

Online Project of Adaptive Control Systems Applied to the Problem of Solar Tracking

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ABSTRACT: The development and application of a method for the online and adaptive design of control systems for the tracking of moving targets is presented in this article. Due to the characteristics of the process the indirect deterministic self-tuning regulator without zeros cancellation was chosen to track the solar irradiation and to position the photovoltaic panel to absorb this energy. In terms of a polynomial approach, the mathematical formulation is presented in the domains of discrete continuous time. Also, the parametric estimation algorithms, the pole allocation method and its minimum degree algorithm for allocation are presented. Finally, the results are compared with a PID controller and the final considerations are presented.

Keywords : Adaptive controller, Discrete time, Estimation, Photovoltaic, Self tuning regulator.

I. INTRODUCTION

Control theory has provided powerful regulator design tools for industrial processes. However, in many practical applications it is difficult to determine the controller parameters since the process and disturbance dynamics are unknown. In this way, process parameters must be estimated [1], [2], [3].

In addition, it is desirable to have a regulator that tunes its parameters in an online manner. Therefore, the designed controller must be composed of three parts: a parametric estimator, a linear controller and a block that determines the estimated process parameters used to calculate the controller parameters.

The main reason for using an adaptive controller is when the process or its environment is continually changing as it is difficult to analyze such systems. To simplify the problem, one can assume that the process has unknown parameters. An important result of the self-adjusting regulator is that the controller parameters converge to the regulator that was designed even though the model structure is incorrect [4].

In tracking control systems it is desirable for the controlled variable to follow the input signal. There are several types of systems for this purpose. One can cite trajectory tracking of manipulating robots in which the robotic arm must follow a given trajectory; Solar thermal and photovoltaic tracker in which both the concentrated solar collector and the photovoltaic collector require the use of solar trackers to increase the area exposed directly by the sun. In photovoltaic systems the solar tracker maximizes energy production. Experimental results have shown that using followers in PV modules has increased the annual energy production by more than 30% compared to the same size and fixed slope photovoltaic systems [5], [6].

The controller type applied in this search is a self-tuning controller. This type of controller automatically adjusts its parameters to obtain the desired properties in closed loop [8]. In several situations it is possible to reparametrize the process such that the model can be expressed in terms of the controller parameters [4]. After realizing the system specifications in closed loop, the parameters of the plant are estimated taking into account these estimated parameters are treated as if they were the true parameters (principle of equivalence to certainty). These estimated parameters are used in the calculation of the control law.

The article is organized into sections that present the proposed methodology, its development, the parametric estimative and adaptive algorithms, together with the results obtained.

In Section 2 the description of the problem with its mathematical formulation is presented in both the continuous time domain and the discrete time domain. In Section 3 the parametric estimation algorithm, the self-adjusting regulator, the pole-allocation design, and the minimum-degree pole-allocation algorithm are presented. Also in this section we present the PID controller that is used for comparison purposes with the STR. Finally, in Section 4, the results obtained with this work are shown and in Section 6 the final considerations are presented.

II. PROBLEM CHARACTERIZATION

In this section, the text is developed around the problem of tracking the solar irradiation and to position the photovoltaic module. The description of the same is presented with its mathematical formulation. In order to

carry out the self-adjusting control of the solar tracker, it is necessary to know the structure of the plant to be controlled and its digital equivalence in order to estimate its parameters and to calculate the control law.

2.1. Problem Description

The methodology presented in this article is applied in a solar tracking system in order to maintain the surface of the PV module perpendicular to the solar incident rays. The dynamic system, represented by the actuator, PV and sensor can be seen in Fig. 1. The actuator of the system is a direct current motor and its armature consists of permanent magnet. The element responsible for measuring the angular position of the PV is a potentiometer coupled to the PV axis.

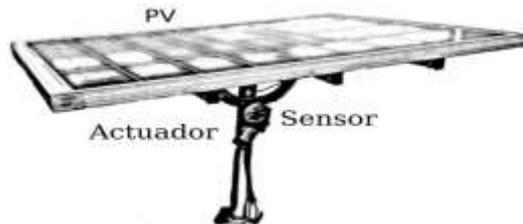


Figure 1: System overview.

For control purposes the PV is modeled as a load on the motor shaft. In Fig. 2, this consideration is taken into account.

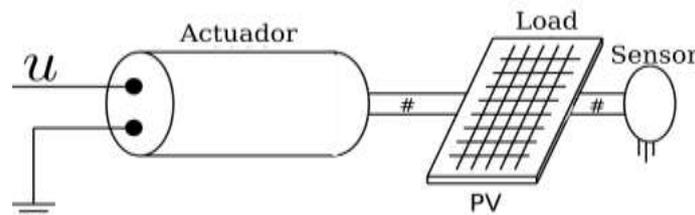


Figure 2: PV as load on the motor shaft.

2.2. Mathematical Formulation of the Problem

The actuator-load assembly model is shown in Fig. 3. Since the actuator armature consists of a permanent magnet, the field circuit is considered constant and the input voltage is applied to the armature circuit.

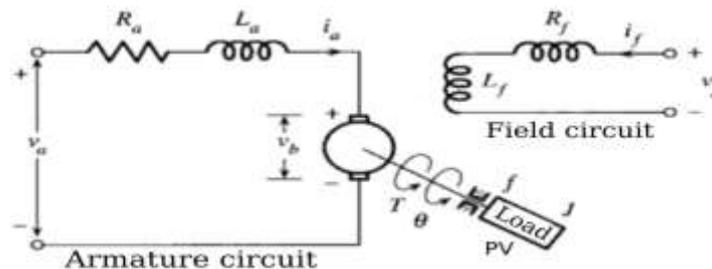


Figure 3: Actuator-load model.

If i_f is constant, the torque applied to the load is given by

$$T(t) = k_t i_a(t) \tag{1}$$

where $k_t = k_i i_f(t)$ a constant. Considering J as the total moment of inertia of the load, axle and motor rotor; θ , the angular displacement of the load; and f , the coefficient of viscous friction of the bearing. So we have to

$$T(t) = J \frac{d^2 \theta(t)}{dt^2} + f \frac{d\theta(t)}{dt} \tag{2}$$

which describes the relationship between the motor torque and the angular displacement of the load.

When the actuator is moving the load, a voltage $v_b(t)$ against electromotive force (fcem) appears in the armature circuit which, in turn, opposes the applied voltage. The voltage $v_b(t)$ is proportional linearly to the angular velocity of the actuator axis:

$$v_b(t) = k_b \frac{d\theta(t)}{dt} \tag{3}$$

Thus the armature circuit of Fig. 3 is described by

$$R_a i_a(t) + L_a \frac{di_a(t)}{dt} + v_b(t) = v_a(t) \tag{4}$$

or

$$R_a i_a(t) + L_a \frac{di_a(t)}{dt} + k_b \frac{d\theta(t)}{dt} = u(t) \tag{5}$$

using (1), (2) and (5), we can find the transfer function of the actuator. The substitution of (1) in (2), and the application of the Laplace transform in (2), assuming zero initial conditions,

$$k_t I_a(s) = Js^2\Theta(s) + fs\Theta(s) \tag{6}$$

$$U(s) = R_a I_a(s) + L_a s I_a(s) + k_b s \Theta(s) \tag{7}$$

The elimination of I_a from these two equations results in

$$\bar{G}(s) = \frac{\Theta(s)}{U(s)} = \frac{k_t}{s[(Js + f)(R_a + L_a s) + k_t k_b]} \tag{8}$$

this is the transfer function from $v_a = u$ to θ .

Using (1), (3), (4) and (6) one can draw the block diagram for the DC motor controlled by the armature as shown in Fig. 4.

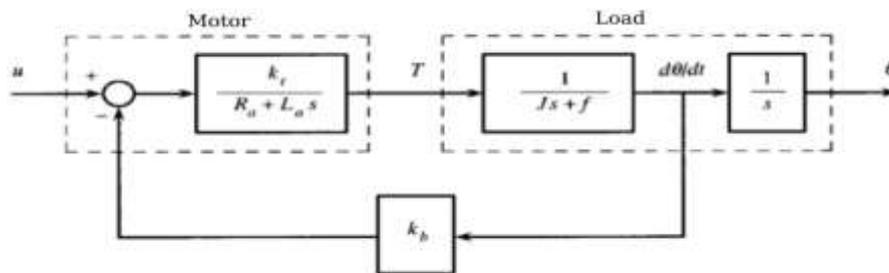


Figure 4: Actuator-load blocks diagram.

In applications, the armature inductance L_a is brought to zero [8]. In this case (8) is reduced to

$$\bar{G}(s) = \frac{\Theta(s)}{U(s)} = \frac{k_t}{s(JR_a s + k_t k_b + fR_a)} = \frac{k_m}{s(\tau_m s + 1)} \tag{9}$$

where

$$k_m = \frac{k_t}{k_t k_b + fR_a} : \text{Gain constant} \tag{10}$$

and

$$\tau_m = \frac{JR_a}{k_t k_b + fR_a} : \text{Time constant} \tag{11}$$

The time constant in (11) depends on the load and the armature circuit. Equations (8) and (9) are often called the transfer function of the motor, although it is effectively the function of transferring the motor and load.

2.3. Model Discretized

Then, the digital equivalent of the PV-axis assembly must be found, since the digital compensators generate a sequence of numbers that must be later transformed into an analog signal to add to the analog plant. This conversion is called the zero-order hold (ZOH).

The transfer function of the zero-order hold is given by

$$G_h(s) = \frac{1 - e^{-Ts}}{s} \tag{12}$$

In digital control a plant is always connected to a zero-order hold (D/A conversion). The transfer function of the discretized model, considering the dynamics of the zero order hold, is given by

$$G(z) = \frac{z-1}{z} \mathcal{Z} \left[\frac{\bar{G}(s)}{s} \right] \tag{13}$$

Multiplying the numerator and denominator of (9) by $1/\tau_m$, one has to

$$\bar{G}(s) = k_m \frac{1/\tau_m}{s(s + 1/\tau_m)} \tag{14}$$

and

$$\frac{\bar{G}(s)}{s} = k_m \frac{1/\tau_m}{s^2(s + 1/\tau_m)} \tag{15}$$

The discrete transfer function of (15) is given by

$$\mathcal{Z} \left[\frac{\bar{G}(s)}{s} \right] = \frac{k_m}{\tau_m} \frac{z \left[\left(\frac{T}{\tau_m} - 1 + e^{-\frac{T}{\tau_m}} \right) z + \left(1 - e^{-\frac{T}{\tau_m}} - \frac{T}{\tau_m} e^{-\frac{T}{\tau_m}} \right) \right]}{(z-1)^2 (z - e^{-T/\tau_m})}$$

and the digital equivalent of the PV-axis assembly results in

$$G(z) = \frac{k_m}{\tau_m} \frac{\left(\frac{T}{\tau_m} - 1 + e^{-\frac{T}{\tau_m}} \right) z + \left(1 - e^{-\frac{T}{\tau_m}} - \frac{T}{\tau_m} e^{-\frac{T}{\tau_m}} \right)}{(z-1)(z - e^{-T/\tau_m})} \tag{16}$$

As the system will stay outdoors it will be subject to changes in environmental conditions such as temperature and direction variation and wind intensity. In addition, the transfer function of the same motor will be different if it drives several loads because the dynamic characteristics of the actuator depend on the load and if the load is not balanced the moment of inertia, J , will be changed as a function of the angular displacement, θ . These situations lead to a variation of the model parameters as a function of time. Another reason for using this methodology is parametric uncertainties. Therefore, in order to achieve and maintain an acceptable level of control system performance in the occurrence of variation of model parameters, an adaptive control approach must be considered [7].

III. ONLINE CONTROL SYSTEM DESIGN

In this section we present the method developed for the control system online project, which is based on the architecture of the self-tuning regulator (STR). In turn, the recursive least squares method is used to compose the parameter estimator. Also, the minimum degree pole placement algorithm (MDPP) is used for the adjustment of the controller parameters.

In the design of adaptive control systems it is assumed that the structure of the process model is specified. The model of the model is estimated online and one can be carried out in a continuous or batch mode. The parametric can be able to be done in several ways. There are also techniques that can be used to design the control system. In this research, the estimated parameters are treated as the controllers. This is called the principle of equivalence to certainty.

3.1. Self-tuning regulator Deterministic

The self-adjusting regulator (STR) automatically adjusts its parameters to obtain the desired properties of the closed-loop system. The choice of the structure of the model and its parameterization are important practices for self-adjusting regulators. Often, the model can be reparametrized such that the controller parameters can be estimated directly. That is, a direct adaptive algorithm is obtained. This approach has been called implicit.

An indirect approach is to estimate the parameters of the process transfer function. This results in an indirect adaptive algorithm. The controller parameters are not updated directly, but indirectly via the estimation of the process model. This method is often called explicit. In Fig. 5, the block diagram based on the indirect self-tuning is shown. It is assumed that the structure of the process model is specified.

The model parameters are estimated recursively online and the "Estimator" block provides an estimate of the process parameters. The "Control Law" block contains calculations that are required to carry out the design of a controller with a specified method and some parameters that can be chosen externally. The "Controller" block is an implementation of the controller whose parameters are chosen from the control project.

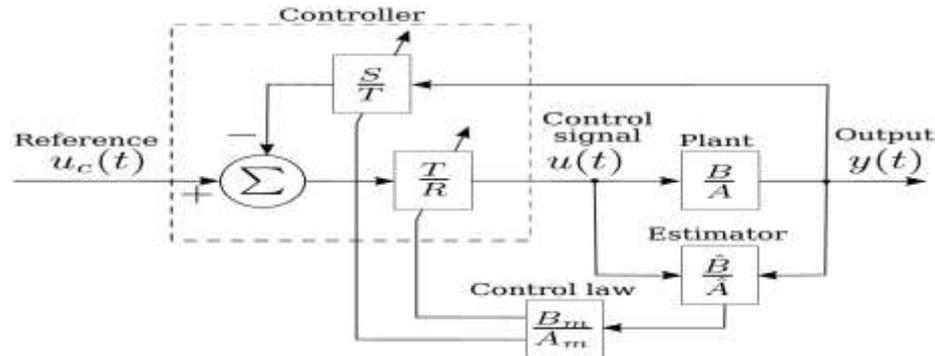


Figure 5: Based on self-tuning indirect regulator.

3.1.1. Parametric Estimation Algorithm

The technique used in this research for parametric estimation is the recursive least squares (RLS). The RLS equations are given by

$$\begin{aligned}
 K(t) &= \frac{P(t-1)\varphi(t-1)}{1 + \varphi^T(t-1)P(t-1)\varphi(t-1)} \\
 \hat{\theta}(t) &= \hat{\theta}(t-1) + K(t)(y(t) - \varphi^T(t-1)\hat{\theta}(t-1)) \\
 P(t) &= (I - K(t)\varphi^T(t-1))P(t-1)
 \end{aligned}
 \tag{17}$$

where K is the gain vector; P the error covariance matrix; φ the vector of regressors and θ is the vector of parameters.

From (16) one can see that the system model is given as follows

$$G(z) = \frac{b_0z + b_1}{z^2 + a_1z + a_2}
 \tag{18}$$

and that in equation to differences results in

$$y(t)z^2 + a_1zy(t) + a_2y(t) = b_0zu(t) + b_1u(t)
 \tag{19}$$

Multiplying (18) by q^{-2} results in

$$y(t) + a_1y(t-1) + a_2y(t-2) = b_0u(t-1) + b_1u(t-2)$$

and (19) can be rewritten as

$$y(t) = \varphi^T(t-1)\theta
 \tag{20}$$

where

$$\begin{aligned}
 \theta^T &= (a_1 \ a_2 \ b_0 \ b_1) \\
 \varphi^T(t-1) &= (-y(t-1) \ -y(t-2) \ u(t-1) \ u(t-2))
 \end{aligned}$$

3.1.2. Design by Pole Placement

The control design method used is the pole allocation method. The idea is to design a controller that provides the desired poles in a closed loop. The process can be described by a single input and single output system (SISO)

$$Ay(t) = B(u(t) + v(t))
 \tag{21}$$

where A and B are polynomials in z and are relatively prime.

A general linear controller can be described by

$$Ru(t) = Tu_c(t) - Sy(t)
 \tag{22}$$

where R , S and T are polynomials.

The elimination of u between the equations (21) and (22) results in the following equations for the closed-loop system

$$y(t) = \frac{BT}{AR + BS}u_c(t) + \frac{BR}{AR + BS}v(t)
 \tag{23}$$

$$u(t) = \frac{AT}{AR + BS}u_c(t) - \frac{BS}{AR + BS}v(t)
 \tag{24}$$

The characteristic polynomial closed loop results in

$$AR + BS = A_c \tag{25}$$

The key idea in the design method is to specify the desired characteristic polynomial in closed loop A_c . The polynomials R and S can be solved from (25) which is called the Diophantine equation.

To determine the polynomial T in the controller (22) it is required that the response of the command signal u_c to the output is described by the dynamics

$$A_m y_m(t) = B_m u_c(t) \tag{26}$$

Follows from (23) and (24) that the conditions must have

$$\frac{BT}{AR + BS} = \frac{BT}{A_c} = \frac{B_m}{A_m} \tag{27}$$

the model-following depends on the model, system and command signal.

The equation (27) implies that there is a cancellation of BT and A_c factors. Factoring the B polynomial as

$$B = B^+ B^- \tag{28}$$

where B^+ is a monic polynomial whose zeros are stable and so well damped that can be canceled by the controller and B^- corresponds to unstable or weakly damped factors that cannot be canceled. It follows that B^- must be a factor of B_m

$$B_m = B^- B_m' \tag{29}$$

since B^+ is canceled, it must be a factor of A_c . The characteristic polynomial in closed loop has the following form

$$A_c = A_o A_m B^+ \tag{30}$$

Since B^+ is a factor of B and A_c , it follows from (25) that

$$R = R' B^+ \tag{31}$$

and the Diophantine equation is reduced to

$$AR' + B^- S = A_o A_m = A_c' \tag{32}$$

By introducing (28), (29) and (30) into (22) results in

$$T = A_o B_m' \tag{33}$$

3.2. Algorithm MDPP

The following one has minimum degree pole placement algorithm (MDPP) [4]

Data: Polynomials A e B

Specifications: Polynomials A_m , B_m , e A_o

Compatibility Conditions:

$$\begin{aligned} \deg A_m &= \deg A \\ \deg B_m &= \deg B \\ \deg A_o &= \deg A - \deg B^+ - 1 \\ B_m &= B^- B_m' \end{aligned}$$

Step 1: Factor B as $B = B^+ B^-$, where B^+ is monic

Step 2: Find the the solution R' and S with $\deg S < \deg A$ from

$$AR' + B^- S = A_o A_m$$

Step 3: Form $R = R' B^+$ and $T = A_o B_m'$, and compute the control signal from the control law

$$Ru = Tu_c - Sy$$

No zeros are canceled: The factorization in **Step 2** also becomes very simple if no zeros are canceled. We have $B^+ = 1$, $B^- = B$, and $B_m = \beta B$, where $\beta = A_m(1)/B(1)$. Furthermore, $\deg A_o = \deg A - \deg B - 1$ and $T = \beta A_o$. The closed-loop characteristic polynomial is $A_c = A_o A_m$, and the Diophantine equation in **Step 2** becomes

$$AR + BS = A_c = A_o A_m$$

3.3. Parallel PID Controller

For the purposes of comparison with the proposed methodology, the remainder of this Section presents the structure of the controller output of three terms, PID (Proportional-Integral-Derivative), in the time domain. This controller offers a proportional term, an integration term and a derivative term and its output can be written as [9]

$$u(t) = K_P e(t) + K_I \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt} \quad (34)$$

The time derivative of $u(t)$ and $e(t)$ in (34) becomes

$$\frac{du(t)}{dt} = K_P \frac{de(t)}{dt} + K_I e(t) + K_d \frac{d^2 e(t)}{dt^2} \quad (35)$$

and the use of the Euler's method in (35) results in the following equation the difference [10]

$$u(k) = u(k-1) + \left(K_P + T K_I + \frac{K_d}{T} \right) e(k) - \left(K_P + 2 \frac{K_d}{T} \right) e(k-1) + \frac{K_d}{T} e(k-2) \quad (36)$$

IV. EXPERIMENTAL RESULTS

The experiments obtained in this work consist of performance results of the control and action methods resulting from the tracking of the moving target. Initially, we present the design specifications, the plant model and the setup of the experiments that are constituted of the parameters the estimator and simulations. The section ends with an analysis of results.

4.1. Design Specifications

The desired system must meet the design specifications that are represented in terms of the damping coefficient, ζ , natural frequency not damped, ω_n , and regime gain. The specifications are given by $\zeta = 0.8$, $\omega_n = 2 \text{ rad/s}$ and steady-state gain unit gain.

4.2. Plant Model

The plant model for purposes of implementing the positioning system are obtained from the motor plate data. Taking into account that $J = 3.2284 \times 10^{-6} \text{ Kg} \cdot \text{m}^2$; $R_a = 4 \Omega$; $k_t = 0.0274 \text{ N} \cdot \text{m} \cdot \text{A}^{-1}$ $k_b = 0.0274 \text{ V} \cdot \text{s} \cdot \text{rad}^{-1}$; and $f = 3.5077 \times 10^{-6} \text{ N} \cdot \text{m} \cdot \text{s}$ results in $\tau_m = 0,0019$ and $k_m = 13,1534$. Thus, the equation (9) is as follows

$$\bar{G}(s) = \frac{2122}{s(s + 59.22)} \quad (37)$$

From (37), the crossing frequency is estimated to be approximately 2 Hz. The sampling frequency should be significantly faster than the plant dynamics. With this, a sampling frequency of 200 Hz is chosen and the digital equivalent of the PV-axis assembly results in

$$G(z) = \frac{0.02409z + 0.02182}{z^2 - 1.744z + 0.7437} \quad (38)$$

4.3. Setup

The setup consists of the initial conditions of the plant parameters, the estimator and command signal. As well, of the parameters of the model-based simulations.

4.3.1. Estimator

The algebraic structure of the estimator is given by (17), the initial conditions of the estimator are: $P = 10^3 \times I$, the initial estimates of $a_1 = -2.5$, $a_2 = 0.5$, $b_0 = 0.03$ and $b_1 = 0.06$.

4.3.2. Command signal

To generate the reference signals of the STR scheme of Fig. 17, it is assumed that the generation of the input data is given by rectangular pulses of variable amplitude and period 125×10^{-3} .

4.4. Parametric Estimator

The evolution of the recursive estimation behavior of the plant parameters by the parametric estimator of the STR schema of Fig. 5 and represented by the polynomials \hat{B}/\hat{A} is in Fig. 6.

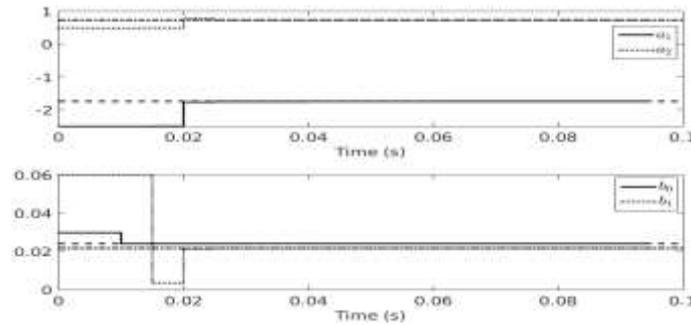


Figure 6: Estimation of parameters.

It is observed in Fig. 6 that the parameters converged to the plant parameters that are given by (38) with a lesser convergence time than 22 ms.

4.5. Tracking Control

In Fig. 7, we have the graph of the STR and PID outputs with the command signal. It is observed that the overhang of the STR controller is smaller when compared to the overhang of the PID controller.

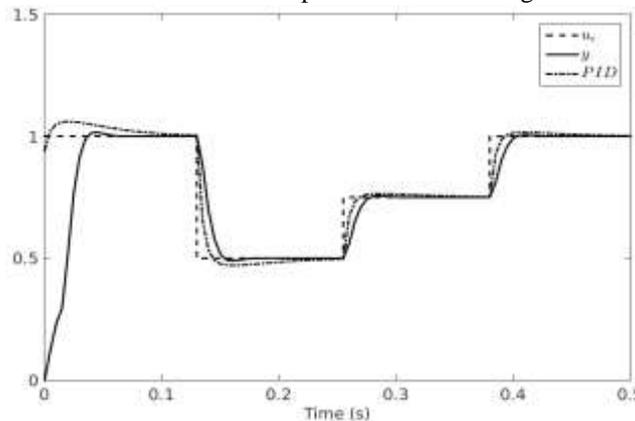


Figure 7: Input and output signal.

In Fig. 8, the control signal of the STR is sent to the actuator

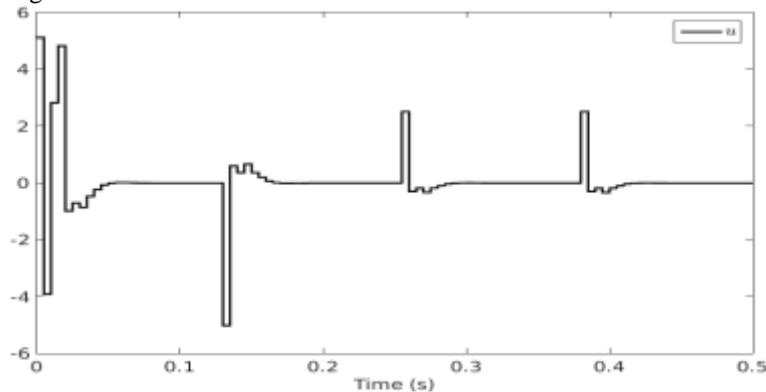


Figure 8: Control signal.

4.5. Performance Analysis

In Fig. 6, it can be seen that the parameters converge with less than 22 ms. The convergence time is influenced by the initial conditions. These parameters are used to calculate the control law polynomials, as described in the MDPP algorithm.

From Fig. 7, it can be seen that the STR and PID output can track the input signal and that there is a salient in the responses. This overshoot comes from the dynamics of the controllers used. In addition, if the choices of the initiator data of the estimator were inadequate the system would present great oscillations at the

beginning of the response, but then the algorithm would be able to make the output of the system follow the input.

The control signal in Fig. 8, presents oscillations only at the beginning of the system response, however in the remaining time of the response the control signal presents a smooth behavior and its value is null in cases in which the system output is equal to the command signal.

V. CONCLUSION

In this paper, we presented the mathematical modeling in the discrete time domain of a photovoltaic tracking system and the algorithm of the indirect self-tuning regulator without zeros cancellation. The parametric estimation algorithm used was the RLS. In addition, the STR was applied in the system under discussion and obtained satisfactory performance. The STR controller has been compared to the PID controller. By means of the obtained results it was noticed that the parametric estimation algorithm converged to its real values and that the output of the system was able to follow the command signal well according to the specifications of the desired system. Finally, it was verified that the signal of the control effort did not reach exaggerated values and did not present severe oscillations, which means that the actuator will not be subject to stress in the operating conditions.

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