Numerical Investigation of Multilayer Fractal FSS

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Abstract: Numerical investigations are presented for a multilayer frequency selective surface with Koch fractal (levels 1 and 2) conducting patch elements. The structure investigated is obtained using two FSS screens separated by an air gap layer. For the proposed investigation were used three different values an air gap height. The results obtained using the numerical method were compared with other technique and using the commercial software Ansoft Designer™. A good agreement was observed in terms of the bandwidth.

Keywords: FSS, Numerical Method, Multilayer Structure, Fractal, Wideband

I. INTRODUCTION

A periodic surface can be defined basically as a set of identical elements arranged in two or three dimensions forming an infinite array. A periodic array formed by conductive patch elements or opening elements is known as Frequency Selective Surface (FSS). FSS are periodic structures that can provide frequency filtering to incoming electromagnetic waves, and their frequency response is entirely determined by the geometry of the structure in one period called a unit cell [1, 2].

Traditional FSS has a long history of development and have been investigated over the years for a variety applications, e.g., frequency filters or diplexers in high performance reflector antenna systems, advanced radome designs, and smart surfaces for stealth applications [3].

The question of operating bandwidth is one of important problems in FSS theory, in some applications a multiband frequency response is desired; however in other applications a wideband frequency response is more preferred. FSS structures have been successfully proven as a mean to increase the communication capabilities of satellite applications. In space mission (Voyager, Galileo and Cassini), for example, the use of dual antenna system with FSS reflectors has made it possible to share the main reflector among different frequency bands, therefore FSS with multiband responses have been studied by several researchers. But, in some applications, such as separating successive frequency bands, FSS are required to have a wide transmission or reflection bands and rapid rolloffs [4-10].

Experimental and numerical investigations on cascading or multilayer FSS have been done, using full and simple (approximate methods) [11-14]. This paper presents a numerical investigation of multilayer or cascaded FSS using a simple but efficient method used to predict the frequency response (transmission coefficient) for these structures. In this work was analyzed a multilayer FSS structure, using conducting Koch fractal elements. This structure was first proposed in [15] and consists of two FSS, called structure 1 and structure 2 mounted on a dielectric isotropic layer, separated by an air gap layer. The dielectric substrate used was the RT-Duroid 3010, with 1.27 mm of height and relative permittivity equal to 10.2 and unit cell periodicity (Tₓ = Tᵧ) equal to 10 mm. We used the commercial software Ansoft Designer™ to obtain the individual scattering parameters of each one FSS structure. To validate the results a comparison with results from other numerical method is performed.

II. FSS THEORY

Through the use of the spectral domain approach to analyzing the response of an FSS, which is assumed to have infinitesimal thickness, one can determine the electromagnetic fields on a plane z = zn, see Fig. 1 and Fig. 2. These electric fields are expressed in terms of a discrete spectrum of plane waves known as Floquet harmonics.
Cwik in [16] has defined a set of scattering parameters that incorporates the vector nature of electromagnetic fields as follows [1]:

\[
S_{11}(m,n,i,j) = \frac{V^{(+,-)}(k_{xm}, k_{yn}, k_{xj}, k_{yj}, z_L) \sqrt{P_{m+}(i,j)}}{P_{m}(i,j)} \\
S_{12}(m,n,i,j) = \frac{V^{(-,+)}(k_{xm}, k_{yn}, k_{xj}, k_{yj}, z_R)}{\sqrt{P_{m}(i,j)}} \\
S_{21}(m,n,i,j) = \frac{V^{(+,-)}(k_{xm}, k_{yn}, k_{xj}, k_{yj}, z_L) \sqrt{P_{m}(i,j)}}{P_{m+}(i,j)} \\
S_{22}(m,n,i,j) = \frac{V^{(-,+)}(k_{xm}, k_{yn}, k_{xj}, k_{yj}, z_R)}{\sqrt{P_{m+}(i,j)}}
\]

where \(z_R\) is the above interface and \(z_L\) is the below interface in Fig. 1, i.e., \(z_R = 0\) and \(z_L = z_{p+q}\). \(V^{\pm,\pm}\) are Floquet voltage waves given by [1]:

\[
V^{(\pm,\pm)}(k_{xm}, k_{yn}, k_{xj}, k_{yj}, z_R) = \tilde{f}(m,n,i,j,z)^{(\pm,\pm)} \sqrt{P(m,n)},
\]
With,

\[ P(m,n) = (k_{xm}^2 + k_{yn}^2)Y_{mn}, \]

Where:

\[ Y_{mn} = \frac{\gamma_{mn}}{j\omega\mu}, \]

The terms \( K_{xm} \) and \( K_{yn} \) (wave numbers of Floquet special harmonics) are defined as:

\[ k_{xm} = \frac{2\pi}{T_x}m + k_{inc}^x, \]

\[ k_{yn} = \frac{2\pi}{T_y}n + k_{inc}^y, \]

The incident harmonics are functions of elevation angle and the azimuthal angle, which are given as [1]:

\[ k_{inc}^x = k_{inc}^x\sin\theta, \]
\[ k_{inc}^y = k_{inc}^y\sin\phi, \]

Referring to the coordinate system of Fig. 1, can be seen that the notation \((\pm, \pm)\) associated with the definition for the voltage waves indicates the direction, both \(+z\) or \(-z\), of the incident and scattered energy in a FSS system. Regarding the scattering parameters, it is more convenient to express the potential vector, used in the definition of the Floquet voltage waves in terms of the electric fields transform.

The potential vector, expressed in terms of total electric fields, can be written as [1]:

\[ f(m,n,i,j,z) = \frac{j(k_{mn}E_{x_{mn}}^{z_{mn}} - k_{mn}E_{x_{mn}}^{z_{mn}})}{k_{xm}^2 + k_{yn}^2}, \]

The total electric field can be found as follows [1]:

\[ E_{(x,y)_{mn}}^{(z_{mn})} = E_{(x,y)_s}^{(x,y)_s} + E_{(x,y)_t}^{(x,y)_t} \delta(k_{x}^{x} - k_{xm}^{x})\delta(k_{y}^{y} - k_{yn}^{y}), \]
\[ E_{(x,y)_{mn}}^{(z_{mn})} = E_{(x,y)_s}^{(x,y)_s} + E_{(x,y)_t}^{(x,y)_t} \delta(k_{x}^{x} - k_{xm}^{x})\delta(k_{y}^{y} - k_{yn}^{y}), \]

Where \( E_{(ref, trans)}^{(x,y)} \) are the reflected and transmitted electric fields, calculated in \( z = 0 \) and \( z = z_{p+q} \). We can write the scattering parameters in terms of the electric field evaluated at the reference planes above \((z_p)\) and below \((z_L)\) of a FSS:

\[ S_{11}(m,n,i,j) = \frac{\sqrt{Y_{mn}}}{Y_{ij}} \frac{j(k_{mn}E_{x_{mn}}^{z_{mn}} - k_{mn}E_{x_{mn}}^{z_{mn}})}{(k_{xm}^2 + k_{yn}^2)(k_{x}^2 + k_{y}^2)}, \]
\[ S_{12}(m,n,i,j) = \frac{\sqrt{Y_{mn}}}{Y_{ij}} \frac{j(k_{mn}E_{x_{mn}}^{z_{mn}} - k_{mn}E_{x_{mn}}^{z_{mn}})}{(k_{xm}^2 + k_{yn}^2)(k_{x}^2 + k_{y}^2)}, \]
\[ S_{21}(m,n,i,j) = \frac{\sqrt{Y_{mn}}}{Y_{ij}} \frac{j(k_{mn}E_{x_{mn}}^{z_{mn}} - k_{mn}E_{x_{mn}}^{z_{mn}})}{(k_{xm}^2 + k_{yn}^2)(k_{x}^2 + k_{y}^2)}, \]
\[ S_{22}(m,n,i,j) = \frac{\sqrt{Y_{mn}}}{Y_{ij}} \frac{j(k_{mn}E_{x_{mn}}^{z_{mn}} - k_{mn}E_{x_{mn}}^{z_{mn}})}{(k_{xm}^2 + k_{yn}^2)(k_{x}^2 + k_{y}^2)} \]
III. NUMERICAL METHOD

Since scattering parameters (scattering matrix) of finite dimension for a FSS are computed, many analytical procedures are available for obtaining multilayer composite representation. It is possible to make direct use of the scattering matrix along with the following equation to obtain the representation of a system with two FSS (multilayer or cascaded FSS), each FSS (individual structure) is viewed as a subsystem [1]. The scattering matrix for the multilayer FSS, using two FSS, is given by [17]:

\[
S^C = \begin{bmatrix}
S_{11}^{(1)} + S_{12}^{(1)}PTS_{11}^{(2)}PS_{21}^{(1)} & S_{12}^{(2)}PTS_{12}^{(2)}
S_{21}^{(2)}P(S_{21}^{(1)} + S_{22}^{(1)}PTS_{11}^{(2)}PS_{21}^{(1)}) & S_{21}^{(2)}PS_{22}^{(1)}PTS_{12}^{(2)} + S_{22}^{(2)}
\end{bmatrix},
\]

Where \(S_{11}^{(1)}, S_{12}^{(1)}, S_{21}^{(1)} \text{ and } S_{22}^{(1)}\) are the scattering parameters which represent the first subsystem, namely, the first FSS, \(S_{11}^{(2)}, S_{12}^{(2)}, S_{21}^{(2)} \text{ and } S_{22}^{(2)}\) are the scattering parameters associated with the second subsystem, and \(S^C\) is the scattering matrix for the system composed by two FSS (multilayer FSS).

And \(T\) is given by [17]:

\[
T = (I - S_{11}^{(2)}PS_{22}^{(1)}P)^{-1},
\]

\(P\) is a diagonal matrix that has \(e^{-j2kd}\) as its elements, \(k\) is the wave number [45], in this work \(k = k_0\) (because the multilayer FSS is formed by two single FSS separated by an air gap layer), and \(I\) is an identity matrix.

To validate the results, that will be presented, a comparison with results from other numerical method called “One Mode Interaction Technique” that can be seen in [17] and obtained by Method of Moments (MoM) using the commercial software Ansoft Designer\textsuperscript{TM} is performed. The numerical results were computed using the commercial software MATLAB\textsuperscript{TM} (MATrix LABoratory).

Using the One Mode Interaction (OMI) technique, the transmission \((C_T)\) and reflection \((C_R)\) coefficients for the multilayer FSS, in this case formed by two single FSS, are [18]:

\[
C_T = \frac{T_1T_2}{1 - R_1R_2e^{-j2kd_1}},
\]

\[
C_R = R_1 + \frac{T_1^2R_2}{1 - R_1R_2e^{-j2kd_1}}e^{-j2kd_2},
\]

Where \(d_1\) is the spacing between the two structures. \(T_1, R_1\) are the transmission and reflection coefficients for the first FSS, and \(T_2, R_2\) are the transmission and reflection coefficients for the second FSS.

IV. MULTILAYER FSS STRUCTURE

The multilayer FSS used in this work consists of two FSS screens called structure 1 and structure 2, respectively, each one using Koch fractal patch elements printed on a dielectric substrate separated by an air gap layer. The first structure with Koch fractal level 1 is defined as structure 1 and the second FSS screen with Koch fractal level 2 is defined as structure 2, as can be seen in Table 1 with their respective resonant frequencies \(f\) and bandwidths \(BW\). The process to generate the Koch fractal elements used in this work can be seen in [14]. The element shapes and the multilayer structure considered in our investigation are shown in Fig. 3 and Fig. 4, respectively.

<table>
<thead>
<tr>
<th>Structure</th>
<th>Element type</th>
<th>(f) (GHz)</th>
<th>(BW) (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Koch fractal level 1</td>
<td>9.115</td>
<td>1.951</td>
</tr>
<tr>
<td>2</td>
<td>Koch fractal level 2</td>
<td>8.333</td>
<td>1.618</td>
</tr>
</tbody>
</table>
V. RESULTS

In order to validate the results obtained with the numerical method presented before, they are compared with results obtained with the OMI technique and the results obtained using the commercial software Ansoft Designer™, the latter uses the full wave method, MoM, to compute its results.

Fig. 5 illustrates the results obtained with the numerical method for the case with an air gap height equal to 2.0 mm. For the multilayer FSS with an air gap height equal to 2.0 mm, the results obtained using the numerical method present a bandwidth, for 20 dB insertion loss reference, equals to 3.420 GHz, a good agreement is observed between the results.

For the multilayer FSS with an air gap height equal to 4.0 mm, the results obtained using the numerical method presents a bandwidth, for 20 dB insertion loss reference, equal to 3.360 GHz, approximately. Fig 6. illustrates a comparison between the results.

Using an air gap height equal to 6.0 mm, the numerical method present a bandwidth, for the 20 dB insertion loss reference, equal to 3.290 GHz, approximately.

![Transmission coefficient for the multilayer FSS structure with an air gap height equal to 2.0 mm.](image-url)
VI. CONCLUSION

In this paper, a multilayer FSS was investigated using a simple numerical method. This multilayer structure was formed by two FSS screens separated by an air gap layer, each one FSS screen using a conducting patch element with fractal geometry in the unit cell. The results were obtained for the numerical method and compared with other numerical technique called “One Mode Interaction”. Moreover, both were compared with the results obtained using the commercial software Ansoft DesignerTM for different values of spacing between the FSS screens. The numerical method is efficient and can be used in conjunction with other methods, like full wave methods such like: Finite Element Method, Wave Concept Iterative Procedure, and Equivalent Circuit Model, among others. For all the cases considered, the frequency responses have characteristics of a high-pass filter with a very large rejection band. A good agreement between the results was observed in terms of bandwidth.

REFERENCES

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