 There is no replacement or repair of the decayed units during the period under consideration.

6 The unit purchase cost is C, h is the holding cost per unit per unit time, K is the ordering cost per order.

7 $I(t)$ is the inventory at any time t.

III. MATHEMATICAL MODELLING AND ANALYSIS

The differential equation governing the system is given by

$$\frac{dI(t)}{dt} = -\lambda\, I(t) - D(p,t) \qquad (1)$$

Solution of (1) is given by after adjusting constant of integration.

$$I(t) = I(0)e^{-\lambda t} + \frac{e^{-\theta t} - e^{-\lambda t}}{d(p)(\theta - \lambda)} \qquad (2)$$

Inventory without decay at time t is given by the differential equation.

$$\frac{d}{dt} I_W(t) = -\frac{e^{-\theta t}}{d(p)} \qquad (3)$$

The solution of which gives

$$I_W(t) = I(0) + \frac{(e^{-\theta t} - 1)}{\theta . d(p)} \qquad (4)$$

The stock loss $Z(t)$ due to decay in $[0,T]$ is given by

$$z(t) = I_W(t) - I(t)$$

$$= I(o) + \frac{e^{-\theta t} - 1}{\theta . d(p)} - I(t) \qquad (5)$$

Using (2), equation (5) reduced to

$$Z(t) = I(t)(e^{\lambda t} - 1) - \frac{e^{-(\theta-\lambda)t} - 1}{d(p)(\theta - \lambda)} + \frac{e^{-\theta t} - 1}{\theta . d(p)} \qquad (6)$$

Total demand D during $(0,T)$ is given by

$$D = \int_0^T \frac{e^{-\theta t}}{d(p)} dt \qquad = \frac{-(e^{-\theta T} - 1)}{\theta . d(p)} \qquad (7)$$

Also $$Z(T) = \frac{e^{-\theta T} - 1}{\theta . d(p)} - \frac{e^{-(\theta-\lambda)T} - 1}{d(p)(\theta - \lambda)} \qquad (8)$$

Lot Size $$Q_T = \frac{1 - e^{-(\theta-\lambda)T}}{d(p)(\theta-\lambda)} \qquad (9)$$

Also $I(0) = Q_T$, then Using (9), (2) reduces to

$$I(t) = \frac{e^{-\theta t} - e^{-\theta T} . e^{-\lambda(t-T)}}{d(p)(\theta-\lambda)} \qquad (10)$$

Cost per cycle becomes

$$C^*(T,p) = K + C.Q_T + h\int_0^T I(t)dt$$

For a fixed price level p cost per unit time $C(T,p)$ is

$C(T, p) = C^*(T, p)/T$

$$= \frac{K}{T} + \frac{C[1-e^{-(\theta-\lambda)T}]}{Td(p)(\theta-\lambda)} + \frac{h}{Td(p)(\theta-\lambda)}\left[\frac{1-e^{-\theta T}}{\theta} - \frac{e^{-(\theta-\lambda)T}(1-e^{-\lambda T})}{\lambda}\right] \qquad (11)$$

By holding p fixed, the necessary conditions for minimizing $C(T,p)$ with respect to T is

$$\frac{\partial\, C(T,p)}{\partial\, T} = 0$$

implies

$$\frac{1}{T^2}\left[\begin{array}{l} -K - \frac{C}{d(p)(\theta-\lambda)} + \frac{(C\lambda+h)e^{-(\theta-\lambda)T}\{T(\theta-\lambda)+1\}}{\lambda d(p)(\theta-\lambda)} - \frac{h\theta T^2}{2d(p)(\theta-\lambda)} - \frac{h}{\lambda d(p)(\theta-\lambda)} \\ + \frac{h\theta^2 T^2}{2\lambda d(p)(\theta-\lambda)} \end{array}\right] = 0$$

$$\Rightarrow \quad \frac{(C\lambda+h)e^{-(\theta-\lambda)T}[T(\theta-\lambda)+1]}{\lambda d(p)(\theta-\lambda)} + \frac{h\theta T^2}{2\lambda d(p)} = K + \frac{C\lambda + h}{\lambda d(p)(\theta-\lambda)} \qquad (12)$$

From this equation by substituting known values $C, \lambda, h, \theta, d(p)$ and K we can find the optimum value of T.

An approximate solution to (12) can be obtained by using a turcated Taylor series expansion for exponential function as θ and λ are very small. Using Taylor series expansion equation (12) reduces to

$$T_p = \left[\frac{2K\lambda\, d(p)}{h\theta - (\theta-\lambda)(C\lambda+h)}\right]^{1/2} \qquad (13)$$

The effect of variation in perishability and price changes on the optimal order decision can be obtained from equation (9) and (13) we can get

$$\frac{Q_{T_p}}{T_p} \simeq \frac{1}{d(p)}\left[1 - (\theta-\lambda)\frac{T_p}{2}\right]$$

The sensitivity of the order rate to change in the perishability is determined by

$$\frac{\partial}{\partial\lambda}\left[Q_{T_p}/T_p\right] = \frac{\lambda}{2d(p)} + \frac{\lambda}{2d(p)} . \frac{\partial\, T_p}{\partial\lambda} > 0$$

For optimal price decision, consider the profit rate function for a fixed period length.

$$f(T,p) = p.\frac{e^{-\theta T}}{d(p)} - C(T,p)$$

Differentiating with respect to p, we get

$$\frac{\partial f}{\partial p} = \frac{e^{-\theta T}}{\{d(p)\}^2}[d(p) - pd'(p)] + \frac{d'p}{[d(p)]^2}\left[\frac{C\{1-e^{-(\theta-\lambda)T}\}}{T(\theta-\lambda)} + \frac{h}{T(\theta-\lambda)}\{(T - \frac{\theta T^2}{2}) + \frac{e^{-(\theta-\lambda)T}(e^{-\lambda T}-1)}{\lambda}\right]$$

Also $\frac{\partial f}{\partial p} = 0$ implies

$$\frac{d(p)}{d'(p)} + e^{\theta T}\left[\frac{C\left\{1-e^{-(\theta-\lambda)T}\right\}}{T(\theta-\lambda)} + \frac{h}{T(\theta-\lambda)}\left\{(T - \frac{\theta\, T^2}{2}) + \frac{e^{-(\theta-\lambda)T}(e^{-\lambda T}-1)}{\lambda}\right\}\right] = p_T$$

IV. EXAMPLE AND TABLES

SPECIAL CASE

Case 1 If θ = 0 and Demand $= d^*(p)(say)$

Then this model reduces to Cohen model (2).

Case 2 If θ = 0, λ = 0 and demand is constant. Then this model reduces to the standard formula for non decaying inventory.

NUMERICAL EXAMPLE

Consider an inventory system such that

K = Rs. 50 per order, h = Rs. 0.50 per unit per week, $d(p)$ = 25-0.5 p.

The optimum or time period.

Table 1 Tabulation of T_p for different values of the parameter.

λ	θ	P*	C	T_p
0.02	0.0	40	30	21.32
0.06	0.02	40	30	17.14
0.06	0.02	30	30	24.25
0.10	0.02	30	40	16.43
0.10	0.06	30	40	21.82

We have found the values of T_p for a fixed set of values K and h varying values of C, p, θ and λ. Table 1 indicates that with the increasing λ, T_p decreases and with the increasing value of θ, T_p increases and the same will increase with the decrease in p.

V. CONCLUSION

In this paper, we developed a deterministic inventory model for deteriorating items when demand is a function of time and price both.

The result of the model is important for formulating the decisions when the inventory decay with constant rate and demand is a function of time and price both.

Two special cases illustrates the effectiveness of the developed model. The future study will incorporate any factual relation that may exists between time and price both in the demand rate function and variable rate of deterioration.

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