SISO MMSE-PIC detector in MIMO-OFDM systems

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ABSTRACT: MIMO-OFDM with bit-interleaved coded modulation (BICM) is an attractive technique for wireless communications over frequency selective fading channels which has gained significant interests as a promising candidate for the 4th Generation wireless communication. The SISO MMSE-PIC based detector is proposed in this paper. The SISO MMSE-PIC detector exchanges soft information with SISO channel decoder through iterative process. For reduction complexity, the Max Log MAP approximations decoder is exploited for iterative BICM decoding of MIMO-OFDM under perfect channel knowledge. Simulation results in the IEEE 802.11 channel model show that the SISO MMSE-PIC iterative detector decoder leads to a clear improvement of the performance than the SISO ZF-PIC based detector with saturation at 4th iterations. For a large number of iterations, the performance improvement is not significant which could be investigated for practical reduction complexity considerations. Furthermore, for a 10⁻² bit error rate BER target, the gain in the performance of the SISO MMSE-PIC is approximately about 3dB when the number of antennas is twice. The performance of the SISO MMSE-PIC under different modulation schemes for MIMO-OFDM systems shows that with lower modulation size, the method can perform better and decreases when the channel delay spreads is increased.

Keywords: BICM, IEEE 802.11, MIMO iterative detection and decoding, MIMO-OFDM, SISO MMSE-PIC.

I. INTRODUCTION

The Multi-input multi-output orthogonal frequency-division multiplexing frequency-division multiplexing (MIMO-OFDM) has gained significant interests as a promising candidate for the 4th Generation (4G) wireless communication. It combines the capacity and diversity gain of MIMO systems with the equalization simplicity of Orthogonal Frequency Division Multiplexing (OFDM) modulation. A higher capacity with high bandwidth efficiency can be achieved over broadband multipath fading wireless channels [1, 2]. The use of multiple antennas at both the transmitter and receiver, which is usually referred to as MIMO communication, can yield large improvements in spectral efficiency and diversity compared to single systems, when using advanced signal processing and coding techniques. OFDM is a multicarrier transmission technique, which divides the available spectrum into many carriers; each one being modulated by a low data rate stream has been recently established for several systems such as American IEEE802.11, the European equivalent HiperLAN/2, digital video and audio broadcasting.

MIMO-OFDM with bit-interleaved coded modulation (BICM) is a promising technique for wireless communications over frequency selective fading channels [3, 4]. Müller-Weinfurtner has demonstrated that for the multiple-input multiple-output (MIMO) system, BICM shows excellent performance in fast-fading channel when maximum likelihood (ML) detection is used [5]. However, as the complexity of ML detection is large, a low complexity solution based on Zero Forcing (ZF) and Minimum Mean Squared Error (MMSE) detection have been proposed. The BICM is incorporated in many modern wireless communication standards such as IEEE 802.11n, IEEE 802.16m and 3rd Generation Partnership Project long term evolution (3GPP LTE) [6,7,8]. It was shown that the full potential of MIMO wireless systems can, in practice, only be achieved through iterative MIMO decoding [9].

In iterative MIMO detection and decoding method, a posteriori probability (APP) MIMO algorithm is the optimal way to calculate the probabilistic soft information of the inner coded bits expressed with Log-Likelihood Ratio (LLR) values [10]. The probabilistic soft information is then further processed in the outer channel decoder based on an optimal (Soft In Soft out) BCJR (MAP) algorithm and fed back to the inner detector [11]. The reliability information is exchanged between the two stages separated by a deinterleaver and an interleaver. However, the computational complexity of the optimum MIMO detection algorithm scales exponentially in the number of spatial streams. Various efficient MIMO-BICM soft detector algorithms providing approximate LLRs have been proposed. Existing approaches use the list extension of the Fincke-Phost Sphere Decoding (LPSD) algorithm as well as algorithms based on Zero-Forcing (ZF) or Minimum Mean Squared Error (MMSE) equalization [12, 13, 14, 15]. The Jacobian logarithm and the so-called log-MAP algorithm reduces the complexity of the original symbol-by-symbol MAP algorithm [11]. A less complex max-log-MAP approximation can also be applied with rather small performance loss compared to the log-MAP [16]. A posteriori probability (APP) detection, optimal but exponentially complex, is usually replaced with Parallel Interference Cancellation (PIC) and Minimum Mean Square Error (MMSE) filtering. MMSE based “soft” successive interference cancellation [17], list sphere detection [18], and list sequential detection [19] are all known achieve performance close to the capacity limit of the MIMO channel while avoiding the prohibitive complexity of a full APP detector.

In this paper, we propose a combination of Parallel Interference Cancellation (PIC) detection with maximum a posteriori (MAP) decoding denoted by SISO MMSE PIC algorithm for MIMO-OFDM systems. The PIC technique applies a linear detector to obtain an initial estimate of the transmitted data layer based on the a priori LLRs obtained from the SISO
channel decoder. Each layer is then nulled with the estimate from other layers. The Interference Plus residual Noise (NPI) term is then equalized using a MMSE filter, followed by computation of per-stream a posteriori LLRs.

The remaining of the paper is organized as follows. In section II, the MIMO-OFDM system model is described. In section III, linear detection schemes, iterative detection and decoding and SISO MMSE Parallel Interference Cancelation (PIC) methods are introduced. Section IV is devoted to simulations and performance evaluation. Finally, Section V concludes the paper.

II. MIMO-OFDM SYSTEM MODEL

A MIMO-OFDM system model based on a bit-interleaved coded modulation (BICM) transmission strategy is depicted in Fig. 1. We consider a multiple antenna system with $N_T$ transmit and $N_R$ receive antennas ($N_R > N_T$). At the transmitter, a stream of information bits $\mathbf{d}$ is first encoded by an outer channel code with rare $R$ and interleaved by a quasi-random interleaver. The resulting stream of coded and interleaved bits $\mathbf{b}$ is then de-multiplexed into $N_T$ sub-streams $\mathbf{b} = [b_1(k),...,b_{N_T}(k)]^T$ and mapped to a sequence of $N_T$ dimensional symbol vectors $\mathbf{s}(k)$. The entries of $\mathbf{s}(k)$ are drawn from a complex QAM (or MPSK) constellation $\Omega$, where $|\Omega| = 2^Q$ and $Q$ is the number of bits per symbol. The length of each symbol vector $\mathbf{s}(k) = [s_1(k),...,s_{N_T}(k)]^T$ is $N_T$ $Q$, where $s_i(k) = \text{Map}(b_{i,q}(k))$ is the $q^{th}$ bit of the $i^{th}$ entry of the symbol vector which takes its value from the QAM alphabet set $\Omega = [a_1,...,a_{2^Q}]$ and the bits $b_{i,q}$ are chosen from the set $\{+1,-1\}$ ($i = 1,...,N_T$ and $q = 1,...,Q$). Then, the modulated signals are passed through an IFFT operation and transmitted via $N_T$ antennas.

The frequency-selective MIMO channel can be decomposed into parallel frequency at MIMO channels. For a MIMO-OFDM system with $N_c$ subcarriers, the received signal for each subcarrier $k$ can be written as:

$$\mathbf{r}(k) = \mathbf{H}(k)\mathbf{s}(k) + \mathbf{\eta}(k) \quad \text{for} \quad 1 \leq k \leq N_c$$

(1)

Where $\mathbf{H}(k)$ is the $(N_R \times N_T)$ frequency domain channel matrix which is assumed to be perfectly known at the receiver, $\mathbf{r}(k)$ is the $(N_R \times 1)$ received signal vector and $\mathbf{\eta}(k)$ is a $(N_R \times 1)$ noise vector whose elements are zero mean independent identically distributed (i.i.d) circular symmetric complex Gaussian random variables with variance $N_0$ observed at the $N_R$ receive antennas.

The channel coefficients of $\mathbf{H}(k)$ for each sub-carrier $k$ are given by the discrete Fourier transform of the channel impulse responses $h_{i,l}(k)$ as:

$$\mathbf{H}^{i,l}(k) = \sum_{l=0}^{L-1} h_{i,l}(k)e^{-j2\pi kl/N_c}$$

(2)

The maximum multipath delay length is equal to $L$ and the length of the Cyclic Prefix $N_c$ is assumed to be long enough to eliminate the inter-symbol interference. In the receiver, the symbols are transformed into frequency domain with the FFT. The soft detector provides soft output LLRs for the decoder.

![Fig. 1. Block diagram of MIMO OFDM transmission](image1)

![Fig. 2. MIMO iterative receiver scheme](image2)
III. LINEAR DETECTION SCHEME

In linear detection such as Zero forcing (ZF) and Minimum Mean Squared Error (MMSE), the receiver symbol vector \( \mathbf{r} \) is multiplied with a linear filter:

1. ZF: \[ \mathbf{s} = \mathbf{G}_{ZF} \mathbf{r} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \mathbf{r} = \mathbf{s} + \eta_{ZF} \] (3)

2. MMSE: \[ \mathbf{s} = \mathbf{G}_{MMSE} \mathbf{r} = \left(\mathbf{H}^H \mathbf{H} + \frac{N_r}{\rho} \mathbf{N}_r\right)^{-1} \mathbf{H}^H \mathbf{r} = \mathbf{s} + \eta_{MMSE} \] (4)

Where \( \rho \) is the Signal to Noise Ratio (SNR).

III.1. Iterative detection and decoding

In Iterative MIMO decoding, the SISO detector has to generate reliability information, or “soft output”, for each of the coded bits \( b_{i,q}(k) \) in the symbol vector \( \mathbf{S}(k) \). Fig. 2 depicts the iterative receiver structure based on the turbo-processing principle [9]. At each iteration, the soft output detector updates and delivers to the channel decoder the extrinsic information for each coded bit. The detector calculates a posteriori soft output values \( L_{e}(b_{i,q}) \) using the a priori information \( L_{a}(b_{i,q}) \). The contribution of the a priori information is subtracted from \( L(b_{i,q}) \) to obtain the extrinsic information \( L_{e}(b_{i,q}) \) as:

\[ L_{e}(b_{i,q}) = L(b_{i,q}) - L_{a}(b_{i,q}) \] (5)

The soft input soft output (SISO) decoder uses the de-interleaved \( L_{e}(b_{i,q}) \) as a priori information \( L_{a}(c) \) to produce the a posteriori values \( L(c) \). The extrinsic information \( L_{e}(c) \) is again obtained by subtracting the a priori values from the a posteriori values. The interleaved \( L_{e}(c) \) can then be used as a priori information for the detector. This iterative process continues until convergence is achieved.

The a posteriori LLR for each coded bit can be written as:

\[ L(b_{i,q}) = \frac{p[b_{i,q}=+1|\mathbf{Y}]}{p[b_{i,q}=-1|\mathbf{Y}]} \] (6)

III.2. SISO MMSE Parallel Interference Cancellation (PIC) detector

In Parallel Interference Cancellation (PIC) detector, a single layer is detected and the corresponding contribution to the received vector is subtracted; the other layers that have not been detected yet are equalized using a ZF or MMSE equalizer. The \( i^{th} \) interference-cancelled received vector is given by:

\[ \hat{\mathbf{Y}}_i = \mathbf{Y} - \sum_{j=1,j \neq i}^{N_T} \mathbf{h}_j \hat{s}_j = \mathbf{h}_i s_i + \hat{\mathbf{w}}_i \] (7)

Where \( \hat{\mathbf{w}}_i \) is the Interference terms plus the residual Noise (NPI).

First, the SISO MMSE-PIC algorithm compute estimates \( \hat{s}_i \) of the transmitted symbols \( s_i \) using a linear filter whose coefficients are given by the a priori LLRs \( L_{a}(b_{i,q}) \) obtained from the SISO channel decoder. These estimates are used to cancel interference in the received vector. The soft symbols estimates \( \hat{s}_i \) are calculated as [20, 21, 22]:

\[ \hat{s}_i = \mathbb{E}[s_i] = \sum_{a \in \mathbb{R}} P[s_i = a] a \quad \text{for} \quad i = 1, ..., N_T \] (8)

Where the a-priori probability of the symbol can be easily derived due to the independence of the bits \( b_{i,q} \):

\[ P[s_i = a] = \prod_{q=1}^{Q} P[b_{i,q} = |a|_q] \]

(9)

\[ = \frac{1}{2^Q} \prod_{q=1}^{Q} \left(1 + \tilde{b}_{i,q} \tanh \left( \frac{L_{a}(b_{i,q})}{2} \right) \right) \] (10)

Where the a priori LLR’s \( L_{a}(b_{i,q}) \) are given by:

\[ L_{a}(b_{i,q}) = \frac{P[b_{i,q}=+1]}{P[b_{i,q}=-1]} \] (11)

With,

\[ P[b_{i,q} = \mp 1] = \frac{\exp \left( \mp \frac{L_{a}(b_{i,q})}{2} \right)}{\exp \left( \frac{L_{a}(b_{i,q})}{2} \right) + \exp \left( \mp \frac{L_{a}(b_{i,q})}{2} \right)} \] (12)

\( [.]_q \) : denotes to the \( q^{th} \) bit associated with the symbol \( a \) and \( \tilde{b}_{i,q} \in \{+1,-1\} \). The reliability of each soft symbol \( \hat{s}_i \) is characterized by its variance:

\[ \text{Var}[s_i] = \mathbb{E}[(s_i - \hat{s}_i)^2] = (\sum_{a \in \mathbb{R}} P[s_i = a]|a|^2) - |\hat{s}_i|^2 \] (13)

Next, In order to further suppress the terms plus the residual noise (NPI), a linear MMSE filter \( \mathbf{G}_i \) is applied to \( \hat{\mathbf{Y}}_i \), to obtain:
\[
\hat{y}_i = G^H_i \hat{Y}_i = G^H_i h_s + G^H_i \bar{w}_i = \tilde{\beta}_i s_i + \tilde{\eta}_i
\]  
(14)

Where the MMSE filter vectors \(G^H_i\) is chosen to minimize the mean squared error between the transmitted symbols at the \(i^{th}\) antenna and the filter vector \(\hat{Y}_i\) as:

\[
G^H_i_{\text{MMSE}} = \arg \min_{G^H} \mathbb{E} \left[ \|G^H_i \hat{Y}_i - s_i \|^2 \right]
\]  
(15)

The solution to this problem is derived in [20] and the MMSE filter vectors is expressed as:

\[
G^H_i_{\text{MMSE}} = \sigma_i^2 (H e_i)^H (H A H^H + N_B I)^{-1}
\]  
(16)

Where \(A_i = \text{cov}(s_i, s_i)\) is a \((N_T \times N_T)\) diagonal matrix having its elements the variance of the symbol \(s_i\) , \(e_i\) is a \((N_T \times 1)\) vector with all elements equal to 0 except the \(i^{th}\) element is equal to 1 and \(\sigma^2_i = \mathbb{E}[|s_i|^2]\) is the symbol energy.

It is shown in [20, 21] that the distribution of the Interference terms plus the residual Noise (NPI) at the output of a linear MMSE detector is well approximated by a Gaussian distribution. Finally, the resulting LLR’s of coded bits \(L(b_{i,q})\) are calculated according to the formula:

\[
L(b_{i,q}) = \log \left( \frac{\sum_{a \in \chi_{i,q}^{-1}} \exp \left( -\frac{[y_i - \tilde{\eta}_i]^2}{\sigma_i^2} - p[y_i = a] \right)}{\sum_{a \in \chi_{i,q}^{-1}} \exp \left( -\frac{[y_i - \tilde{\eta}_i]^2}{\sigma_i^2} - p[y_i = a] \right) + p[y_i = a]} \right)
\]  
(17)

Where \(\chi_{i,q}^{-1}\) and \(\chi_{i,q}^+\) denotes the subset of symbol vectors that have the \(q^{th}\) bit in the label of the \(i^{th}\) symbol equal to -1 and +1, respectively. \(\tilde{\beta}_i\) is the bias introduced by the equalizer and \(\tilde{\sigma}_i^2\) represents the total variance of the interference terms plus the residual noise, such that:

\[
\tilde{\sigma}_i^2 = \text{Var}[y_i] = G^H_i (\sum_{a \neq q} \sigma_i^2 h_i h_i^H + N_B I N_B) G_i
\]  
(18)

The complexity of the previous relation can be reduced by using the Logarithm Jacobian defined by:

\[
\log(\exp(-x) + \exp(-y)) = -\min(x, y) + \log(1 + \exp(-|x-y|))
\]  
(19)

This can be approximated by:

\[
\log(\exp(-x) + \exp(-y)) = \min(x, y)
\]  
(20)

The resulting intrinsic LLRs are then computed as:

\[
L(b_{i,q}) = \min_{a \in \chi_{i,q}^{-1}} \left\{ \frac{-[y_i - \tilde{\eta}_i]^2}{\tilde{\sigma}_i^2} - \frac{\tilde{b}_{i,q} L_a(b_{i,q})}{2} \right\} - \min_{a \in \chi_{i,q}^+} \left\{ -\frac{y_i - \tilde{\eta}_i]^2}{\tilde{\sigma}_i^2} - \frac{\tilde{b}_{i,q} L_a(b_{i,q})}{2} \right\}
\]  
(21)

Then, the extrinsic a posteriori LLRs of the SISO MMSE-PIC can be obtained by using the equation:

\[
L_e(b_{i,q}) = L(b_{i,q}) - L_a(b_{i,q})
\]  
(22)

IV. SIMULATION RESULTS

Consider a 4x4 BICM MIMO-OFDM system based on MMSE-PIC detector. Our simulations are based on the following system parameters. The frame size is 1000 information bits. The convolutional encoder of rate 1/2 is used with generator polynomials \(g_0=133_8\) and \(g_1=171_8\). The coded bits are then interleaved by a pseudo-random permutation. The number of subcarriers is \(N=64\) and the modulation is M-QAM. The outer decoder of the receiver used is an optimal (soft-in-soft-out) BCJR (MAP) decoder. Perfect CSI at the receiver is assumed.

Fig. 3 shows the performance of iterative MIMO-OFDM decoding using the SISO MMSE –PIC based detector for various number of iterations. The IEEE 802.11 channel model with RMS delay spread of 25ns is used. As we can conclude, the iterative detector decoder leads to a clear improvement of the performance with saturation at 4th iterations. For a large number of iterations (> four), the performance improvement is not significant which could be investigated for practical reduction complexity considerations. The gain in performance attains more than 6dB for 10⁻² bit error rate BER with four iterations. Similar performance was observed with all modulation schemes. In Fig. 4, the BER versus SNR performance of the MMSE-PIC method is plotted for QPSK modulation. Fig. 5 quantitatively illustrates the performance improvement of MMSE detection compared with ZF method, in terms of SNR gain.
Fig. 3. BER vs. SNR performance of MIMO-OFDM systems based SISO MMSE-PIC detector with 16QAM modulation scheme and various iterations. The BCJR decoder is used.

Fig. 4. BER vs. SNR performance of MIMO-OFDM systems based SISO MMSE-PIC detector with QPSK modulation scheme and 4x4 number of antennas.

Fig. 5. BER vs. SNR performance comparison of MIMO-OFDM systems based ZF and SISO MMSE-PIC detectors with 16QAM modulation scheme and various iterations. The BCJR decoder is used.
Fig. 6 shows the performance of the proposed method compared with different number of coding rate (1/2, 2/3, 3/4). The performance improvement is obtained with the lowest coding rate and is enhanced with the number of iterations.

In Fig. 7, the performance of the SISO MMSE-PIC under different modulation schemes for MIMO-OFDM systems is considered. We can conclude that with lower modulation size, the method can perform better. For QPSK and 16QAM or 64QAM, the improvement is approximately 8dB for 10^{-2} bit error rate BER target.

Fig. 8 show the BER versus SNR performance for a \( N_T = N_R = 2 \) and \( N_T = N_R = 4 \) MIMO-OFDM system, respectively, with QPSK data 16QAM modulation schemes. The number of iterations is set to four. For a 10^{-2} bit error rate BER target, the gain in the performance of the SISO MMSE-PIC is approximately about 3dB when the number of antennas is twice.
In Fig. 9, two indoor channel models with RMS delay spreads of 25ns and 50ns values corresponding to the IEEE 802.11 channel model are used. As expected, the later model decreases the performance of the MIMO-OFDM.

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**V. CONCLUSION**

A MIMO-OFDM system model based on a bit-interleaved coded modulation (BICM) transmission strategy is presented in this paper. Simulations in the IEEE 802.11 channel model have shown that attractive performance is reached using SISO MMSE-PIC based detector in iterative manner with the BCJR decoder. The BICM decoder requires log likelihood ratios (LLRs) whose exact computation is extremely costly. In our simulations, the complexity is reduced by using the Max Log MAP approximations without a significant loss in the performance.
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