International Journal of Modern Engineering Research (IJMER)
www.ijmer.com Vol. 3, Issue. 6, Nov - Dec. 2013 pp-3577-3581 ISSN: 2249-6645

Controlling Of Chaos in Modified Nicholson-Bailey Model

Debasish Bhattacharjee[1], Tarini Kumar Dutta[2]
[1]B. Borooah College; Guwahati 781007; INDIA
[2]Gauhati University; Guwahati 781014; INDIA

Abstract: *In this paper, a non- linear ecological mapmay be called as modified Nicholson-Bailey map is considered,which becomes chaotic in nature with the increase of the control parameter. As in most of the caseschaos is an unwanted phenomenon, so controlling of chaos becomes a necessary part of study. First of all Chau's method is applied on this map and the chaotic region is controlled forming periodic trajectories. AgainOGY method is applied on the map to have chaos controlled. Lastly, the model has been modified to Chau's form which generates a set of fixed points which have been stabilized by OGY method.*

Key Words: *Chaos/Periodic orbits/Accumulation point/Control of chaos 2010 AMS Classification: 37G15, 37G 35, 37C 45*

I. Introduction

The Nicholson-Bailey model describes the population dynamics of host-parasite (predator-prey) system and is described as follows:

$$x_{n+1} = L\, x_n e^{-a\, y_n}$$
$$y_{n+1} = x_n(1 - e^{-a\, y_n}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.1.1)$$

Where x_{n+1} represents the number of hosts (or prey) at stage n and y_{n+1} represents number of parasites(or predator) at n^{th} stage. It has been observed**[21]**that the model fulfils the fact that equilibrium state never occurs for predator system in nature.Hence a modified version has been introduced by Dutta,T.K.,et al [**6**] to restrict the unlimited growth of host(or prey),which arises in Nicholson Bailey model.The modified model is as follows:

$$x_{n+1} = L\, x_n e^{-a\, y_n - x_n^2}$$
$$y_{n+1} = x_n(1 - e^{-a\, y_n}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.1.2)$$

In fact various modified forms have been discussed in the literature**[3,6,11,14].**

It has been shown that with the increase of the control parameter, the model (1.1.2) follows a period doubling bifurcation route to chaos[**6**]. As chaos is observed as undesirable part in engineering control. So, a desirable task is to control the chaotic region. Since 1990 after the discovery of OGYmethod**[22]**,chaos control problem attracts various researchers.

We use the periodic pulse method to control chaos which was discussed by N.P. Chau, which restricts the way of choosing initial point resulting periodic orbit. Further, OGY method is applied such that the known unstable periodic point is made stable with a small disturbance in the control parameter.

II. Controlling of chaos in the map by Chau's method

Chau'stheory **[2]** applied to the map is as follows:
The model can be written as follows:

$X_{n+1} = F(X_n)$, where X_n is a vector in R^2 .Let $G = KF^p$, wher K is a diagonal matrix having the diagonal elements say k_1, k_2, and F^p is the composition map of F up to p times. If X is a fixed point of G i.e. $KF^p(X) = X$, then the fixed point will be stable if the absolute value of the largest eigen value of the Jacobian matrix of G is less than 1.The next step is to get the value of k_1, k_2,such that chaos is controlled.
For p=1,

The Jacobian matrix of G is as follows:

$$\begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix} \begin{pmatrix} \boldsymbol{e^{-x^2-ay}(1-2x^2)L} & \boldsymbol{-aLxe^{-x^2-ay}} \\ 1-e^{-ay} & axe^{-ay} \end{pmatrix}$$

i.e.

$$\begin{pmatrix} \boldsymbol{k_1 e^{-x^2-ay}(1-2x^2)L} & \boldsymbol{-aLxe^{-x^2-ay}} \\ 1-e^{-ay} & k_2 axe^{-ay} \end{pmatrix}$$

The fixed points will be stable if $|\lambda| < 1$, **(1.2.1)**
whereλ is the eigen value of the Jacobian matrix.
If X=(x,y) is the fixed point of G, then
$k_1 L\, xe^{-a\, y - x^2} = x$, consideringx$\neq 0$, we have $k_1 = \frac{1}{Le^{-ay-x^2}}$ Similarly $k_2 = \frac{y}{x(1-e^{-ay})}$
(1.2.2) We choose (x,y) such that absolute value of λ is less than 1.

The basin of (x,y) chosen such that it satisfies both (1.2.1) and (1.2.2) is shown graphically as follows:

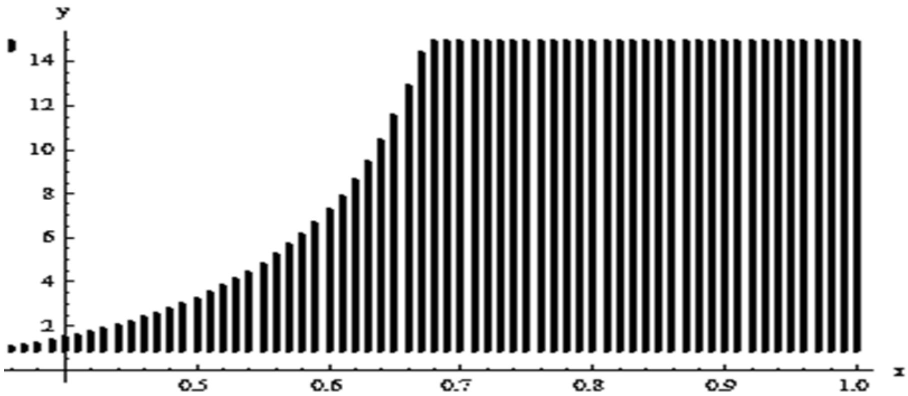

Fig: 1.2.1 Above picture shows the the value of x,ywhich for particular value of k_1, k_2 and p=1 becomes a stable fixed point for L=3.85 and a=0.1;

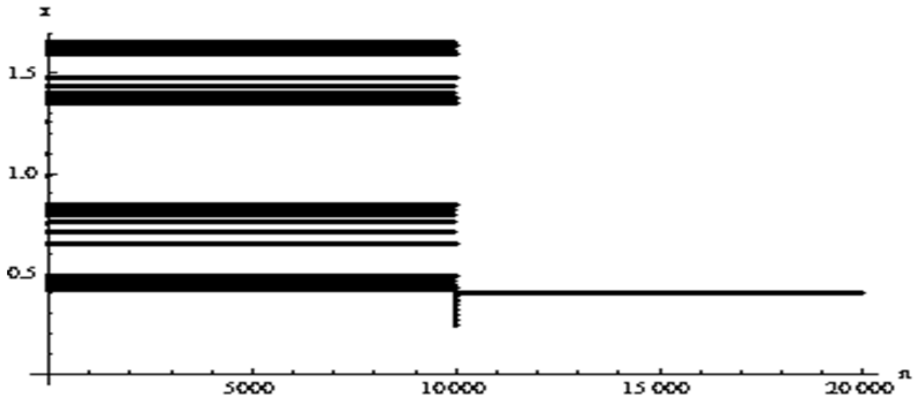

Fig: 1.2.2: Above figure shows the chaotic region up to 10000 iterations after which chaos control key k_1, k_2 is activated to make x fixed.

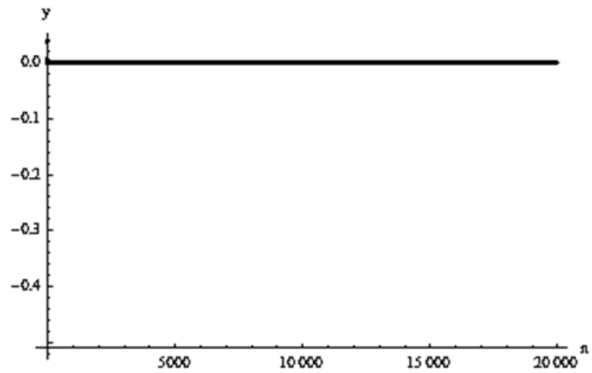

Fig: 1.2.3: In the above figure it is clear that y co-ordinate already achieves the fixed value as iteration proceeds.

Thus as a whole it is clear that once the chaos controlling switch is turned on, the trajectory points (x,y) achieves stability to form a fixed point as iteration proceeds.

If it is desired to obtain a periodic trajectory of period p, we have,

$$k_1 = \frac{x}{x_{p-1} e^{r\left(1-\frac{x_{p-1}}{k}\right)-b y_{p-1}}}, \; k_2 = \frac{x}{1-e^{-a y_{p-1}}}$$

Where x_{p-1}=first component of f^{p-1} and y_{p-1} is the second component of f^{p-1}. And the Jacobian matrix is given as

$$\begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix} \begin{pmatrix} \frac{\partial x_p}{\partial x} & \frac{\partial x_p}{\partial y} \\ \frac{\partial y_p}{\partial x} & \frac{\partial y_p}{\partial y} \end{pmatrix}$$

For p=2;

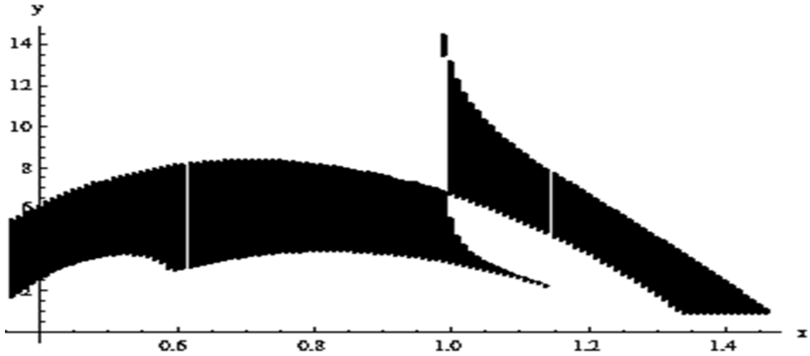

Fig: 1.2.3 Above picture shows the value of x, y which for particular value of k_1, k_2 and p=2 becomes a stable fixed point for L=3.85 and a=0.1;

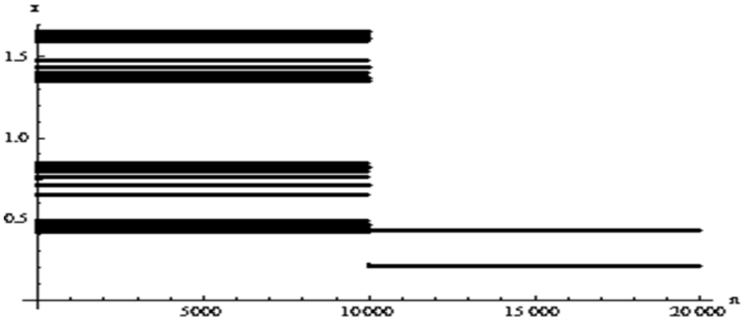

Fig: 1.2.4: Above figure shows the chaotic region up to 10000 iterations after which chaos control key k_1, k_2 is activated to make x fixed.

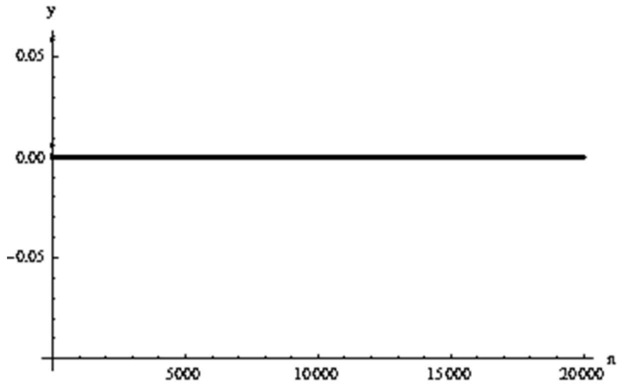

Fig: 1.2.5: In the above figure it is clear that y co-ordinate already achieves the fixed value as iteration proceeds.

For p=4, we have

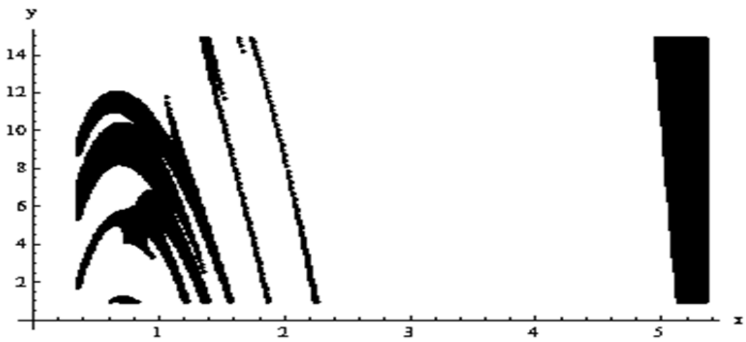

Fig: 1.2.6: Above picture shows the the value of x,y which for particular value of k_1, k_2 and p=2 becomes a stable fixed point for L=3.85 and a=0.1;

It has been observed that the pool of data as shown in the figure above although serve the purpose of a stable periodic point of period two, they are not always able to attract the trajectory towards them as the radius of the basin of attraction is small enough. Thus although stable periodic points are created by the kicking method, chaos may not be controlled unless the starting point lies in the basin of attraction.

Similarly the basin of periodic points of period 8 is created which are stable with the help of a suitable computer program.

For p=8;

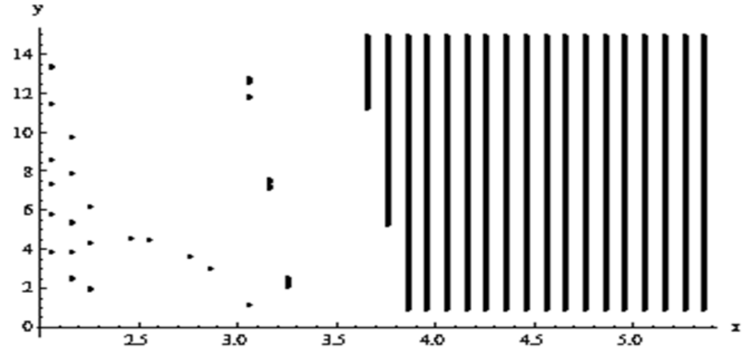

Fig:1.2.7: Above picture shows the the value of x,y which for particular value of k_1, k_2 and p=2 becomes a stable fixed point for L=3.85 and a=0.1;

III. Controlling of chaos in the map by OGY method

The OGYmethod[**16,22**] applied is as follows:

Let the two dimensional map (1.1.2) be written as :

$Z_{n+1} = f(Z_n, L)$. Let $Z_s(L)$ be an unstable fixed point of equation (1.1.2).For values of L near L_0(say) in a small neighborhood of$Z_s(L_0)$.The map can be approximated bya linear map given by

$Z_{n+1} - Z_s(L_0) = J(Z_n - Z_s(L_0)) + C(L - L_0)$……………………………………………………………….(1.3.1)

Where J is the Jacobian and C is $\frac{\partial f}{\partial L}$, at the point $Z_s(L_0)$.Assuming that in a small neighbourhood around the fixed point

$L - L_0 = -K(Z_n - Z_s(p_0))$,where K is a constant vector of dimension 2 to be determined. Then the equation (1.3.1) becomes

$Z_{n+1} - Z_s(L_0) = (J - CK)(Z_n - Z_s(L_0))$………………………………………………………………………(1.3.2)

Using equation (1.3.2) at the parameter value L=3.85, a time series plot is shown below, where after 300 iterations, chaos control is switched on.

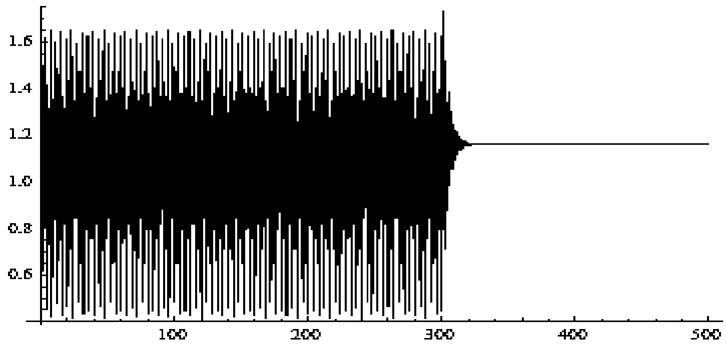

Fig: 1.3.a: Time series graph showing chaos control.

IV. Applying OGY method on Chau's method

Let (x*,y*) be a point and let the modified equation of (1.1.2) be

$$x_{n+1} = \frac{x_n e^{-a\,y_n - x_n^2}}{e^{-a\,y^* - x^{*2}}}$$

$y_{n+1} = \frac{x_n(1-e^{-a\,y_n})y^*}{x^*(1-e^{-ay^*})}$……………………………………………………………………….(1.4.1)

Which may be written as h(x,y)=(f(x,y),g(x,y)), where f(x,y)=$\frac{xe^{-a\,y-x^2}}{e^{-a\,y^*-x^{*2}}}$,g(x,y)=$\frac{x(1-e^{-a\,y})y^*}{x^*(1-e^{-ay^*})}$,clearly h(x*,y*)=(x*,y*).We consider x*=f_1(L),y*=f_2(L), which helps to calculate C = $\begin{pmatrix} \frac{\partial f}{\partial L} \\ \frac{\partial g}{\partial L} \end{pmatrix}$.Now applying equation (1.3.2) we have

$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \begin{pmatrix} x^* \\ y^* \end{pmatrix} + (J - CK)\begin{pmatrix} x_n - x^* \\ y_n - y^* \end{pmatrix}$…………………………………………….(1.4.2) Equation (1.4.2) says that $\begin{pmatrix} x^* \\ y^* \end{pmatrix}$ is a fixed point whose stability will be determined the eigen values of J-CK, where $K = \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}$.If λ_1, λ_2 be two eigenvalues then k_1, k_2 are to be determined in such a way that $-1 < |\lambda_1|, |\lambda_2| < 1$.Calculating J-CK, we have

J-CK=$\begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix}$,where

$$A_1 = 1 - 2f(L)^2 - k_1 f(L)(2f(L)f'(L) + a\,g'(L))$$
$$B_1 = -af(L) - k_2 f(L)(2f(L)f'(L) + a\,g'(L))$$
$$C_1 = \frac{g[L]}{f[L]} - k_1\left\{-\frac{g(L)f'(L)}{f(L)} + g'(L) - \frac{a\,e^{-a\,g[L]}g(L)g'(L)}{(1-e^{-a\,g(L)})}\right\}$$
$$D_1 = \frac{a\,e^{-a\,g(L)}g(L)}{(1-E^{-a\,g(L)})} - k_2\left\{-\frac{g(L)f'(L)}{f(L)} + g'(L) - \frac{a\,e^{-a\,g[L]}g(L)g'(L)}{(1-e^{-a\,g(L)})}\right\}$$

Let $a_1 = 1 - 2f(L)^2$,$a_2 = f(L)\Big(2f(L)f'(L) + a\,g'(L)\Big)$,
$b_! = -af(L)$,$b_2 = f(L)(2f(L)f'(L) + a\,g'(L))$,
$c_1 = \frac{g[L]}{f[L]}$,$c_2 = -\frac{g(L)f'(L)}{f(L)} + g'(L) - \frac{a\,e^{-a\,g[L]}g(L)g'(L)}{(1-e^{-a\,g(L)})}$
$d_1 = \frac{a\,e^{-a\,g(L)}g(L)}{(1-E^{-a\,g(L)})}$,$d_2 = -\frac{g(L)f'(L)}{f(L)} + g'(L) - \frac{a\,e^{-a\,g[L]}g(L)g'(L)}{(1-e^{-a\,g(L)})}$

Then the matrix J-CK=$\begin{pmatrix} a_1 - k_1\,a_2 & b_1 - k_2 b_2 \\ c_1 - k_1 c_2 & d_1 - k_2 d_2 \end{pmatrix}$

Whose eigen values are

$$\lambda_{1,2} = \frac{1}{2}\,(\mathrm{a}_1 + \mathrm{d}_1 - \mathrm{a}_2\mathrm{k}_1 - \mathrm{d}_2\mathrm{k}_2 \pm \sqrt{(\mathrm{a}_1 + \mathrm{d}_1 - \mathrm{a}_2\mathrm{k}_1 - \mathrm{d}_2\mathrm{k}_2)^2 + 4\left(-\mathrm{a}_1\mathrm{d}_1 + \mathrm{b}_1(\mathrm{c}_1 - \mathrm{c}_2\mathrm{k}_1) + (-\mathrm{b}_2\mathrm{c}_1 + \mathrm{a}_1\mathrm{d}_2 + \mathrm{b}_2\mathrm{c}_2\mathrm{k}_1)\mathrm{k}_2 + \mathrm{a}_2\mathrm{k}_1(\mathrm{d}_1 - \mathrm{d}_2\mathrm{k}_2)\right)}$$

Taking x*=y*=L/3,and following the above theory, we have controlled chaos at L=3.85,a=0.1.

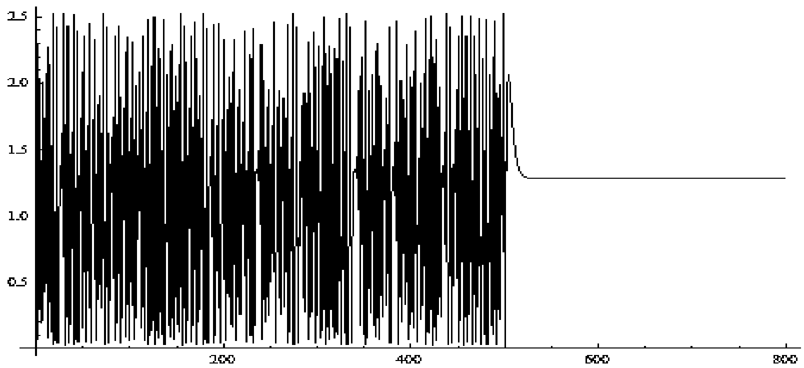

Fig: 1.4.a representing chaos upto iteration 500 and then control switch is turned on till 800 iteration converging to the stable x-coordinate L/3.

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