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Sequential Imperfect Preventive Maintenance Policy of a Deteriorating Repairable System based on Age and Hazard Rate Correction factors

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Abstract: *This paper presents a mathematical model for a sequential imperfect preventive maintenance policy which considers stochastic factors affecting the failure rate and service life of a repairable deteriorating system. The failure rate and maintenance expenses depend on a number of influencing factors, such as the age, condition of the equipment, plant capacity utilization, mode of operation as well as the applied maintenance concept. With the failure rate following the Weibull distribution, the model investigates the combination of the age reduction and failure rate increase coefficients to determine the optimal intervals between preventive maintenance schedules and overhauls in order to minimize the expected costs over an infinite time horizon. The optimal policies are presented and numerical examples are also given.*

Keywords*: Preventive Maintenance (PM), Age Reduction, Hazard rate, Correction Factors, Cost Minimization*

I. INTRODUCTION

The most commonly used maintenance models for the failure process of a deteriorating system are perfect maintenance actions which restore the system to ‘as good as new’ and imperfect maintenance which restores a system to a state between ‘good as new’ and ‘bad as old [1].The link between the two concepts is done by corrective maintenance (CM) or minimal repair which restores the system to the state it was prior to failure. In practice, the reality is between the two extremes of perfect and imperfect maintenance. The perfect maintenance may not yield a proper functioning system which is ‘as good as new’ and the imperfect maintenance seems to be too pessimistic in repair strategies as it depends on the availability of resources. Timely preventive maintenance actions can inhibit further equipment degradation and reduce the frequency of failures thereby extending its service life. Ideal maintenance policies aim to optimize system reliability, availability and safety performance at optimum costs [2],[3],[4],[5].

The optimization of maintenance for repairable systems was initiated by [6]. Since then, a number of maintenance models and strategies for maintainable systems have appeared in literature. An imperfect preventive maintenance model was introduced by[7] which described the age reduction effect and that preventive maintenance reduces the failure rate of a system to a fraction of its value just before PM. Then another model by [8] where a system undergoes PM with a different failure distribution between preventive maintenance actions was considered. An age-dependent preventive maintenance model [9] was presented and it was observed that, the selection of an appropriate maintenance strategy directly depend on the previous operating parameters.

In [10] the problem of optimal PM policies was investigated and assumed that after each repair the system’s age becomes zero(as good as new) while the failure rate increases by the increasing of the number of repairs carried out.

A periodic PM model [11] for which the maintenance slows the degradation process of a system, while the hazard rate is monotone is discussed. Authors in [12] formulated maintenance cost policy where PM is performed based on predetermined maximum failure rate, and failures are corrected by minimal repair.

Improvement factors in the hazard-rate function were adopted in [13] and analysed two corresponding imperfect PM models: PM reduces the hazard rate while it increases with the number of PM actions and that PM reduces the age. In [1] are listed methods and techniques used for modelling imperfect preventive maintenance and observed that, maintenance models would be valuable if they are capable of incorporating all factors that affect a repairable system.

A hybrid PM model comprising a combination of the hazard rate and age was proposed in [14]. They further, [15], studied the problem of optimizing PM for repairable systems with two categories of failure. A similar work was proposed by [16], they developed an age-based hybrid model for imperfect PM with the two categories of failure modes, i.e. maintainable and non-maintainable failure modes. An optimal periodic PM policy for the deteriorating system based on [11] was proposed in [17], i.e, a maintenance model where PM slows down system degradation while the failure rate remains monotonically increasing was analysed. Optimal maintenance options [18] for continuously monitored multi-component systems based on Markov deteriorating processes were studied. In [19] the author considered the imperfect effect of PM activities and gave an availability-centred preventive maintenance model for multi-unit systems, which is based on sequential PM theory.

Several preventive maintenance models that considered the effect of imperfect maintenance on effective age of a repairable system are presented in [20]. And [21] further built a model where each PM action reduces the effective age but not restoring it to an 'as good as new' state. A sequential imperfect preventive maintenance policy with random maintenance quality under reliability limit was proposed by [22]. And [23] proposed improvement factors in a sequential maintenance model. Similar work was done by [24] while considering phasic sequential PM. A stochastic maintenance policy for a degrading system over a finite period was considered by [25].

The general form of maintenance optimization analysed is either based on cost, age reduction, and failure rate or reliability/availability analysis. This paper looks at a specific function using both the concept of increasing failure rate and age reduction for a deteriorating maintainable system to determine intervals between maintenance actions in order to minimize the related maintenance costs.

II. MODEL DESCRIPTION

In [14] adopted correction factors for failure rate, $h(t)$, and time interval between preventive maintenance actions, i.e., the failure rate during the subsequent period is $ah(t)$, where $h(t)$ was the failure rate prior to the last maintenance action, $a \geq 1$ is a correction factor and $t \geq 0$ is the time after the previous maintenance action which is reset to zero at each PM. Time interval between two maintenance actions is reduced to bt after each maintenance action, where $b \leq 1$ is the reduction factor. According to [13], for a failure rate $h(t)$, $t \in (0, t_1)$, the preventive maintenance action at t_1 determines a new failure rate $g(t)$, $t \in (t_1, t_2)$, which depends on the previous failure rate and the quality of maintenance action, i.e. failure rate reduction. The proposed form of $g(t)$ uses both the concept of increasing failure rate and reduction of preventive maintenance interval according to system usage. The failure rate model assumes that the failure rate immediately after preventive maintenance action increases more quickly than it did before the PM action. The age model assumes that there is an effective age reduction immediately after the PM action, and the failure rate continues to be a function of the effective age. Based on the above observations, the model captures both by how much the effective age is reduced just after PM action and how much faster the failure rate function increases when the system is back into operation. The model failure rate function after the first PM is written as:

$$g(t_1 + x) = ah(bt_1 + x) \tag{1}$$

Where $x \in (0, t_2 - t_1)$. For $a = 1$ the model is resized to the one of life reduction, and $b = 0$ to increasing the failure rate.

Using the failure rate in (1), optimal preventive maintenance policies are developed, with a major implication, i.e. giving up the classic, non-economic approach of constant intervals of preventive maintenance execution in order to avoid costly sudden failures, with the following assumptions.

a. The repairable system under consideration deteriorates with increased usage and age and is regarded as a single-unit system without consideration of dependencies between different components. The planning horizon is infinite.
b. The equipment undergoes minimal repair upon failures between two adjacent PM intervals. The times for PM and minimal repair are negligible.
c. The imperfect PM is performed at a sequence of time t_1, t_2, …. t_{N-1} and is overhauled at t_N.
d. The failure rate of the system is increasing despite the PM actions.

In the proposed maintenance model, the following notations will be used:

-$h(t)$ is the failure rate and $H(t)$ is cumulative failure rate of the system,
-λ is the maximum value for failure rate;
-x_k, $k = 1,2, \dots, N,$ length of scheduled intervals between PM actions,
-y_k is the effective age of the system just before k^{th} preventive maintenance action,

-N is the number of preventive maintenance actions, (N^{th} action is perfect maintenance which is overhaul)
-a_k is the failure rate adjustment factor after k^{th} preventive maintenance action with
$1 = a_0 \leq a_1 \leq a_2 \leq \quad \ldots \leq a_{N-1}; \quad and \quad A_k = \prod_{i=0}^{k-1} a_i \,, k = 1,2,\ldots,N$

-b_k is effective age adjustment factor after k^{th} preventive maintenance action,
$0 = b_0 \leq b_1 \leq b_2 \leq \cdots \leq \quad b_{N-1} < 1;$
-c_m is the corrective maintenance cost; c_p is preventive maintenance cost; c_r is overhaul cost; C is average cost rate.

It is assumed that the system is subjected to preventive maintenance at times $t_1, t_2 \ldots t_{N-1}$ and overhauled or replaced at t_N. Corrective maintenance is executed if failures appear between adjacent preventive maintenance actions. The overhaul at the time t_N doesn't restore the system to 'as good as new' state. The system has a failure rate $A_k h(t)$ between the number t_{k-1} and t_k preventive maintenance actions, i.e. in the range (t_{k-1}, t_k). The effective age of the system is $b_{k-1}y_{k-1}$ immediately after the k-1 preventive maintenance action, and it becomes $y_k = x_k + b_{k-1}x_{k-1} + \ldots + b_{k-1}b_{k-2}\ldots b_2 b_1 x_1$ after the number k preventive maintenance i.e. the age changes from $b_{k-1}y_{k-1}$ to y_k in the range (t_{k-1}, t_k).

$y_k = x_k + b_{k-1}y_{k-1}$
$x_k = y_k - b_{k-1}y_{k-1}$

Given that

$x_1 = y_1$
$x_2 = y_2 - b_1 y_1$
$x_3 = y_3 - b_2 y_2$
$x_N = y_N - b_{N-1}\, y_{N-1}$

Then

$$Cycle = \sum_1^N x_k = \sum_{k=1}^{N-1}(1 - b_k)y_k + y_N \qquad (2)$$

Average cost

$$C(y_1, y_2 \ldots y_N) = \frac{c_r + c_p(N-1) + c_m \sum_{k=1}^{N} A_k [H(y_k) - H(b_{k-1}y_{k-1})]}{[\sum_{k=1}^{N-1}(1 - b_k)y_k + y_N]} \qquad (3)$$

2.1. Model A – Cost Minimization with Failure Rate limit

The objective of this model is to perform preventive maintenance action before the failure rate, λ, exceeds a prescribed value, i.e. at the time t_k (k = 1, 2, ..,N). As the system failure rate is strictly increasing, preventive maintenance intervals will also keep changing to avoid system sudden failure.
That is,

$$\lambda = A_k h(y_k), k = 1,2,\ldots,N \qquad (4)$$

Solving equation (4), we get y_k *(k = 1, 2, ..,N)* as a function of λ. Substituting (4) into (3) and differentiating with respect to $\lambda = 0, \left(i.e. \frac{\delta C}{\delta \lambda} = 0\right)$,leads to

$$\frac{\sum_{K=1}^{N-1} \frac{h(y_k) - a_k b_k h(b_k y_k)}{h(y_k)} + \frac{h(y_N)}{h(y_N)}}{\sum_{k=1}^{N-1} \frac{1-b_k}{A_k h(y_k)} + \frac{1}{A_N h(y_N)}} = \frac{C}{c_m} \qquad (5)$$

Solving (5) with respect to λ and get λ as a function of *N*, we find *N* which minimizes the *left hand side* of equation (5).
Based on the above, the preventive maintenance interval algorithm is derived as follows:
1. Solve equation (4) and find y_k with respect to λ;
2. Substitute solutions in step *1* into equation (5) and determine λ;
3. Choose N to minimize the *left hand side* of (5),where y_k *(k = 1, 2,..,N)* is from step 1 & 2;
4. Compute y_k (k = 1, 2... N) from steps 1 and 2, for the value of N from step 3;
5. Compute $x_k = y_k - b_{k-1}y_{k-1}$, k = 1, 2... N.

2.2. Model B – Cost Rate Minimization with Age Reduction

In the age-reduction model, it is assumed that the effective age of the system after the k^{th} PM is reduced to $b_k y_k$ if it was y_k before the k_{th} PM as the number of PM actions on the system increases. The decision variables are N and the maintenance ages $y_1, y_2 \ldots y_N$ *that* minimizes $C(y_1, y_2 \ldots y_N)$ in equation (3).

differentiating equation (3), with respect to y_k, $\frac{\partial C}{\partial y_k} = 0$, we get

$$A_k h(y_k) - A_{k+1} b_k h(b_k y_k) = A_N (1 - b_k) h(y_N), \; k = 1, 2, \ldots, N-1 \quad (6)$$

And $$c_m A_N h(y_N) = C \quad (7)$$

Substituting each solution y_k of equation (6) into (7), we get

$$A_N h(y_N)\left[\sum_{k=1}^{N-1}(1 - b_k) y_k + y_N\right] - \sum_{k=1}^{N} A_k \left[H(y_k) - H(b_{k-1} y_{k-1})\right] = \frac{c_r + c_p(N-1)}{c_m} \quad (8)$$

Where each y_k (k = 1, 2... N - 1) is a function of y_N.

The algorithm to find the optimal maintenance intervals is the same as in the failure rate limit model described earlier.

III. NUMERICAL EXAMPLES

The repairable system under consideration is an Ore Grinding Mill which is subject to preventive maintenance actions every 630 operating hours and follows the Weibull life distribution whose scale and shape parameters are, $\beta = 6{,}148 \times 10^{-9}$ and $\alpha = 2{,}462$ respectively.

$$h(t) = \beta t^{\alpha - 1}$$

3.1 For model A,

solving the failure rate equation (4) with respect to y_k, we have the solutions as expressions of λ:

$$y_k = \left(\frac{\lambda}{A_k \beta}\right)^{\frac{1}{\alpha-1}} \quad k = 1, 2, \ldots, N \quad (9)$$

Substituting the values of y_k in (9) into equation (5) and solve it with respect to λ, we get the values of λ with respect to N.

$$\lambda = \beta^{\frac{1}{\alpha}} \left[\frac{c_r + c_p(N-1)}{\left(1 - \frac{1}{\alpha}\right) c_m E(N)}\right]^{\frac{\alpha-1}{\alpha}} \quad (10)$$

Where $$E(N) = \sum_{k=1}^{N-1}(1 - a_k b_k^{\alpha}) A_k^{-\frac{1}{\alpha-1}} + A_N^{-\frac{1}{\alpha-1}}$$

The *left hand side* of equation (5) becomes

$$\beta^{\frac{1}{\alpha}} \left[\frac{c_r + c_p(N-1)}{\left(1 - \frac{1}{\alpha}\right) c_m}\right]^{\frac{\alpha-1}{\alpha}} \frac{[E(N)]^{\frac{1}{\alpha}}}{F(N)} \quad (11)$$

Where $$F(N) = \sum_{k=1}^{N-1}(1 - b_k) A_k^{-\frac{1}{\alpha-1}} + A_N^{-\frac{1}{\alpha-1}}$$

Minimization of equation (11) is equivalent to minimization of:

$$Q(N) = \frac{[c_r + c_p(N-1)]^{\frac{\alpha-1}{\alpha}} [E(N)]^{\frac{1}{\alpha}}}{F(N)}$$

The optimal value N* for N is found by the following inequalities:

$$Q(N+1) \geq Q(N) \text{ and } Q(N) < Q(N-1)$$

This implies: $$W(N) \geq \frac{c_r}{c_p} \; and \; W(N-1) < \frac{c_r}{c_p}$$

Where $$W(N) = \frac{[E(N+1)]^{\frac{1}{\alpha-1}} [F(N)]^{\frac{\alpha}{\alpha-1}}}{[E(N)]^{\frac{1}{\alpha-1}} [F(N+1)]^{\frac{\alpha}{\alpha-1}} - [E(N+1)]^{\frac{1}{\alpha-1}} [F(N)]^{\frac{\alpha}{\alpha-1}}} - (N-1)$$

3.2 For model B

According to the described model, equation (6) becomes:

$$A_k \beta y_k^{\alpha-1} - A_{k+1} b_k \beta (b_k y_k)^{\alpha-1} = A_N (1-b_k) \beta y_N^{\alpha-1}, \qquad k = 1,2,\ldots,N-1$$

Solving the above equation we get

$$y_k = \left[\frac{A_N(1-b_k)}{A_k - A_{k+1} b_k^{\alpha}}\right]^{\frac{1}{\alpha-1}} y_N, \; k = 1,2,\ldots,N-1 \tag{12}$$

Substituting (12) into (7) leads to

$$y_N = \frac{\left[c_r + c_p (N-1)^{\frac{1}{\alpha}}\right]}{\left\{c_m \beta \left(1-\frac{1}{\alpha}\right)\left[A_N + A_N^{\frac{\alpha}{\alpha-1}}\right] \sum_{k=1}^{N-1} d_k\right\}^{\frac{1}{\alpha}}} \tag{13}$$

where $d_k = \left[\frac{(1-b_k)^{\alpha}}{A_k - A_{k+1} b_k^{\alpha}}\right]^{\frac{1}{\alpha-1}}$

Then $A_N h(y_N)$ becomes:

$$\beta \frac{A_N\left[c_r + c_p (N-1)^{\frac{\alpha-1}{\alpha}}\right]}{\left\{c_m \beta \left(1-\frac{1}{\alpha}\right)\left[A_N + A_N^{\frac{\alpha}{\alpha-1}}\right] \sum_{k=1}^{N-1} d_k\right\}^{\frac{\alpha-1}{\alpha}}}$$

Minimization of $A_N h(y_N)$ is equivalent with minimization of the function

$$B(N) = \frac{c_r + c_p (N-1)}{A_N^{-\frac{1}{\alpha-1}} + \sum_{k=1}^{N-1} d_k}, \qquad N = 1,2,\ldots$$

Optimal value N^* for N is found solving the inequalities:

$$B(N+1) \geq B(N) \text{ and } B(N) < B(N-1).$$

The two inequalities involve:

$$D(N) \geq \frac{c_r}{c_p} \text{ and } D(N-1) < \frac{c_r}{c_p} \tag{14}$$

$$\text{where } D_N = \frac{A_N^{-\frac{1}{\alpha-1}} + \sum_{k=1}^{N-1} d_k}{A_{N+1}^{-\frac{1}{\alpha-1}} - A_N^{-\frac{1}{\alpha-1}} + d_N}$$

The optimal preventive maintenance interval has the following form $x_k = y_k - b_{k-1} y_{k-1}, k = 1,2,\ldots,N$ where y_k is given in (12) and (13). For accurate results of x_k, (k = 1, 2... N), the economic parameters of c_m, c_p and c_r are taken from the repairable system operation data. For analytical purposes, the ratios c_r/c_p *and* c_r/c_m are used.

For this analysis we use the coefficients $a_k = (6k+1)/(5k+1)$; $b_k = k/(2k+1)$, $c_r/c_p = 2, 5, 10, 20, 50$; $c_r/c_m = 4$; recommended in [14].

The two models show the reduction of maintenance intervals between two successive actions which consequently reduces the average cost in equation (3). Table 1 and Table 2 gives the optimal preventive maintenance schedules for both models and same input parameters are considered in both cases.

Table 1. Optimal PM Schedules for Model A

Cr/Cp	2	5	10	20	50
N	1	2	3	4	5
Scheduled Preventive Maintenance Intervals (hours)					
x_1	1663.3	2292.6	2619.5	3089.4	3991.3
x_2		1231.7	1484.2	1750.4	2261.5
x_3		1006.3	1159.9	1367.9	1767.3
x_4		893.5	968.4	1142.1	1475.6
x_5			828.5	977.1	1262.4
x_6			717.05	845.7	1092.6
x_7			620.6	736.6	951.7
x_8				644.1	832.1
x_9				564.6	729.4
x_{10}				495.7	640.4
x_{11}					562.9

Table 2. Optimal PM Schedules for Model B

Cr/Cp	2	5	10	20	50
N	1	2	3	4	5
Scheduled Preventive Maintenance Intervals (hours)					
x_1	1520	2599.2	2951.1	3839.9	5421.1
x_2		1386.7	1575.1	2049.4	2893.3
x_3		1079.1	1225.8	1594.9	2259.2
x_4		1466.1	1020.4	1327.7	1874.4
x_5			870.9	1133.2	1599.9
x_6			752.4	979.0	1382.1
x_7			1118.4	684.5	1202.1
x_8				597.8	1049.8
x_9				522.1	919.3
x_{10}				798.04	806.5
x_{11}					708.5
x_{12}					623.02
x_{13}					548.3
x_{14}					482.8
x_{15}					752.4

Table 1, shows the reduction of maintenance intervals with increase of the system failure rate in order to avoid sudden failure. This shows that, the studied equipment is subject to degradation during its operation, and the preventive maintenance, minimal repair and subsequent overhaul actions do not restore the system to 'as good as new' state. The failure rate and maintenance costs should be analysed at the beginning of each operation cycle depending on prevailing operating conditions.

From Table 2, it is observed that, the scheduled preventive maintenance intervals decreases for all values of x_k except the latest time element which shows an increase. The system reaches an optimum age beyond which the cost of preventive maintenance will not be economical as the intervals become shorter and the maintenance cost start increasing. This coincides with [13], "It would be reasonable to do frequent preventive maintenance with age, and it is recommended to do the last preventive maintenance as late as possible because the system should be overhauled or replaced at the last PM". This implies that, during the last operation interval, equipment can be utilized as long as it satisfies the minimum operating requirements as it awaits overhaul or replacement.

IV. CONCLUSIONS

This paper presents the ways to develop an appropriate preventive maintenance policy for a deteriorating repairable system. Two models, which minimize the expected total cost, for an Ore Grinding Mill whose failure rate is according to Weibull distribution law, were studied. The analysis combined the effect of the age reduction and the failure rate increase coefficients. Therefore, maintenance actions do not only reduce equipment life but also change the failure rate distribution. The proposed approaches provide maintenance managers with decision making techniques in the light of all available relevant information and data about the system deterioration. The outcome of the case study doesn't show significant variations between the two models. However, use of the failure rate limit model can ensure that reliability of the equipment is kept above a predetermined level to avoid the risk of sudden functional failure so that the costly repair and downtime expenses are minimized.

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