Heat Source Effects in Heat and Mass Transfer Of Nano Fluid Flow past a Sheet

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ABSTRACT: The Heat source/sink and suction/injection effects are studied during the Heat and Mass transfer through copper, water nano-fluid along an inclined permeable oscillating flat sheet. The governing equations are solved and the influence of various parameters is analyzed. The Rate of heat transfer for volume fraction against heat source is also analyzed.

Keywords: Nano - fluid, MHD, Inclined plate, Method of line.

List of symbols:

- \( B_0 \): Constant applied magnetic field (Wb m\(^{-2}\))
- \( C_p \): Specific heat at constant pressure (J kg\(^{-1}\) K\(^{-1}\))
- \( g \): Gravity acceleration (m s\(^{-2}\))
- \( J \): Current density
- \( M \): Dimensionless magnetic field parameter
- \( n \): Dimensionless frequency
- \( Nu \): Local Nusselt number
- \( Pr \): Prandtl number
- \( Q \): Dimensional heat source (kJ s\(^{-1}\))
- \( Q_H \): Dimensionless heat source parameter (kJ s\(^{-1}\))
- \( s \): Dimensionless suction parameter
- \( t \): Dimensionless time (s)
- \( T \): Local temperature of the nano-fluid (K)
- \( T_W \): Wall temperature (K)
- \( T_\infty \): Temperature of the ambient nano-fluid (K)
- \( u_i \): Dimensionless velocity components (m s\(^{-1}\))
- \( U_0 \): Characteristic velocity (m s\(^{-1}\))
- \( w_0 \): Mass flux velocity
- \( k \): Thermal conductivity
- \( D_f \): Diffusivity of water
- \( D_s \): Diffusivity of copper

Greek symbols:

- \( \alpha \): Thermal diffusivity (m\(^2\) s\(^{-1}\))
- \( \beta_T \): Thermal expansion coefficient (K\(^{-1}\))
- \( \beta_c \): Molecular expansion coefficient
- \( \varepsilon \): Dimensionless small quantity (<< 1)
- \( \phi \): Solid volume fraction of the nano-particles
- \( \mu \): Dynamic viscosity (Pa s)
\[ \psi \text{ \ Kinematic viscosity (m}^2\text{s}^{-1}) \]
\[ \theta \text{ \ Dimensionless temperature} \]
\[ \sigma \text{ \ Electrical conductivity (m}^2\text{s}^{-1}) \]
\[ \sigma_1 \text{ \ Stefan-Boltzmann constant} \]
\[ \delta \text{ \ Mean absorption Coefficient} \]
\[ \gamma \text{ \ Inclination angle of the plate} \]
\[ \rho \text{ \ Density} \]
\[ \nu \text{ \ Dimensionless diffusion} \]

Superscript
- Dimensional quantities

Subscripts
- \( f \) Fluid
- \( s \) Solid
- \( nf \) Nano-fluid

I. INTRODUCTION

Research in the field of Heat and Mass transfer challenging the cooling of many systems used in day to day life of mankind. The heat and mass transfer enhances enormously when nano-particles are suspended in liquids like water, ethylene glycol etc. This has substantiated by Das, Choi and Patel (2006) in their review paper. In this scenario cooling systems demand the very low heat and mass transfer rate through nano-fluids and heat mass energy systems like automobiles demanding the high heat and mass transfer rate through nano-fluids.

Kuznetsov and Nield (2010) studied the classical problem of free convection boundary layer flow of a viscous and incompressible fluid (Newtonian fluid) past a vertical flat plate in the presence of an applied magnetic field with constant heat source. We consider a Cartesian coordinate system \((x,y,z)\). The flow is assumed to be in the \( x \) direction, which is taken along the plate, and \( z \) - axis is normal to the plate. We assume that the plate has an oscillatory movement on time \( \tilde{t} \) and frequency \( \tilde{n} \) with the velocity \( u_0(t) = U_0 (1 + \varepsilon \cos(\tilde{n}t)) \), where \( \varepsilon \) is a small constant parameter \( (\varepsilon \ll 1) \) and \( U_0 \) is the characteristic velocity. We consider that initially \( (t < 0) \) the fluid as well as the plate is at rest. A uniform external magnetic field \( B_0 \) is taken to be acting along the \( z \)-axis. We consider the case of a short circuit problem in which the applied electric field \( E = 0 \), and also assume that the induced magnetic field is small compared to the external magnetic field \( B_0 \). The surface temperature is assumed to have the constant value \( T_w \) while the ambient temperature has the constant value \( T_\infty \), where \( T_w > T_\infty \). The conservation equation of current density \( \nabla J = 0 \) gives \( J_x = \text{constant} \). Since the plate is electrically non-conducting, this constant is zero. It is assumed that the plate is infinite in extent and hence all physical quantities do not depend on \( x \) and \( y \) but depend only on \( z \) and \( \tilde{t} \).
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]

It is further assumed that the regular fluid and the suspended nano-particles are in thermal equilibrium and no slip occurs between them. Under Boussinesq and boundary layer approximations, the boundary layer equations governing the flow and temperature are,

\[ \frac{\partial w}{\partial z} = 0 \] (1)

\[ \frac{\partial u}{\partial t} + w \frac{\partial u}{\partial z} = \rho_n f \frac{\partial^2 u}{\partial z^2} + \left( \rho \beta_C \right)_n f \frac{\partial T}{\partial z} + g \right( T - T_{\infty} \cos \theta + \left( \rho \beta_C \right)_n f \frac{\partial c}{\partial z} \cos \theta - \sigma B_n f^2 u \right) \] (2)

\[ \frac{\partial T}{\partial t} + w \frac{\partial T}{\partial z} = \frac{\partial^2 T}{\partial z^2} - \left( \rho c_p \right)_n f \right( T - T_{\infty} \) \) (3)

\[ \frac{\partial c}{\partial t} + w \frac{\partial c}{\partial z} = D_n f \frac{\partial^2 c}{\partial z^2} \] (4)

The appropriate initial and boundary conditions for the problem are given by

\[ u(z,t) = 0, T = T_{\infty}, c = c_{\infty} \text{ for } t < 0 \forall z \]

\[ u(0,t) = U_0 \left[ 1 + \frac{\varepsilon}{2} \left( e^{i \omega t} + e^{-i \omega t} \right) \right], T(0,t) = T_w, c(0,t) = c_w \]

\[ u(x,t) \rightarrow 0, T(\infty,t) \rightarrow T_{\infty}, c(\infty,t) \rightarrow c_{\infty}, \epsilon << 1 \] (5)

Thermo-Physical properties are related as follows:

\[ \rho_n f = (1-\phi) \rho_f + \phi \rho_s, \mu_n f = \frac{\mu_f}{(1-\phi)^2}, \alpha_n f = \frac{k_n f}{(\rho c_p)_n f} \]

\[ (\rho c_p)_n f = (1-\phi) (\rho c_p)_f + \phi (\rho c_p)_s \]

\[ (\rho \beta)_n f = (1-\phi) (\rho \beta)_f + \phi (\rho \beta)_s \]

\[ k_n f = k_f \left[ \frac{k_f + 2k_f - 2\phi(k_f-k_s)}{k_s + 2k_f + \phi(k_f-k_s)} \right] \] (6)

The thermo-physical properties (values) of the materials used are as follows.

| Table I |
|-----------------|-----------------|
| Physical Properties | Water | Copper (Cu) |
| \( C_p (J/kg \cdot K) \) | 4,179 | 385 |
| \( \rho (kg/m^3) \) | 997.1 | 8,933 |
| \( \kappa (W/m \cdot K) \) | 0.613 | 400 |
| \( \beta_f \times 10^{-5} (1/K) \) | 21 | 1.67 |
| \( \beta_c \times 10^6 (\omega^2/\lambda) \) | 298.2 | 3.05 |

We consider the solution of Eq. (1) as \[ w = -w_0 \] (7)

Where the constant \( w_0 \) represents the normal velocity at the plate which is positive for suction \( (w_0 > 0) \) and negative for blowing or injection \( (w_0 < 0) \). Thus, we introduce the following dimensionless variables:
\( z = \left( \frac{\psi_f}{U_0} \right) \psi_f, \quad t = \left( \frac{\psi_f}{U_0^2} \right)^n, \quad n = \left( \frac{U_0^2}{\psi_f} \right) \eta, \quad u = U U_0, \quad \theta = \frac{T - T_\infty}{T_\infty - T_W}, \quad c = \frac{c_r - c_w}{c_r - c_w} \quad \text{………(8)} \)

Using equations 5.6.7.8 the Esq. 2.3&4 can be written in the following dimensionless form:

\[
\left[ 1 - \phi + \frac{(\rho c_p s)}{(\rho c_p f)} \right] \left( \frac{\partial U}{\partial \tau} - \frac{S}{\delta Z} \frac{\partial U}{\partial Z} \right) = \frac{1}{\alpha} \left[ 1 - \phi + \frac{(\rho \beta f s)}{(\rho \beta f f)} \right] \left( \frac{\partial \theta}{\partial \tau} - \frac{S}{\delta Z} \frac{\partial \theta}{\partial Z} \right) - \frac{1}{\alpha} \frac{Q H}{\delta Z} \theta \quad \text{………(9)}
\]

\[
\left[ 1 - \phi + \frac{(\rho c_p s)}{(\rho c_p f)} \right] \left( \frac{\partial c}{\partial \tau} - \frac{S}{\delta Z} \frac{\partial c}{\partial Z} \right) = \frac{1}{S c} \left[ \frac{\partial^2 c}{\partial Z^2} \right] \quad \text{………(11)}
\]

Where the corresponding boundary conditions (5) can be written in the dimensionless form as:

\[
\begin{align*}
U(z,t) &= 0, \quad \theta(z,t) = 0, \quad c(z,t) = 0 \quad \text{for} \quad t < 0 \quad \forall \quad z \\
U(0,t) &= U_0 \left[ 1 + \frac{\xi}{2} e^{i\beta t} + e^{-i\beta t} \right], \quad \theta(0,t) = 1, \quad c(0,t) = 1 \quad \forall \quad t \geq 0 \quad \text{………(12)}
\end{align*}
\]

Here \( p_r \) is the Prandtl number, \( S \) is the suction \( (S > 0) \) or injection \( (S < 0) \) parameter, \( M \) is the magnetic parameter, \( Q_H \) \( (> 0) \) is the heat source parameter or \( Q_H \) \( (< 0) \) is the heat sink parameter, \( S c \) is the Schmidth number, which are defined as:

\[
p_r = \frac{\psi_f}{\alpha f}, \quad S = \frac{w_0}{U_0}, \quad M = \frac{\sigma B_0^2 \psi_f}{\rho f U_0^2}, \quad Q_H = \frac{Q \psi_f^2}{k_f U_0^2}, \quad S c = \frac{\psi_f}{d_f}
\]

Where the velocity characteristic \( U_0 \) is defined as

\[
U_0 = \left[ g \beta f (T_W - T_\infty) \right]^{1/3} \quad \text{………(13)}
\]

The local Nusselt number \( Nu \) in dimension less form:

\[
Nu = - \frac{k_{nf}}{k_f} \theta^*(0) \quad \text{………(14)}
\]

**III RESULTS AND DISCUSSIONS**

The governing equations are solved by using Method of lines with the help of Mathematica package. The variations of velocity \( U \) and temperature \( \theta \) are graphically exhibited and the Heat Transfer rate \( (Nu) \) is exhibited in Table – II for various values of \( \phi \), \( S \), \( M \), \( \alpha \), \( Q_H \) by keeping \( Pr = 6.2 \), \( nt = \pi / 2 \) and \( \epsilon = 0.02 \). The effect of various parameters is as follows.

The increase of solid volume fraction reduces the velocity Fig.1 and enhances the temperature Fig.6. The thickness of momentum and the thermal boundary layers decreases with increase in \( \phi \). From Fig.2&7 the momentum and thermal boundary layers decreases for injection or suction. From Fig. 3 & 8 the momentum and thermal boundary layers decreasing for heat sink or source \( Q_H \). The variations of velocity and temperature with magnetic parameter \( M \) are depicted in Figs. 4 & 9. The effects of inclination angle \( \alpha \) on velocity and temperature
are exhibited in Figs. 5 & 10. The increase in inclination reduces the velocity and enhances the temperature. From Fig.11 it is observe that the increase in diffusivity or the decrease in viscosity increases the velocity. The same is observed in diffusion with variation of Sc Fig.12.

![Fig.1 Variation of U with $\phi$](image1)

![Fig.2 Variation of U with S](image2)

![Fig.3 Variation of U with $Q_H$](image3)

![Fig.4 Variation of U with M](image4)

![Fig.5 Variation of U with $\alpha$](image5)

![Fig.6 Variation of $\theta$ with $\phi$](image6)
Fig. 7 Variation of $\theta$ with $S$

Fig. 8 Variation of $\theta$ with $Q_H$

Fig. 9 Variation of $\theta$ with $M$

Fig. 10 Variation of $\theta$ with $\alpha$
Nusselt Number:

\[ S = 1; \ M = 5; \ \alpha = \pi/3; \ Sc = 0.6 \]

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