

## Stress Intensification & Flexibility in Pipe Stress Analysis

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**Abstract:** A typical piping system consists of combination of pipes and various fitting components like bends, Tees, O'lets etc. The plant piping systems are subjected to various types of loading due to weight, pressure, temperature, wind, water hammer etc. causing possible failure modes, based on type of loading, as plastic, rupture, fatigue, creep etc. In addition, pipe exhibits different geometric characteristics at fittings which have notable effect on the flexibility of the piping system. This in turn has influence on stress concentration at fittings and the loads produced due to it. This paper attempts to explain basic concepts of flexibility, stress intensification factors and their equations provided in ASME B31 codes. Authors have few observations on B31 SIF equations and hence attempted to compare the results of B31 SIF results against Finite Element Analysis providing the results at the end.

**Keywords:** Flexibility characteristics, Flexibility factor, Stress intensification factor.

### I. INTRODUCTION

In a typical piping system two pipes can be connected to each other directly as pipe to pipe joint or by means of various fittings viz. bends, Tee's, O'lets etc. Simple beam theories which can be applied to straight pipe may not be able to reflect true behavior of the piping fittings due to varying cross sections, thickness, curvatures etc. Hence it is essential to consider additional stresses at the fittings by introducing Stress Intensification Factor (SIF). This paper mainly discusses about the stress intensity calculations followed in Process Piping Plants referring to code ASME B31.3<sup>[1]</sup> based on Markl's<sup>[2]</sup> great work in this domain.

### II. FLEXIBILITY CHARACTERISTICS, FLEXIBILITY FACTOR & STRESS INTENSIFICATION FACTOR

To elaborate the concept of Stress Intensification factor (SIF), an example of bend has been considered.

#### Abbreviations:

$h$  =Flexibility characteristics  
 $\bar{T}$  =Nominal wall thickness of header pipe or bend, in  
 $R_1$  =Bend radius, in  
 $r_2$  =Mean radius of matching pipe, in  
 $S_b$  =Bending stress, PSI  
 $M$  =Bending moment, lb-in  
 $Z$  =Section modulus of pipe, in<sup>3</sup>  
 $i$  =Stress intensification factor  
 $N$  =Number of load cycles

#### 2.1 Flexibility Characteristics, $h$

It is a geometric characteristics based on the nominal wall thickness and mean radius of the fitting. ASME B31.3 defines it as a unit less number calculated based on type of fitting.

Example: for a bend

$$h = \bar{T} R_1 / r_2^2 \quad (1)$$

Flexibility characteristics is used to calculate Flexibility factor and SIF. It is in inverse proportion to Flexibility factor and SIFs.

#### 2.2 Flexibility factor, $k$ <sup>[3]</sup>

The most common fitting used in Piping system is 'Bend' due to its inherent flexibility characteristic which results due to its ability to ovalize under the action of bending moment. Consider a straight pipe (refer Figure 1) with length 'l' which will produce rotation ' $\theta$ ' under the action of bending moment 'M'. A bend having same diameter and thickness with same arc length 'l' under the action of same bending moment 'M' will exhibit ' $k \theta$ ' rotation. In nutshell, bend shows k times flexibility than the straight pipe, called as Flexibility factor.

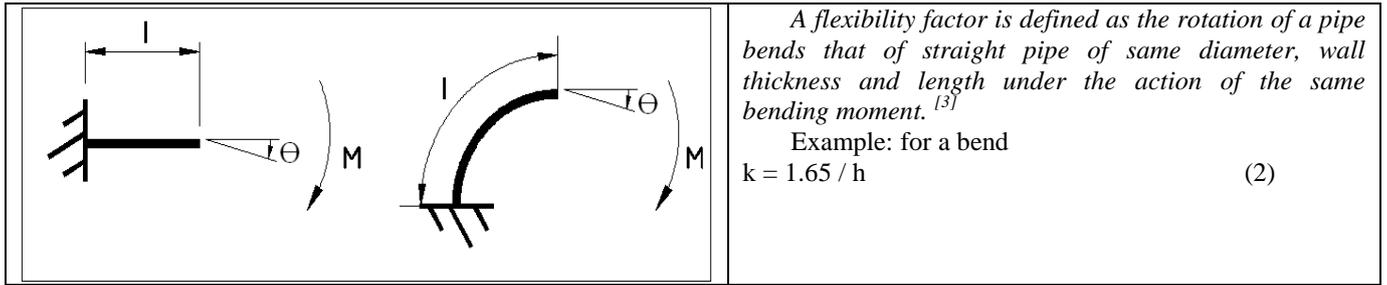


Figure: 1

**2.3 Stress Intensification Factor (SIF)**

The behavior of a straight pipe and a bend under the externally applied bending moment is different. Straight pipe acts like a beam retaining the cross section as circular whereas, the bend takes oval shape (Figure 2). Due to ovalization of the bend the outer fiber comes closer to the neutral axis reducing moment of inertial and subsequently the section modulus of the bend which in turn enhances bending stress.

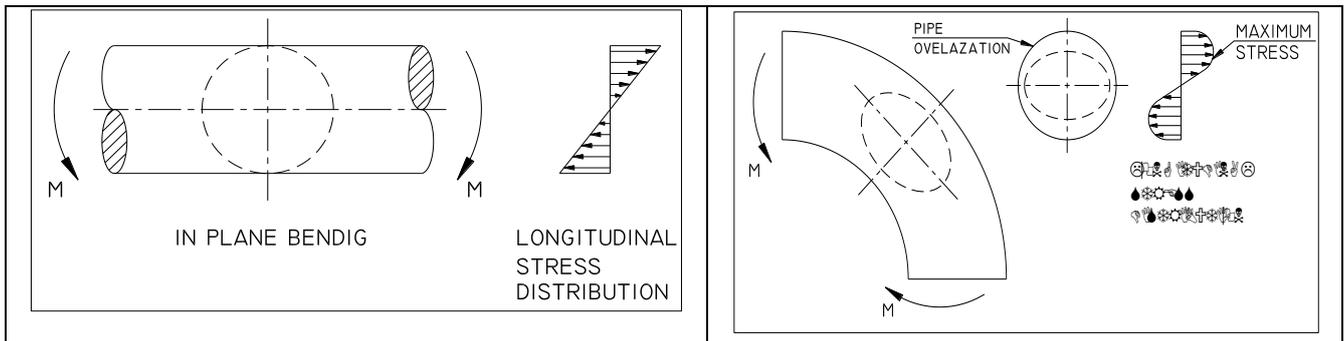


Figure 2

The bending stress in a straight pipe is calculated as

$$S_b = M / Z$$

The bending stress in a bend is calculated as

$$S_b' = M / Z'$$

where Z' is reduced section modulus.

Thus the stresses in the bend are higher compared to straight pipe of same size due to the reduced cross section.

$$\text{The SIF of Bend} = S_b' / S_b.$$

To quote Markl, SIF can be defined as 'the relation of rotation per unit length of the part in question produced by a moment, to the rotation per unit length of a straight pipe of the same nominal size and schedule or weight produced by the same moment' [3] or 'simply Actual Bending Stress to the Calculated Bending Stress'

Example: for a bend

$$i_{\text{in-plane}} = 0.9 / h^{2/3} \tag{3}$$

$$i_{\text{out-plane}} = 0.75 / h^{2/3} \tag{4}$$

**2.4 In-plane & Out-plane Bending moments**

Simply, 'the bending moment which causes elbow to open or close in the plane formed by two limbs of elbow is called in-plane bending moment.' and 'the bending moment which causes one limb of elbow to displace out of the plane retaining other limb steady is called out-plane bending moment.'

Figure 3 elaborates the said concepts of In-plane and Out-plane bending moments.

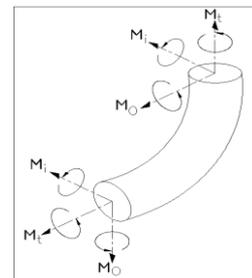


Figure 3

**III. EXPERIMENTAL EVALUATION & THEOROTICAL CALCULATIONS OF SIF [4]**

**3.1 Markl's Experiment**

Markl performed fatigue testing of various piping components in late 1940's. Most of the experiments were performed on 4 inch, SCH40, A106 Grade B pipes with Class 600 flanges. In one of the experiments two size on size un-

reinforced Tee's were tested and it was observed that they failed below ten thousand cycles of reversal displacement. Welds were tested in the as-welded condition. The experimental set up was as shown in the figure 4.

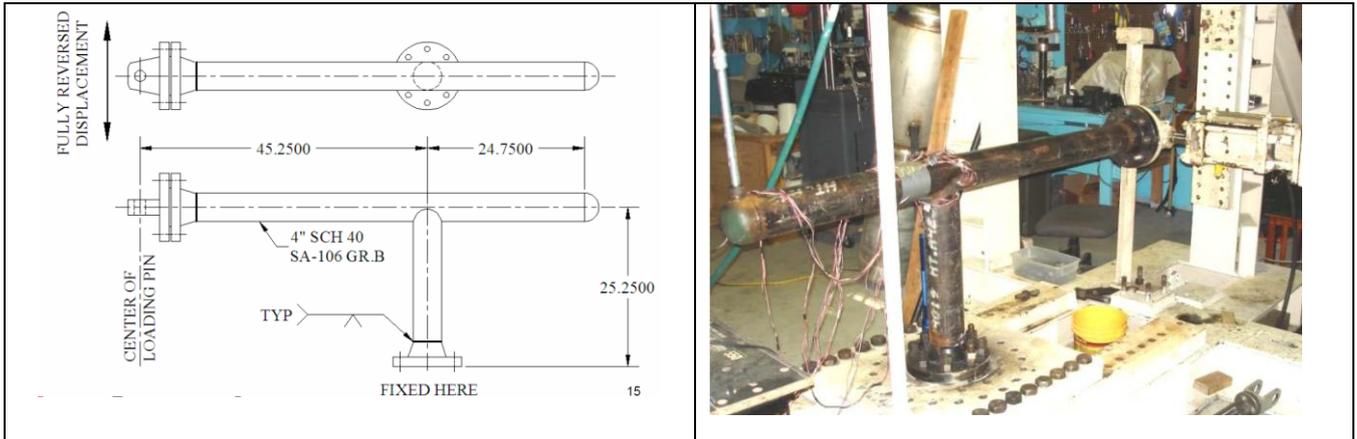


Figure 4

Markl's original work was based on the following equation (in PSI)

$$i. S_f = 490000N^{-0.2} \tag{5}$$

Where  $i$  = SIF,  $S_f$  = Stress range to failure,  $N$  = no. of cycles to failure

### 3.2 ASME B31 code equations for SIF <sup>[5][9]</sup>

Considering the example of bend under moment, the ovalization of pipe generates bending on the pipe wall which creates a high circumferential bending stress on the pipe wall. Since the pipe is oval at the bend and not circular, there cannot be direct comparison with non-ovalized bend. Hence the bending stress at bend is compared with the circular cross section of pipe.

The theoretical SIF's for circumferential stresses are <sup>[6]</sup>

$$i_{ci} = 1.8 / h^{2/3} \text{ for in-plane bending} \tag{6}$$

$$i_{co} = 1.5 / h^{2/3} \text{ for out-plane bending} \tag{7}$$

Markl and others observed that the theoretical SIF's are consistent with the test data. But the test performed on commercial pipe implied theoretical SIF of 2.0 against polished pipe which is mainly due to three factors – girth welds (welded or grinded), clamping - supporting effects and defects, surface roughness. Hence, in attempt of simplifying the analysis the SIF of commercial girth weld had been considered as unity modifying equations of SIFs in B31 codes as-

$$i_i = 0.9 / h^{2/3} \text{ for in-plane bending} \tag{8}$$

$$i_o = 0.75 / h^{2/3} \text{ for out-plane bending} \tag{9}$$

Table-1 below provides a part of Appendix D of ASME B31.3 indicating  $h$ ,  $k$  and SIF values for different fittings.

Table-1

Description	Flexibility Factor, $k$	Stress Intensification Factor [Notes (2), (3)]		Flexibility Characteristic, $h$	Sketch
		Out-of-Plane, $i_o$	In-Plane, $i_i$		
Welding elbow or pipe bend [Notes (2), (4)-(7)]	$\frac{1.65}{h}$	$\frac{0.75}{h^{2/3}}$	$\frac{0.9}{h^{2/3}}$	$\frac{TR_1}{r_2^2}$	
Unreinforced fabricated tee [Notes (2), (4), (9), (11)]	1	$\frac{0.9}{h^{2/3}}$	$\frac{3}{4}i_o + \frac{1}{4}i_i$	$\frac{T}{r}$	

**3.3 SIF calculation example**

Based on different types of fittings ASME B31 code provides empirical formulae to calculate h, k and SIFs. Consider an Unreinforced fabricated Tee junction having following data-

Header outside diameter:	Dh = 12.75"
Header nominal thickness:	$\bar{T} = 0.375$ "
Branch outside diameter:	Db = 10.625"
Branch nominal thickness:	t = 0.365"
Mean radius of header:	r <sub>2</sub> = 6.1875"

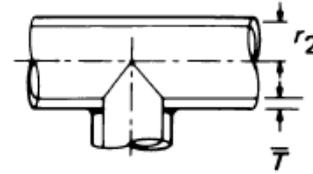


Figure 5

Flexibility characteristics:  $h = \bar{T} / r_2 = 0.375 / 6.1875 = 0.06$

Flexibility factor: k = 1

(as per ASME B31, Appendix D)

Out-plane SIF:  $i_o = 0.9 / h^{2/3} = 0.9 / 0.06^{2/3} = 0.9 / 0.154 = 5.83$

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(10)

In-plane SIF:  $i_i = 0.75 * i_o + 1/4 = 0.75 * 5.83 + 0.25 = 4.62$

(11)

The above results have compared with CAESAR II<sup>®</sup> [7] output and found consistent.

NODE	Axial Stress lb./sq.in.	Bending Stress lb./sq.in.	Torsion Stress lb./sq.in.	Hoop Stress lb./sq.in.	Max Stress Intensity lb./sq.in.	SIF In Plane	SIF Out Plane	Code Stress lb./sq.in.	Allowable Stress lb./sq.in.	Ratio %	Piping Code
20	-119.7	1005.1	0.0	0.0	1124.7	1.000	1.000	1005.1	49125.6	2.0	B31.3
30	-119.7	10716.9	0.0	0.0	10836.5	4.625	5.833	10716.9	45002.4	23.8	B31.3
30	0.0	0.0	0.0	0.0	0.0	4.625	5.833	0.0	42169.6	0.0	B31.3
40	0.0	0.0	0.0	0.0	0.0	1.000	1.000	0.0	49054.8	0.0	B31.3

**IV. VALIDATING B31 SIF EQUATIONS AGAINST FEA ANALYSIS**

Besides other observations on B31 SIF equation, authors have concentrated on one important observation that the SIF value of any fitting or elbow is irrespective of the branch properties i.e. branch diameter and branch thickness. Authors selected the unreinforced fabricated tee junction, similar to one of the Markl's experiment as shown in Figure 4 to find out effect of various header and branch properties on SIF. Number of FEA models were analyzed changing one variable at a time out of four variables affecting SIF viz. Header outside diameter, header thickness, branch outside diameter and branch thickness

Table-2: Experiment summary

Factors	EXP-1	EXP-2	EXP-3	EXP-4
Header diameter	C	C	C	V
Header thickness	C	C	V	C
Branch diameter	C	V	C	C
Branch thickness	V	C	C	C

V = varying parameter, C = parameter kept constant

**4.1 FEA Model**

Ansys<sup>[8]</sup> software used to simulate experiment set up shown in Fig. 6. To achieve the surface geometry at the junction, a 2D element 'shell 63' with real constant as pipe thickness has been used. The boundary conditions are as shown in Figure 4. A force applied at flange end and the stresses were checked at the Unreinforced Tee junction. These stresses were compared against pipe with circular cross section to get SIF.

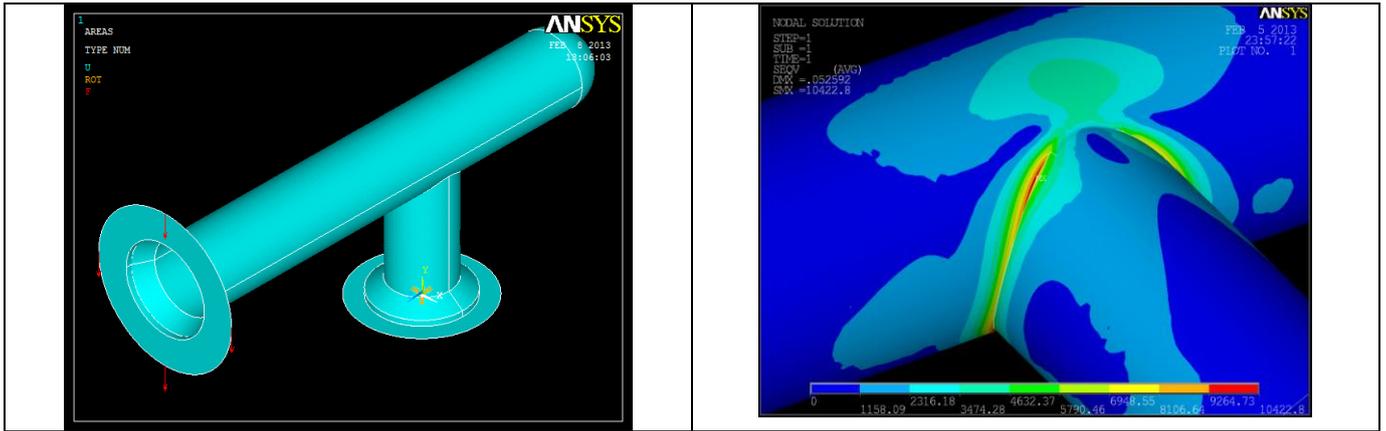
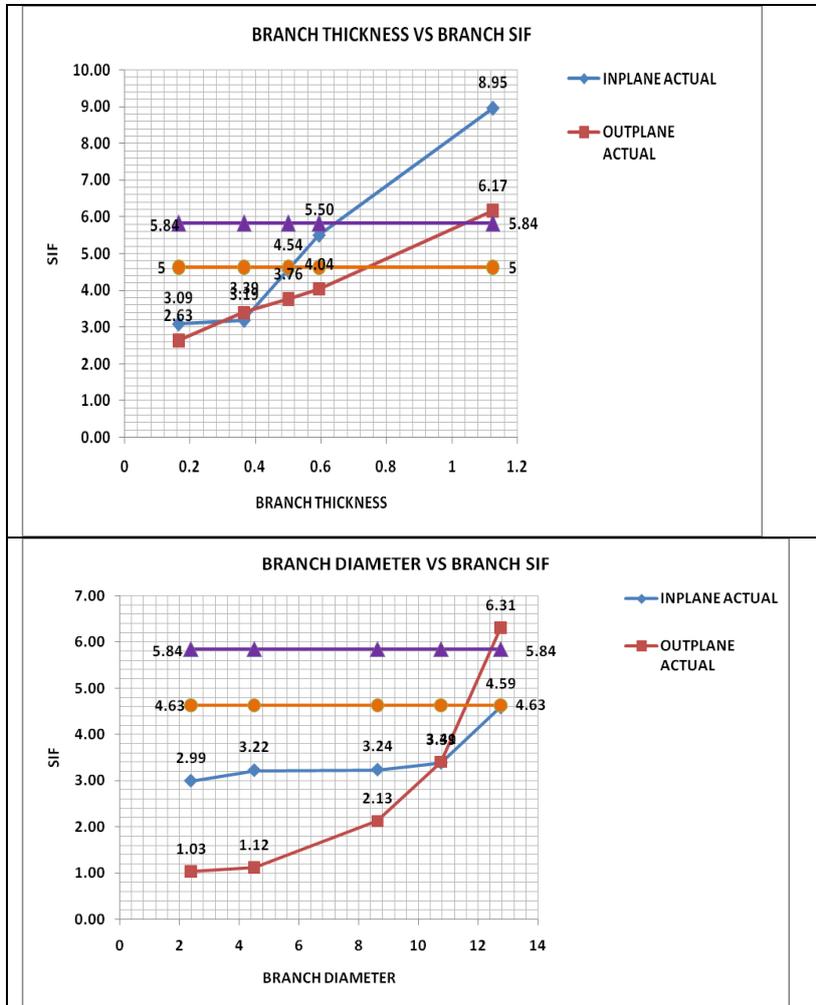
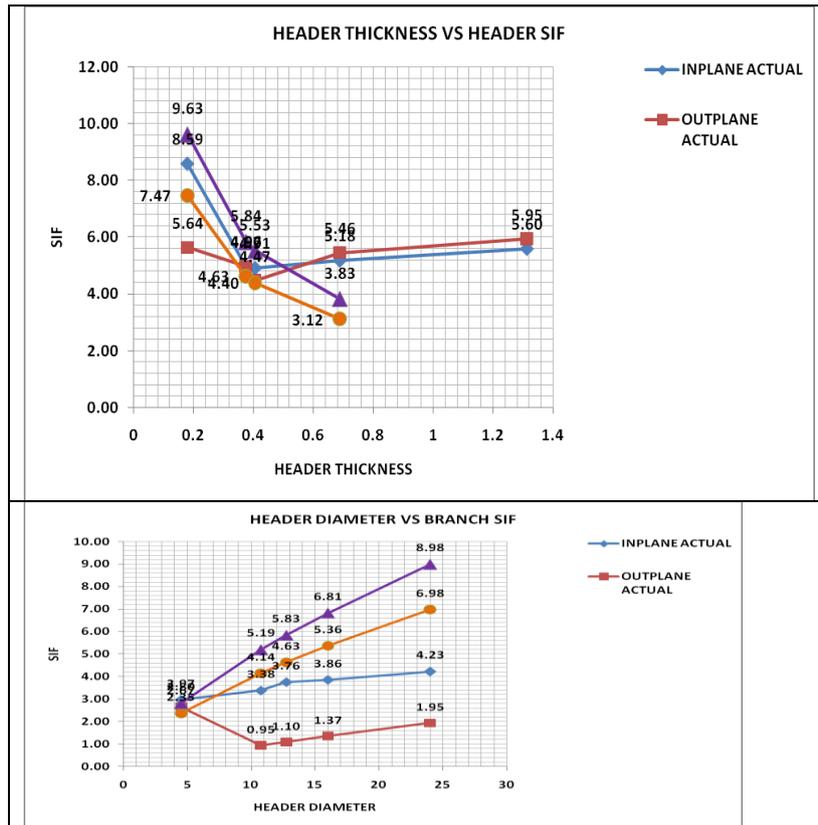


Figure 6

**4.2 Results based on ASME B31 equations**

The results based on B31 equations were obtained using CAESAR II analysis and that based on FEA analysis were obtained using ANSYS. Both the results have been graphically plotted in Fig.7 below.





**The observations based on B31 equations and that of FEA analysis are listed below-**

1. B31 SIFs (In-plane & Out-plane) remain constant irrespective of branch thickness. FE analysis indicates branch In-plane and Out-plane SIF increases as branch thickness increases but the values are less compared to B31 equation. However, header SIFs decreases.
  2. B31 SIFs remain constant irrespective of branch diameter. FE analysis indicates as branch diameter decreases header SIFs increase but branch SIFs decrease.
  3. B31 SIF decreases as header thickness increases. FEA analysis indicates as header thickness increases the header in-plane SIF decreases, header out-plane SIF initially decreases and then increases, branch SIFs decreases.
  4. B31 SIF decreases as header diameter decreases. FE analysis indicates that as header diameter decreases header SIFs decrease and branch in-plane SIF decreases, out-plane SIF initially decreases and then shows increasing trend.
- These observations indicate that B31 code provides same SIF value for header and branch. However FE analysis provides different SIFs for branch and header. Even though in general B31 equations provide conservative values of SIF compared to FEA; at few occasion FE analysis provides higher SIF's. Hence, in case of critical systems in depth study is required before adapting any SIF value.

**Additional points to be noted about SIF**

- a) The values of SIF for the same component can be different in different codes.
- b) Different edition of same code can provide different value of SIF for the same component.
- c) The multiplication fraction to be applied for SIF of same fitting can be different in different code.

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