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Analysis of Dual Resonant Solid State Tesla Transformer

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ABSTRACT: *In this paper, Analysis of dual resonant solid state tesla transformer (DRSSTS) is carried out. The fundamental difference between this types of tesla transformer and spark gap tesla transformer, is using of one or more power electronic switch instead of spark gap. Likewise DRSSTS is different from the solid state tesla transformer due to the addition of a primary tank capacitor. In this paper, tesla transformer is fed by full bridge inverter that generates square wave. For the sake of simplicity, tesla transformer is modeled only by lumped elements and resistive losses are neglected. Then, analytical equations for Tesla transformer's output voltage and primary side current, are calculated and conditions of increased tesla transformer's output voltage are also investigated. Using Euler numerical integration method and switching function concept, the effect of tesla transformer's operation, on inverter dc side voltage and current have been evaluated. The test system is simulated in MATLAB/Simulink software, and validity of calculated equations, is verified by simulation results.*

Keywords:*Tesla transformer, single phase inverter, square wave, switching function, numerical integration.*

I. INTRODUCTION

A Tesla transformer is a resonant air core transformer invented by Nikola Tesla around 1891 that is used for producing high-voltage, low-current, high frequency alternating-current pulses. Tesla transformer increases the voltage in two steps: first by conventional step-up iron core transformer and second by an air core resonant transformer that increases voltage range up to several hundred kV. Usually a high voltage capacitor is connected to primary winding of air core transformer and comprises a resonant circuit after that the secondary winding of air core transformer with its self-capacitance form secondary resonant circuit. To achieve the best performance of the Tesla transformer resonance frequency of the primary and secondary windings must be equal [1,2].In Fig.1 spark gap based Tesla transformer is shown. Spark gap (G), which is composed of two electrodes and an air gap, operates as voltage-controlled switchand when potential difference across it exceeds threshold of air breakdown then the arc willbe produced and tesla transformer operates in dual resonant mode. To achieve this and improving tesla transformers performance, usually a conventional iron core is also used for increasing voltage in the first step which has a neon sign transformer (NST) because of its great leakage reactance. In Figure 1 the capacitor C_2 shows the distributed capacitance between ground and top load. In addition of need of step up transformer, tesla transformer of figure 1 has another major disadvantage, such as: High voltage capacitors are difficult to find, and you usually have to make. And if static spark gap is used, generated noise and light is extremely annoying, and rotary spark gap needs motor. Because of nonlinear nature of air ionization phenomena and practical limitations of spark gap, increasing in operating of frequency of system is not possible.Numerous articles and books have been published in dual tuned resonant transformer and analysis and implementation of spark gap based tesla transformer [1-8] and [11-13], For example, analysis of triple tuned resonant coils and tesla transformer based of iron core transformer instead of air core transformer are carried out in [9] and [10] respectively. Analysis of series resonant converter in order to producing high voltage is studied in [14].According to spark gap based tesla transformers defects, use of Tesla transformer based on power electronic devices have been proposed [6]. The main idea is replacing of spark gap with one or more power electronic switches such as IGBT or MOSFET. In [15] measurement of resonance frequency and other parameters of single ended Tesla transformer in laboratory prototype system is performed. Investigation on occurrence of resonance phenomena in two coils with different structures when one of them is stimulated with square wave is carried out in [16] but theoretical details of inverter is not considered. A Dual Resonant Solid State Tesla Transformer is different from the Solid State Tesla Transformer due to the addition of a primary capacitor. In this paper, analysis of Tesla transformer fed by square wave that generated by full bridge inverter is studied and Tesla transformer is modeled by lumped elements and analytical equations are provided. Also switching frequency of inverter to generate maximum is determined. In this basis, first output voltage and primary current of Tesla transformer are calculated analytically, then dc link current and voltage of inverter are calculated by using switching function and Euler integration method. Obtained equations are compared with MATLAB/Simulink simulation results and are shown that they are in good agreement.

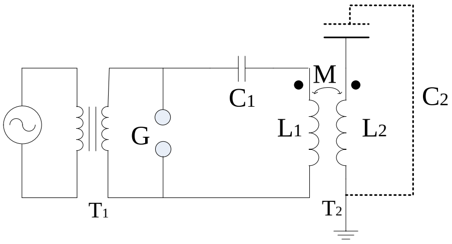


Fig.1 spark gap based tesla transformer

II. INVERTER BASED TESLA TRANSFORMER

Tesla transformer using lumped element is fed by full bridge inverter is shown in below.

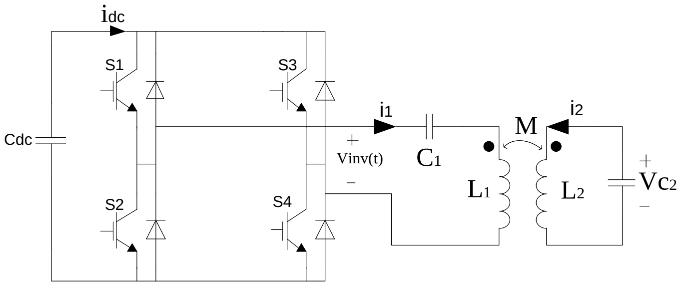


Fig.2 tesla transformer fed by full bridge inverter

In figure 2, inverter generates square wave with duty cycle $\frac{t_1}{T}$,that is shown in figure 3. Considering the Voltage Source $V_{inv}(t)$ as the voltage produced by the inverter and T equivalent circuit of coupled inductor, circuit of figure 2 converts to Figure 4.

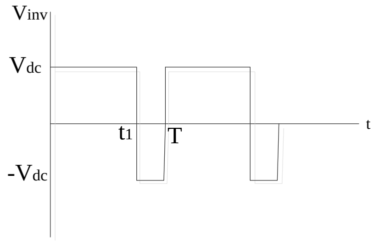


Fig.3 output voltage of full bridge inverter

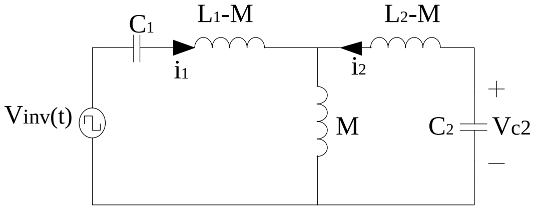


Fig.4 equivalent circuit of system

By neglecting resistive losses and using KVL:

$$\frac{1}{C_1}\int i_1 dt + L_1\frac{di_1}{dt} + M\frac{di_2}{dt} = V_{inv}(t)$$
$$\frac{1}{C_2}\int i_2 dt + L_2\frac{di_2}{dt} + M\frac{di_1}{dt} = 0 \tag{1}$$

In figure 4, characteristic equation of system is equal to:

$$(1-k^2)s^4 + (\omega_1^2+\omega_2^2)s^2 + \frac{1}{C_1C_2} = 0$$
$$\omega_1 = \frac{1}{\sqrt{L_1C_1}},\quad \omega_2 = \frac{1}{\sqrt{L_2C_2}},\quad k=\frac{M}{\sqrt{L_1L_2}} \tag{2}$$

In Eq.2 ω1 and ω2 are resonance frequency of primary and secondary coils respectively, and k is the coefficient of coupling. if inverter produces $V_{inv}(t)$, then fourier series of $V_{inv}(t)$is equal to:

$$v_{inv}(t) = \sum_{n=0}^{\infty} a_n\cos(n\omega_0 t) + b_n\sin(n\omega_0 t) \tag{3}$$

Inabove equation,$\omega_0 = \frac{1}{T}$is fundamental frequency of inverter and coefficients of fourier series are shown below:

$$a_0 = (\frac{2t_1}{T} - 1)V_{dc}$$
$$a_n = \frac{2V_{dc}}{\pi n}\sin(\frac{2\pi n t_1}{T})$$
$$b_n = \frac{-2V_{dc}}{\pi n}[\cos(\frac{2\pi n t_1}{T}) - 1]$$

Differential equations of system in Laplace domain are equal to:

$$(L_1 s + \frac{1}{C_1 s})I_1 + MsI_2 = \sum_{n=0}^{\infty} a_n \frac{s}{s^2 + n^2\omega_0^2} + b_n \frac{n\omega_0}{s^2 + n^2\omega_0^2}$$
$$(L_2 s + \frac{1}{C_2 s})I_2 + MsI_1 = 0 \tag{4}$$

Combining the above equations and after tedious calculations, finally output of tesla transformer in time domain is calculated as:

$$V_{c2} = \frac{-M\omega_0}{L_1 L_2 C_2 \sigma^2}\sum_{n=1}^{\infty}\{n\frac{y_{1n}}{x_1}[a_n\cos(x_1 t) + b_n\sin(x_1 t)] + n\frac{y_{2n}}{x_2}[a_n\cos(x_2 t) + b_n\sin(x_2 t)] + \frac{y_{3n}}{\omega_0}[a_n\cos(n\omega_0 t) + b_n\sin(n\omega_0 t)]\} \tag{5}$$

That parameters of Eq.5are given below:

$$\sigma^2 = 1 - \frac{M^2}{L_1 L_2}, \quad x_1^2 = \frac{\omega_1^2 + \omega_2^2}{2\sigma^2} + \sqrt{(\frac{\omega_1^2 + \omega_2^2}{2\sigma^2})^2 - \frac{\omega_1^2\omega_2^2}{\sigma^2}}, \quad x_2^2 = \frac{\omega_1^2 + \omega_2^2}{2\sigma^2} - \sqrt{(\frac{\omega_1^2 + \omega_2^2}{2\sigma^2})^2 - \frac{\omega_1^2\omega_2^2}{\sigma^2}}$$
$$y_{1n} = \frac{-x_1^2}{(x_1^2 - x_2^2)(x_1^2 - n^2\omega_0^2)}, \quad y_{2n} = \frac{x_2^2}{(x_1^2 - x_2^2)(x_2^2 - n^2\omega_0^2)}, \quad y_{3n} = \frac{n^2\omega_0^2}{(x_1^2 - n^2\omega_0^2)(n^2\omega_0^2 - x_2^2)}$$

In case of equality of ω1 and ω2, $x_1^2$and x_2^2 are simplified as:

$$x_{1,2}^2 = \frac{\omega_1^2}{1 \mp k} \tag{6}$$

According toy_{1n},y_{2n} and y_{3n}, it is obvious that with increasing n,y_{1n}, y_{2n} and y_{3n} will decrease with square reverse rate of n, and coefficients of Fourier series will decrease with reverse rate of n (Eq.3). Therefore, it is reasonable to approximate the output of Tesla transformer with low values of n. Considering n = 1 the output voltage Tesla transformer is equal to:

$$V_{c2} = \frac{-M\omega_0}{L_1 L_2 C_2 \sigma^2}\{\frac{y_1}{x_1}[a_1\cos(x_1 t) + b_1\sin(x_1 t)] + \frac{y_2}{x_2}[a_1\cos(x_2 t) + b_1\sin(x_2 t)] + \frac{y_3}{\omega_0}[a_1\cos(\omega_0 t) + b_1\sin(\omega_0 t)]\} \tag{7}$$

If duty cycle of inverter is 50%, then an become zero and V_{c2}calculated as following equation:

$$V_{c2} = \frac{-M\omega_0}{L_1 L_2 C_2 \sigma^2}[\frac{y_1}{x_1}b_1\sin(x_1 t) + \frac{y_2}{x_2}b_1\sin(x_2 t) + \frac{y_3}{\omega_0}b_1\sin(\omega_0 t)] \tag{8}$$

According to y_{1n}, y_{2n} , y_{3n} and Eq.6 it is ascertained that, when k is small, if ω_0 approaches to resonance frequency of each primary or secondary, output voltage will increase. But when k is almost unity, for generating higher voltage, ω_0 should approach to x_1 or x_2. Theoretically maximum value of Eq.18 and primary current of Tesla transformer are calculated as Eq.9 and Eq.10.

$$V_{c2}^{\max} = \frac{M\omega_0 b_1}{L_1 L_2 C_2 \sigma^2}(\left|\frac{y_1}{x_1}\right| + \left|\frac{y_2}{x_2}\right| + \left|\frac{y_3}{\omega_0}\right|) \tag{9}$$

$$\begin{aligned} i_1 = \frac{\omega_0}{L_1 L_2 C_2 \sigma^2}\sum_{n=1}^{\infty} & y_{1n}(L_2 C_2 - \frac{1}{x_1^2})n[-a_n\sin(x_1 t) + b_n\cos(x_1 t)] + \\ & y_{2n}(L_2 C_2 - \frac{1}{x_2^2})n[-a_n\sin(x_2 t) + b_n\cos(x_2 t)] + \\ & y_{3n}(L_2 C_2 - \frac{1}{n^2\omega_0^2})n[-a_n\sin(n\omega_0 t) + b_n\cos(n\omega_0 t)] \end{aligned} \tag{10}$$

In general, dc link current of inverter, is calculated by convolution of primary current of tesla transformer (Eq.10) and in the frequency domain, or by multiplying of two mentioned functions in the time domain [17].

$$i_{dc}(t) = s(t) i_1(t) \tag{11}$$

Where s(t) is switching function of inverter in time domain and is calculated as follows:

$$\begin{aligned} s(t) &= \sum_{n=0}^{\infty} c_n \cos(n\omega_0 t) + d_n \sin(n\omega_0 t) \\ c_0 &= (\frac{2t_1}{T} - 1) \\ c_n &= \frac{2}{\pi n} \sin(\frac{2\pi n t_1}{T}) \\ d_n &= \frac{-2}{\pi n} [\cos(\frac{2\pi n t_1}{T}) - 1] \end{aligned} \tag{12}$$

And the dc link voltage can be calculated as follows:

$$V_{dc}(t) = V_0 - \frac{1}{C_{dc}} \int_0^t i_{dc}(t) dt \tag{13}$$

In above equation, V_0 is the initial voltage of capacitor. Because of nonexistence analytical equation for dc link current (Eq.11), Eq.13 is solved using Euler numerical integration method. Using Euler method, first order differential equation as a Eq.14 is written in discrete domain as a Eq.15:

$$\frac{d}{dt} y = f(y, t) \tag{14}$$

$$y(t_{k+1}) = y(t_k) + \Delta t . f(y(t_k), t_k) \tag{15}$$

In Eq.15, y(t_{k+1}) is value of y in t_{k+1} and Δt is and the time step of integration that is equal to $t_{k+1} - t_k$.

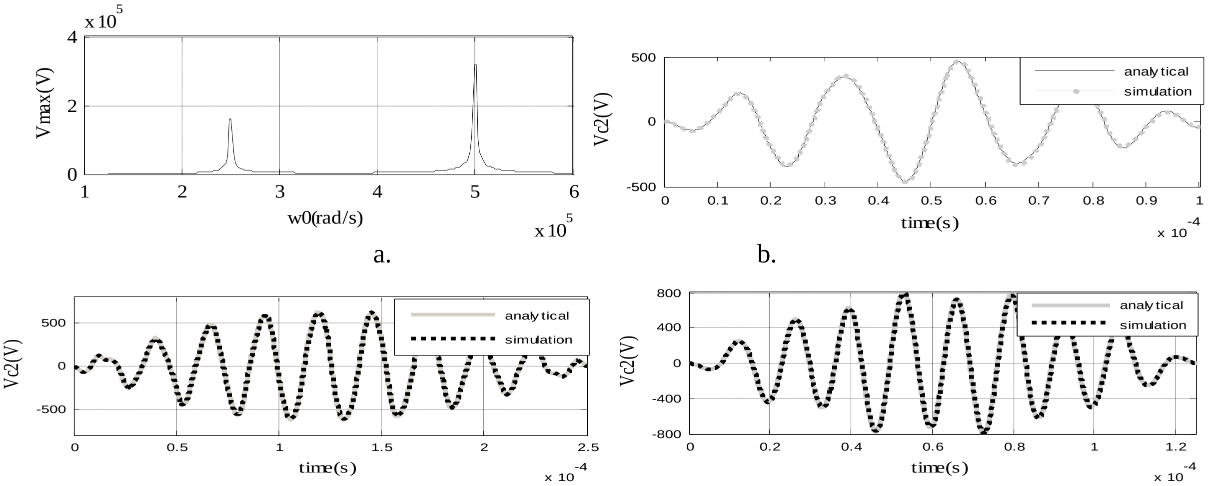
III. SIMULATION RESULTS

To verify the obtained equations in the previous section, the test system in figure 2, is simulated with MATLAB/SIMULINK software and simulation results are compared with analytical and numerical results. It should be noted that in all of obtained equations to the 80th harmonic in the Fourier series is considered and other parameters of test system are given in table.1.

Table.1 simulation parameters

L_1=10μH	L_2=1mH	K=0.6	f=50kHz
C_1=1μF	C_2=10nF	$\frac{t_1}{T} = 0.5$	V_{dc}=12V

In this case $x_1$and x_2 are approximately equal to 5×105 rad/s and 2.5×105 rad/s respectively and V_{c2}^{Max} (Eq.9) in terms of ω0 is shown in fiure5.a.Both of simulation and analytical output voltage of tesla transformer (Eq.5) in the case of$\omega_0 = \omega_1$, $\omega_0 = 0.9x_2$ and $\omega_0 = 0.9x_1$are shown in fiure5.b-d. According to fiure5.b-d it is obvious that maximum of tesla transformers output voltage, when ω_0 approaches tox_1, is larger than when $\omega_0$approaches to x_2. Comparison results Eq.8, primary current, dc link current and voltage in the case of $\omega_0 = \omega_1$, Cdc=0.1mF and Δt=0.1μs are shown in fig.6a-d. As shown in fig.6c with decreasing Δt, insignificant difference between the numerical results and simulation will decrease.

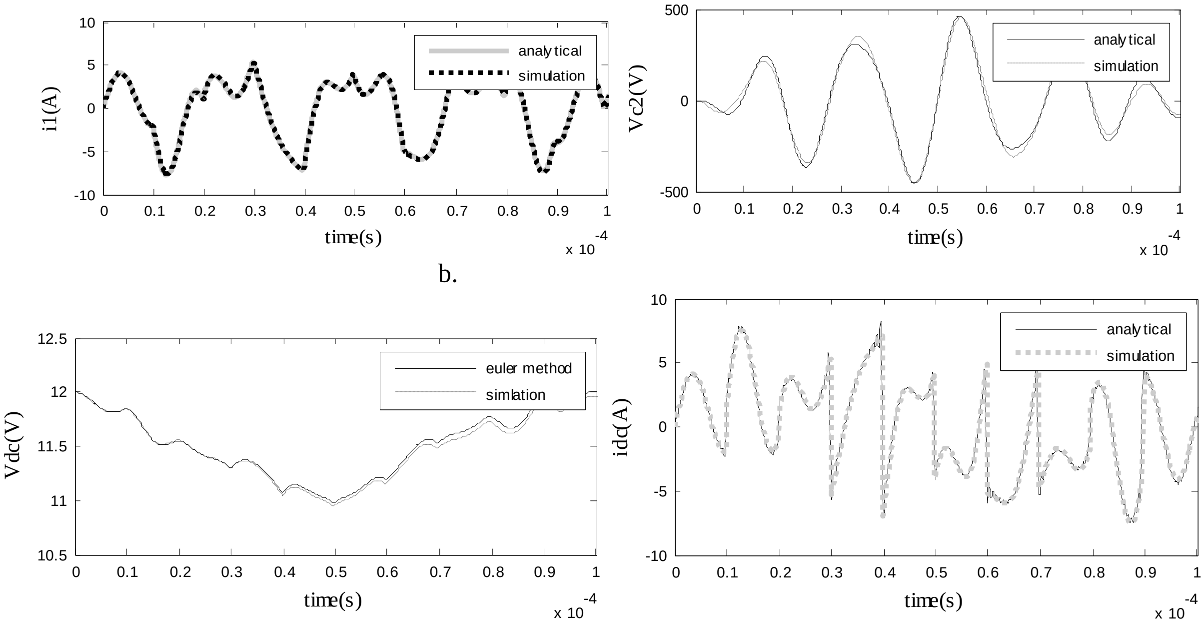


a. b.

c. d.

Fig.5 a:maximum output voltage of tesla transformer in terms of ω_0.
b: ω_0= ω_1. c: ω_0=0.9x_2. d: ω_0=0.9x_1.

Comparison results Eq.8, primary current, dc link current and voltage in the case of $\omega_0 = \omega_1$, Cdc=0.1mF and Δt=0.1μs are shown in fig.6a-d. As shown in fig.6c with decreasing Δt, insignificant difference between the numerical results and simulation will decrease.



a. b.

c.d.

Fig.6 a: approximated output voltage of tesla transformer.
b: primary current. C: dc link current. D: dc link voltage

Figure.7 shows the results of numerical and simulation of the dc link voltage in the case of $C_{dc} = 50\mu F$.

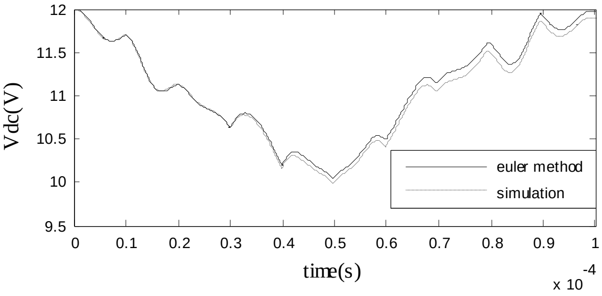


Fig.7 dc link voltage:C_{dc}=50μF

IV. CONCLUSION

According to the discussions, it was shown that the calculated equations are sufficiently accurate, And a closed equation for the maximum output voltage Tesla transformer is calculated and operating frequency of inverter that cause increment in output voltage of tesla is investigated. Also effect of tesla transformer operation on dc link voltage and current are studied in this paper and a method for sizing of dc link capacitor is proposed.For further works, impact of other parameters such as duty cycle of inverter and coupling coefficient and etc on output voltage of Tesla transformer and other variables of the system can be studied.

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