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Strehl Ratio with Higher-Order Parabolic Filter

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Abstract: *In all the branches of science, engineering and technology, it is known that the output due to an input impulse function, spatial or temporal, is never an impulse. There is a spread of the input impulse function in the output due to the noise introduced by the physical device. It was Strehl who first introduced the important image-quality assessment parameter* ***"Definitionshelligkeit"*** *or simply known as the Strehl Ratio (SR) after his name. In this paper, we have studied this parameter for an optical system apodised with the higher-order super-resolving parabolic filters. The results obtained have been discussed graphically.*

Key-words: *Mathematical Optics, Higher order Parabolic Filters, Fourier Optics, Strehl Ratio ..... etc.*

I. Introduction

It is well-known that the image of a point object obtained even with a diffraction limited system is not a point. There is a spread of light flux over a considerable region of space in the focus of the image plane, the actual nature of the spread, known as the Point Spread Function is controlled by the size and shape of the aperture and the type of the non-uniformity of transmission. The importance of this was first realized by LOMMEL [1] and he developed the theory of the distribution of light at and near the focus of an optical system with a circular aperture. In the present paper, we shall present the results of our studies on one of the most important image-quality assessment parameters, the Strehl ratio which is based on the point-spread function of the optical system and is apodised with higher-order parabolic filters. Initially, the Strehl ratio was introduced as **"Definitionshelligkeit"** by its originator Strehl himself. In its original nomenclature, the term **"definition"** was used to mean **"distinctness"** of an outline or **detail** in the image.

II. Previous Studies on Strehl-Ratio

Strehl ratio[2,3] is an important quality assessment parameter for imaging systems and its maximization by the use of amplitude filters has been attempted by several workers. BARAKAT [4], in his study on solutions to Lunenburg's apodization problems, investigated the Strehl ratio for both circular and slit apertures. It is not a physically measurable quantity in the strict sense of the word but nevertheless is a common measure of theoretical performance of the system. WILKINS [5], while solving the modified Lunenburg apodization problems discussed the Strehl ratio. BARAKAT and HOUSTON [6] computed Strehl ratio for an annular aperture possessing third-order and fifth- order spherical aberration. They have adopted the approach of MARECHAL [7] to minimize the mean square deviation of the wave front and hence maximize the Strehl ratio.

DEVELIS [8], in his study of comparisons of methods of evaluation, discussed the Strehl ratio and its relation to Marechal tolerance. HOPKINS [9] stated that for highly corrected optical systems, that is those substantially satisfying the Rayleigh quarter-wave criterion, the Strehl ratio may be used as diffraction based criterion of image quality. Strehl ratio, for circular apertures with a ring- shaped π - phase change, has been investigated by ASAKURA and MISHINA [10]. This work has been extended by ASAKURA and NAGAI [11] to modify annular and annulus apertures. It has been found that the Strehl ratio is always reduced in comparison with that of a clear aperture as long as the semi-transparent and phase annulus aperture is used.

KUSAKAWA [12] has studied the problem of finding the pupil function which minimizes the **dispersion factor** (Excluded energy), subject to the condition that the Strehl ratio, must have a certain prespecified value. The relation between the minimum obtainable **second order-moment** and the prespecified Strehl ratio has been discussed by them. HAZRA [13] studied the problem of maximization of Strehl ratio for the more general case of partially space-coherent illumination. Hazra restated the criterion of "**maximization of Strehl ratios"** as the criterion of **"maximization of effective central illumination within a circle of infinitesimally small radius around the centre of the diffraction pattern"**. The apodization problem of finding the diffraction pattern has specified Sparrow limit of resolution and the maximum possible Strehl criterion has been solved by PENG and WILKINS [14], for both incoherent and coherent illumination, respectively. MAHAJAN [15] calculated the Strehl ratio, quite accurately from the phase aberration variance. KIBE and WILLIAMS [16] have studied Strehl ratio for a specified Rayleigh limit and for maximum central irradiance. McCUTCHEN'S theorem has been used by LOHMANN and OJEDA CASTANEDA [17], to derive the condition for axial symmetry and periodicity of Strehl ratio, which may serve as a focus criterion.

Formulae for estimating the Strehl, coefficient in the presence of third and Fifth-order aberrations as well as defocusing have been obtained by GRAMMATIN and OKISHEVA [18]. RAMNATHAN [19] examined the effect of Kaiser Pupils on the Strehl ratio. MURTY [20], used co-sinusoidal filters and investigated the influence of apodization and defocusing, with both circular and annular apertures on Strehl ratio. SURENDAR [21] has evaluated the Strehl ratio for apodised optical systems, circular and annular, using Lanczo's filters and determined that apodisations in combination with obscuration further lowers the Strehl ratio. KARUNASAGAR [22] has evaluated the Strehl ratio for both circular and annular apertures apodised with generalized Hanning filters for the first, second, third and the fourth orders of the filter considered. A good account and a comprehensive review on Strehl ratio can be found in the reference [23].

III. Definition of Strehl ratio

STREHL suggested the use of the **relative intensity** of the diffraction as a measure of the image quality. **The strehl ratio (SR)** is defined as the ratio of the central intensity of the PSF of the system and that of the uniform pupil function for diffraction limited system.

$$SR = \frac{I_p(0,0)}{I_A(0,0)} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

Where the subscripts P and A referred to the parabolic and Airy pupils respectively. I_p (0,0) represent the intensity point spread function at centre (0,0) of the diffraction pattern due to the optical system used and I_A (0,0) represent the same for the diffraction-limited perfect system. According to the above expression for SR can be written in terms of respective pupil function as follows. Therefore,

$$SR = \frac{\left|G_p(0,0)\right|^2}{\left|G_A(0,0)\right|^2} \quad \ldots\ldots\ldots\ldots\ldots (2)$$

Where the symbol G_P and G_A stand for the point spread function for the actual optical system used and the perfect system respectively, thus,

$$SR = 4\left[\int_0^1 f(r)\, r\, dr\right]^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Where f(r)= $(\alpha + \beta r^2)^N$ where N=2,3,4&5.

Thus finally,

$$SR = 4\left[\int_0^1 (\alpha + \beta r^2)^N r\, dr\right]^2 \quad \ldots\ldots\ldots\ldots\ldots (4)$$

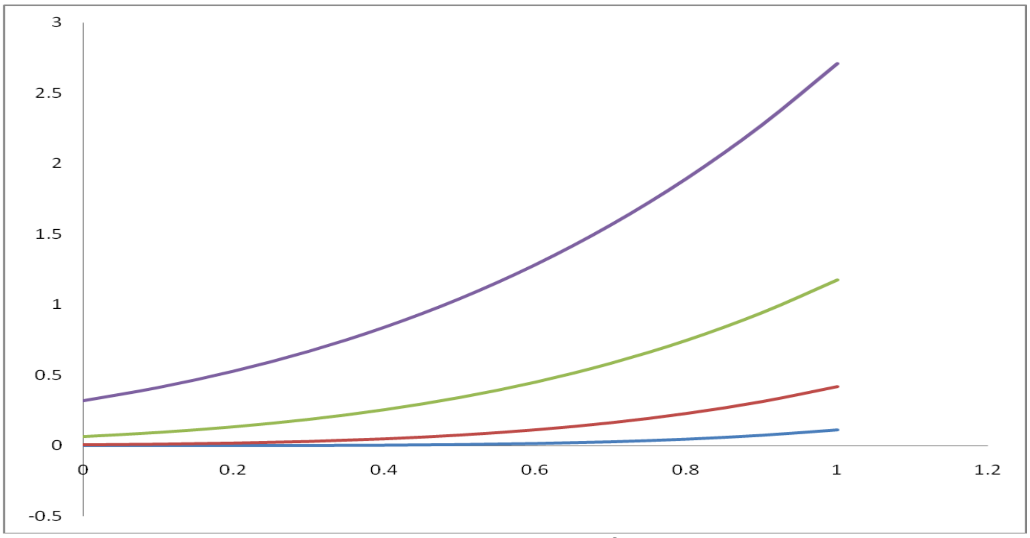

Fig:1 Second order: Variation of SR with β for α=0, 0.25, 0.5& 0.75

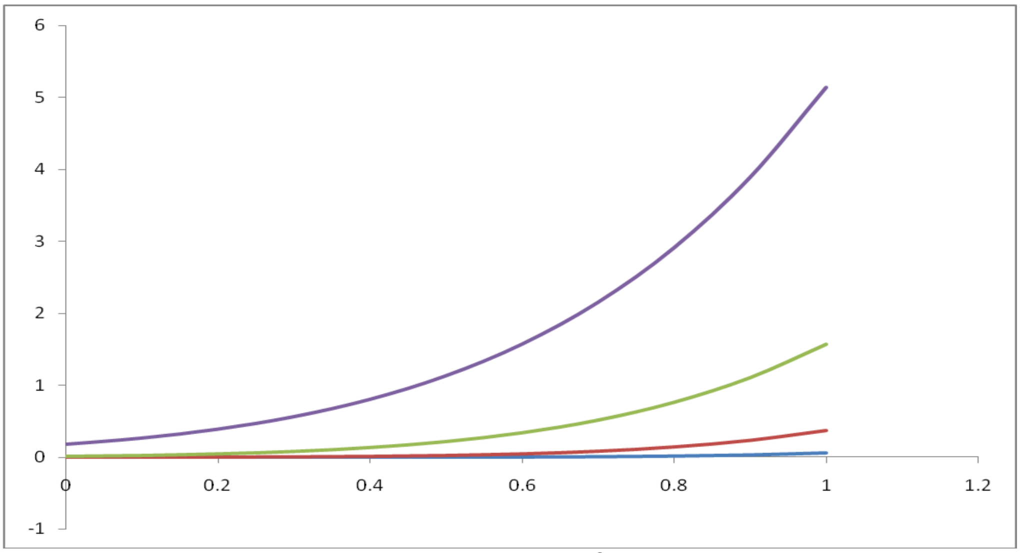

Fig:2 Third order: Variation of SR with β for α=0, 0.25, 0.5& 0.75

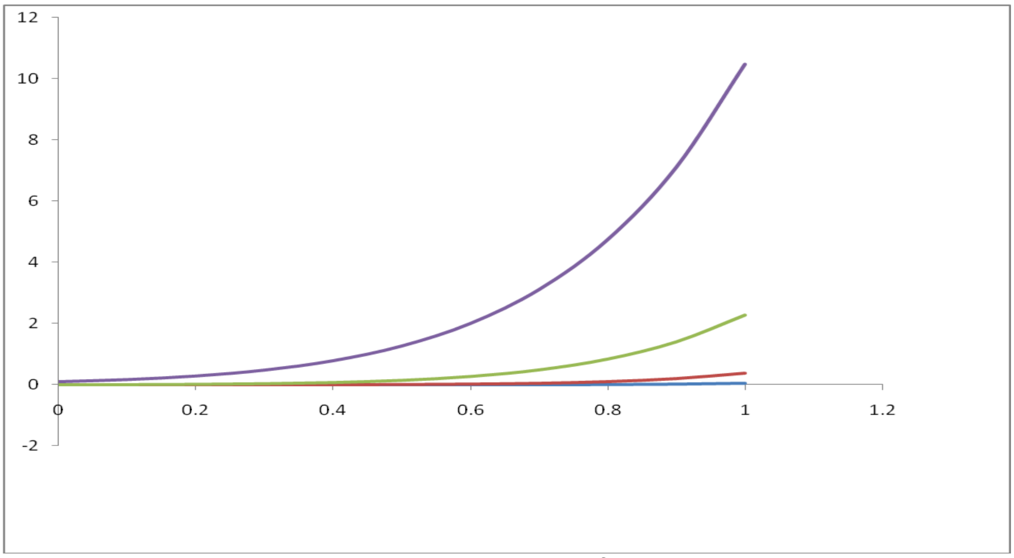

Fig:3 Fourth order: Variation of SR with β for α=0, 0.25, 0.5& 0.75

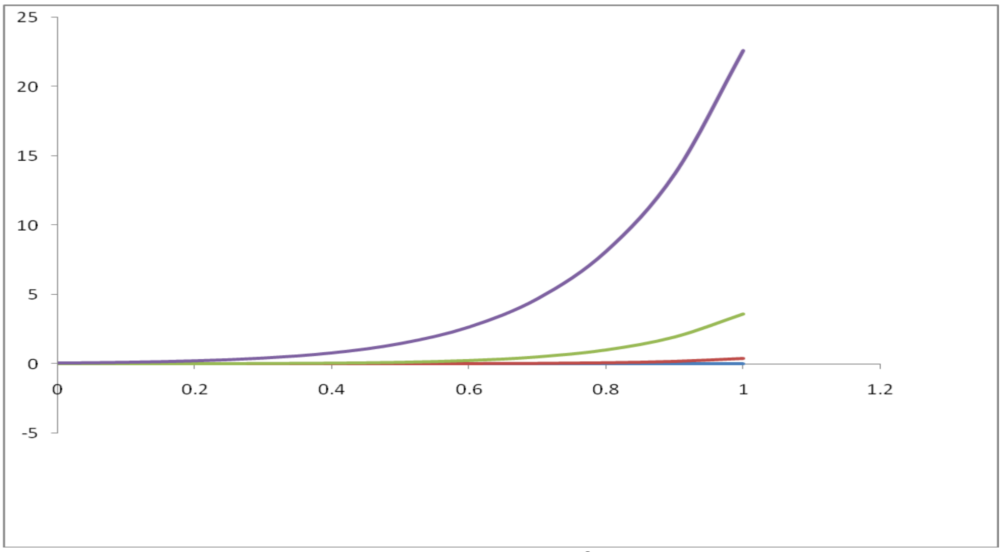

Fig:4 Fifth order: Variation of SR with β for α=0, 0.25, 0.5& 0.75

IV. Results and Discussions

Fig 1 to 4 represents the SR curves for values of $\alpha = 0, 0.25.0.5 \& 0.75$ with β=0 to 1 with deference 0.1. It is observed from the figures that for all the values of α, as the values of β are increased, over all curves increases, maintaining of course, their super-resolving parabolic shape. The most important feature to be observed in this figure is that the order increasing then all the curves starts from origin i.e. for second order the curves start from deferent values and the fourth and fifth order the curves start from origin. If the order increasing the α =0 curve coincide with the β -axis.

In the figure 1,,2,3&4 we have shown variation of Strehl ratio with various values of apodisations parameter and for various values of the D.C.bias α =0,0.25,0.5,0.75 it is observed from the figures that the various SR curves for all the values of α are parabola curves. These curves can therefore, be mathematically represented by the following equation.

$$SR=\alpha + \beta r^2 \quad \text{..........(5)}$$

Where m is the slope of SR curve and α is its intercept on the SR-axis. The important point to be mentioned here is that the effect of β on the SR values is quantitatively different. Quantitatively, however, the SR values depend prominently on the α values. Because, higher is the value of α the Strehl ratio values are quantitatively higher than those for lower values of α. However, it must be pointed out that we can not increase the value of α indefinitely in order to keep the over-all value of f(r) ≤ 1, in order to satisfy the fundamental passivity condition of an optical system.

REFERENCES

[1]. LOMMEL, E., Abh.Bayer Akad., vol.15, Abth.2, 1885
[2]. STREHL, K., (1895), Z.f Instrummkede, 15, 364
[3]. STREHL, K., (1902), Z., f Instrumkde, 22, 213
[4]. BARAKAT, R. (1962), J. Opt, Soc. Am., 52, 264, 276 & 985.
[5]. WILKINS, J. E., Jr., (1963), J, Opt, Soc, Am., 53, 420.
[6]. BARAKAT, R. and HOUSTON, A., (1963), J. Opt. Soc. Am., 53, 1244.
[7]. MARECHAL, A., (1947), Rev, Opt., 26, 257
[8]. DEVELIS, J.B., (1965), J. Opt. Soc. Am., 55,165
[9]. HOPKINS, H. H., (1966), Opt, Acta. 13, 343.
[10]. ASAKURA, T., MISHINA, H., (1970), Jap. J. Appl. Phys., 9,195
[11]. ASAKURA, T., NAGAI, S., (1971), J, Appl, Phys., (Japon), 10,879.
[12]. KUSAKAWA, T., (1972), Jap, J, Appl, Phys., 11,1632.
[13]. HAZRA, L.N., (1975), j.Opt, (India), 4, 51.
[14]. PENG, W. P., WILKINS, J.E., Jr, (1975), J, Opt, Sci, Am., 65, 1292
[15]. MAHAJAN, V.N., (1983), Appl, Opt., 22 3035

[16]. KIBE, J.N., WILKINS, J.E., Jr., (1984), J, Opt, Soc, Am, A., 1,337.
[17]. LOHMANN, A.W., OJEDA-CASTNEDA, J, (1984), Opt, Acta, 31, 603
[18]. GRAMMATIN, A.P., OKISHEVA, E.V., (1986), Sov, J, Opt, Techno (USA), 53,460
[19]. RAMANATHAN, S., (1986), Ph.D, Thesis, p, 41. Osmania Univ, Hyderabad.
[20]. MURTY, P.V.V.S., (1992), p.32, in Thesis entitled " Studies on Diffracted Field and Imaging Characteristics of Optical Systems with Co sinusoidal Apodisations Filters", presented to Osmania University, for Ph.D.
[21]. SURENDAR, K., (1993), P, 69, Ph.D Thesis, "Studies on Diffracted Field and Image Characteristics of Optical Systems Apodised with Lanczo Filters" SURENDAR, K., GOUD, S, L., MONDAL, P.K., (1992), Acta ,Cienca Indica 18, P, 6134
[22]. KARUNASAGAR, D., (2003), p, 62, Ph.D Thesis: "Studies on the Performance of Optical Systems Apodised with Generalized Hanning amplitude Filters"
[23]. RATHNAM, C., "Fourier Analytical Investigations on the performance of multiple-Annuli coded Aperture in multiplexed Tomography", Ph.D thesis, Osmania University, Hyderabad, A.P, India, 2005.
[24] **P Thirupathi 1** , Sreehari Pagidipally 2 "Strehl Ratio of Point Spread Function with First-Order Parabolic Filter" International Journal of Engineering Research & Technology (IJERT) ISSN: 2278-0181 Vol. 3 Issue 3, March – 2014.