

## Design Method Prediction for Flexural Torsional Buckling Resistance of Steel Circular Arches- Box-section

Sanket A. Potdar<sup>1</sup>, M. N. Bajad<sup>2</sup>

<sup>1</sup>Department of Civil Engineering, Sinhgad College Of Engineering, Pune, India

<sup>2</sup>Associate Professor Departments of Civil Engineering, Sinhgad College Of Engineering, Pune, India

**ABSTRACT :** *The distinguishing characteristics of an arch are the presence of horizontal reactions at the ends and the considerable rise of the axis at the center span which differ arch from beams. In case of beams supporting uniformly distributed load the bending moment increases with the square of span and hence they become uneconomical for long span structures. Long span structure arches have an advantage as they would develop horizontal reactions which reduce the design bending moment. An arch has tendency to buckle in its plane of the arch loading due to axial compression. Due to curved profile arches have high-efficient in-plane load carrying capacity but steel arches are prone to out-of-plane buckling that controls its design strength rather than in-plane flexural buckling. For pinned arches, in practical use arch ends can be treated as in-plane free to rotate but with respect to out-of-plane bending they are actually semi-rigidly restrained rather than fully restrained. That is why in practical engineering design; much attention needs to be paid to the case of arch with elastic end bending restrained. In practice arches having uniformly distributed vertical loading slightly bending forces and mainly compressive forces are induced on the arch structure. That is why in practical engineering design much attention needs to be paid in the case of arches having uniformly distributed vertical loading, and design method needs to be proposed for this case. The aim of the study is to predict the flexural torsional buckling resistance design of steel circular arch loaded in-plane uniformly distributed vertical loads that buckles out-of-plane in flexural torsional mode before in-plane failure. This study shows that, due to insertion of normalized slenderness ratio in the analysis of arches, the design method can be predicted for arches having uniformly distributed vertical loads and end supports with elastic end bending restraining conditions.*

**Keywords:** *Flexural torsional buckling; Out-of-plane buckling; In-plane buckling, uniformly distributed*

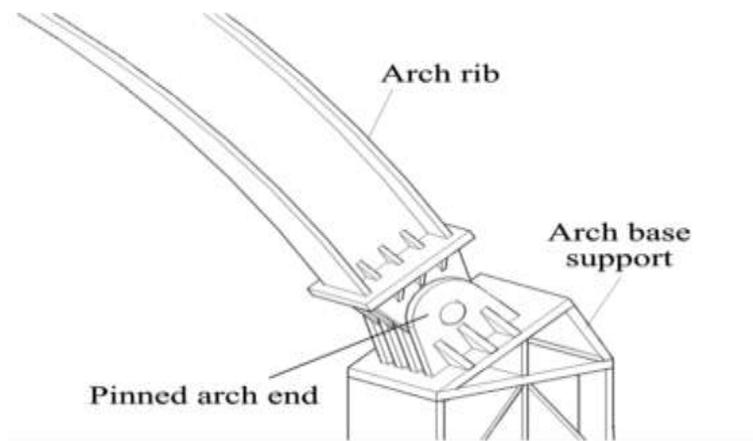
### I. INTRODUCTION

Arch structure is aesthetically pleasant structure; as they represent the primary structural components in important, eye pleasant and expensive structures, many of which are unique. Current trends in modern architectural designer heavily rely on curved building components due to their strengths and architectural appeal. Primarily arch type structures are used in construction of bridges and industrial roofing. The distinguishing characteristics of an arch from beam are the presence of horizontal reactions at the ends and the considerable rise of the axis at the center span which differ arch from beams. In case of beams supporting uniformly distributed load the bending moment increases with the square of span and hence they become uneconomical for long span structures. Long span structure arches have an advantage as they would develop horizontal reactions which reduce the design bending moment. Horizontal reactions also develop normal thrust at the arch ends, due to these arches may buckle and fail out-of-plane and twist about its longitudinal axis. This out-of-plane buckling controls its design strength rather than in-plane buckling.

Steel arches have in-plane load carrying capacity is higher, due to their curved profile, but it has tendency to buckle out-of-plane that control its design strength rather than in-plane load carrying capacity. Due to this behavior of steel arches much attention needs to be paid on this problem. Many factors are affecting on out-of-plane buckling and flexural torsional buckling, in which rise-to-span ratio and geometric imperfections is affecting on flexural torsional buckling behavior mostly. Due to increase in rise of arch the horizontal reaction induced at support get increased, this horizontal reaction acts as a compressive axial force, and due to increase in axial force structure become unstable and deflects out of its plane. End conditions of supports also affecting on steel arches. For pinned arches, in practical use arch pin ends have to be dealing with in-plane free to rotate but

with respect to out-of-plane bending they are actually semi-rigidly restrained rather than fully restrained. That is why in practical engineering design; much attention needs to be paid to the case of arch with elastic end bending restrained. Therefore, analytical investigation for arches with elastic end bending restraints and having uniformly distributed vertical loads needs to be investigated.

Under the action of uniformly distributed radial loads, circular arches are in ideal status of uniform compression without in-plane bending moment Dou Chao et.al. 2015. Several investigations have been carried out into flexural-torsional inelastic buckling and design of circular arches in uniform compressions. Pi and Trahair 1998 and Pi and Bradford 2005 explored the flexural torsional buckling strength and proposed design equations for pinned and fixed circular arches by using a self-developed finite element (FE) program. Dou and Guo 2012 adopted a commercial finite element package for elasto-plastic stability analyses to establish design approaches for steel circular arches. The author modelled circular arches with straight beam element (BEAM188) and analyzed which shows good argument with practical conditions and previous experimental results. Despite these previous researches were carried out for ideal end boundary conditions. The end boundary conditions are also affecting on steel arches i.e. hinged arches, pinned arches and fixed arches, in which the degree of freedom (dof) at arch end was either fully restrained or fully free. However in practical conditions behaviour of pinned arches are in-plane free to rotate as shown in fig. 1 and out-of-plane buckling is semi-rigidly restrained. The experimental study was conducted by Dou Chao et al. 2015 indicate that, for fixed arches all dofs at arch ends can be fabricated rigidly restrained which was close to the assumption in an ideal model. However for pinned arches arch ends can be treated as in-plane free to rotate but out-of- plane it is semi-rigidly restrained that's why much attention needs to be paid on arch end with elastic end bending restrained.



**Fig. 1** Practical pin end condition

Arches are widely used in bridges, satial structures and long-span structures because of their high-efficient in-plane load-carrying capacity, especially under full span length uniformly distributed vertical load when the arch ribs are primarily in compression and bending. The arch is always supported in such a way that the spreading of two ends outward is prevented, which introduces compressive and lesser bending action in the arch rib Dou Chao et al. 2015. Similar to axially-loaded straight compression members, buckling resistance design is prevented for circular arches in compression and slightly bending actions are influenced by parameters such as residual stresses, slenderness ratio, end restraint conditions, geometric imperfections, and section types. Apart from these, the rise-to-span ratio which is an important factor, which distinguishes arches from columns. And due to this rise in its elevation horizontal reaction is produced at the arch structure which minimizes bending moments at the arch structure

This research focuses on studying the behavior of arches with elastic end bending restraints and having uniformly distributed vertical loads on the freestanding arch structure. The aim of the study is to predict the flexural torsional buckling resistance design of steel circular arch loaded in-plane uniformly distributed vertical loads that buckles out-of-plane in flexural torsional mode before in-plane failure.

## **II. METHODOLOGY**

Similar to axially-loaded straight columns, buckling resistances of circular arches in uniform compression are influenced by parameter such as rise-to-span ratio. Rise-to-span is an important factor which distinguishes arches from column. In this paper, steel circular arches of uniform hot-rolled hollow Box sections with elastic end bending restraints under the action of uniformly distributed vertical load are studied. The rise-to-

span ratio  $f/L$  varies from 0.10 to 0.50, in which  $f$  denotes the rise of the arch and  $L$  denotes the span, all measured on the basis of the centroid axis of cross section. The out-of-plane slenderness  $\lambda_y = S/i_y$  varies from 40 to 240, in which  $S$  is the developed length of cross-sectional centroid axis of the arch and  $i_y$  is the gyration radius of the cross section with respect to out-of-plane bending. It was found that in practical use all the degrees of freedom (dofs) at the arch ends were close to the assumption in an ideal calculation model (either free or fully restrained) except the out-of-plane bending degree of freedom which is actually semi-rigidly restrained rather than fully restrained. Thus in this study, at the arch ends the translations in three directions, torsion along the tangent direction and warping deformation of the arch axis are fully restrained, and the bending in-plane is set free, but the bending out-of-plane is semi-rigidly elastically restrained. The elastic end bending rigidity is defined as  $k_a$  and it is same for both end conditions. The finite element package *ANSYS R13.0* is adopted to perform the flexural-torsional buckling analyses of arches. The arch is divided into elements and is modeled with *BEAM188* element, and only global stability is involved in the analysis without local buckling of the plates. The arch material is perfectly elastic plastic. The first in-plane flexural buckling mode and the first out-of-plane flexural-torsional buckling mode in eigenvalue buckling analyses of the arches were selected as failure mode of the arch and from the obtained load factor from the eigenvalue buckling analysis, reduction factor is calculated which defines the arch load carrying capacity in the compression. The Newton-Raphson iterative method is used to track the large deformation elasto-plastic load-displacement development and achieve the ultimate buckling resistances. For eigenvalue buckling analyses, the Block-Lanczos method is used for the eigenvalue extraction.

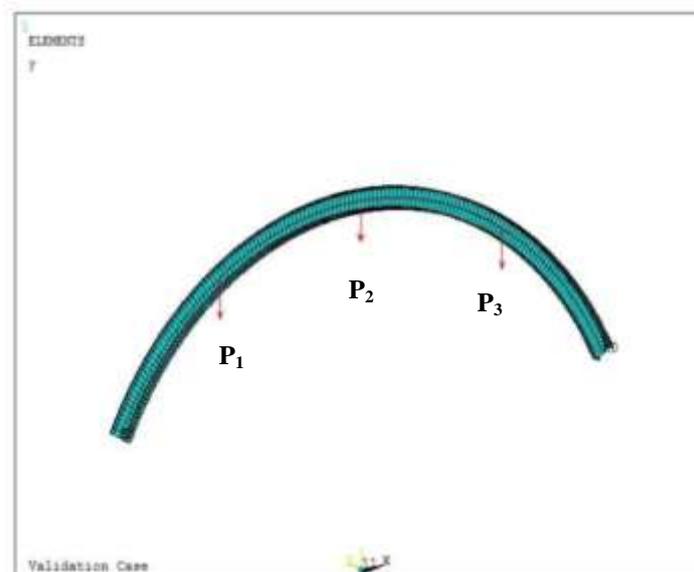
### III. VERIFICATION OF NUMERICAL MODEL

Numerical model was prepared on a basis of previous research paper by Dou Chao et.al. (2015) a. from that paper, three points loading case P-3-0.30 is used here for a numerical modeling in ANSYS. Doubly symmetric welded I-section is used here. Dimensions of I-section are  $h \times b \times t_w \times t_f = 200 \times 100 \times 8 \times 12$ . Where,  $h$ ,  $b$ ,  $t_w$ , and  $t_f$  are overall height, width of flange, thickness of web and thickness of flange respectively. The dimensions are listed below in Table 1

**Table 1 Dimensions of numerical model**

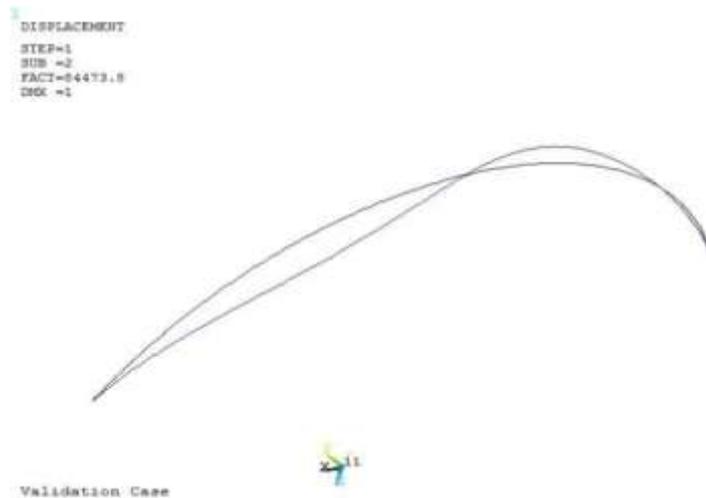
| Specimen | Rise-to-span ratio ( $f/L$ ) | Load case | Radius (mm) | Included Angle (degree) | Out-of-plane slenderness |
|----------|------------------------------|-----------|-------------|-------------------------|--------------------------|
| P-3-0.3  | 0.3                          | 3 points  | 3400        | 123.8                   | 320                      |

Also geometric imperfections are also included in the numerical model as per the paper. Here geometric imperfections are included is the  $1/300$  of the actual arc length ( $S$ ) of the arch used in the modeling. The material properties of steel taken for the analysis are, yield stress of material  $f_y = 285$  MPa, Modulus of elasticity  $E = 202$  GPa and ultimate tensile strength  $f_u = 440$  MPa are included in numerical model. For modeling of arch segment *BEAM188* and *COMBIN39* elements are used. The results obtained are at each point load with in-plane deflection, out-of-plane deflection and twist rotation of section. The points in which point load is applied namely  $P_1$ ,  $P_2$  and  $P_3$ . Numerical model shown in fig. 2



**Fig. 2** Geometry of numerical model

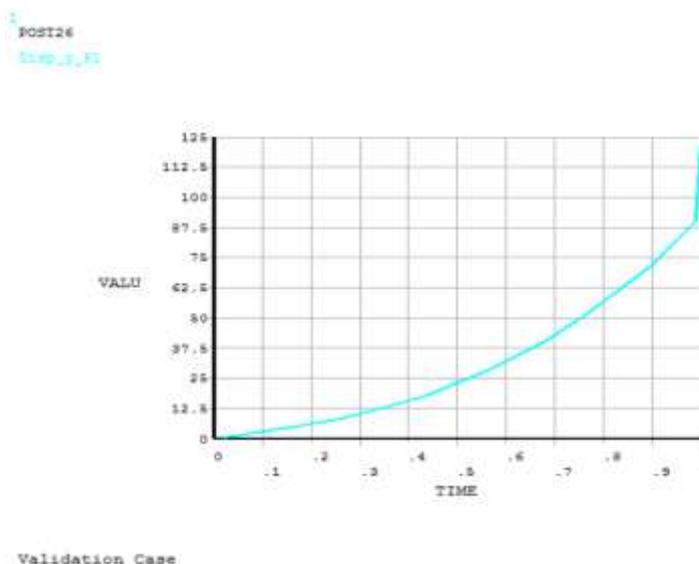
The arch is modelled with *BEAM188* and *COMBIN39* element in *ANSYS*. The material properties and dimensions of arch segment are shown above. Geometric Material non-linear imperfection analysis is carried out, and then obtained results shown in fig. 3 below,



**Fig. 3** S-shaped flexural torsional buckling failure mode

#### IV. RESULTS OF NUMERICAL MODEL

The numerical result shows good agreement with experimental results. It shows that the in-plane deflections in numerical are pretty much closer, the error between numerical value is 12.5%. The out-of-plane deflection at point  $P_1$  are also shows the error of only 3.2% with the experimental results. Numerical results of twist rotation at point  $P_1$  give 12.72% of error in comparison with experimental results. The buckling failure mode by numerical and experimental approaches is S-shaped failure. Overall results shows that the numerical model made by *ANSYS* elements i.e. *BEAM188* and *COMBIN39* is compatible for finite element method analysis and can give good prediction of the out-of-plane flexural-torsional ultimate resistance of steel circular arches. The numerical results obtained in graphical format by software analysis shown in Fig. 4 below,



**Fig. 4** Out-of-plane displacement of  $P_1$

*BEAM188* and *COMBIN39* an element shows good predictions and gives accurate results and good prediction of the flexural torsional buckling, therefore these elements are used for this study of flexural torsional buckling of arch structure with different loading conditions. Numerical and experimental results obtained and gathered in table shown below in Table 3

**Table 3** Comparison of results

| Deflection Parameter | Experimental results by Dou Chao et.al. 2015 |                |                | Numerical results obtained in this paper |                |                |
|----------------------|--|----------------|----------------|--|----------------|----------------|
|                      | P <sub>1</sub>                               | P <sub>2</sub> | P <sub>3</sub> | P <sub>1</sub>                           | P <sub>2</sub> | P <sub>3</sub> |
| In-plane (mm)        | 15   | 24             | 12.5           | -  | 21             | -              |
| Out-of-plane (mm)    | 125  | 25             | -152           | 121                                      | 0.01           | -121           |
| Twist rotation (rad) | 0.22   | 0.01           | -0.25          | 0.192                                    | 0.001          | -0.192         |

**V. ARCH WITH UNIFORMLY DISTRIBUTED VERTICAL LOADS**

From Eigenvalue buckling analysis we get the load intensity factor, from that factor we can calculate the ultimate crippling load of steel circular arches. From this crippling load we can found out the reduction factor for semicircular arches in uniformly distributed vertical loading. The linear buckling analysis is carried out for these arches and reduction factor is going to be finding out ( $\phi$ ) and ultimate crippling load. The reduction factor for arches under uniform compression is defined as,

$$\phi = N_u / N_y = q_u \cdot R / f_y \cdot A$$

Where,  $N_u$  = Axial compressive force

$N_y$  = Squash load of cross-section

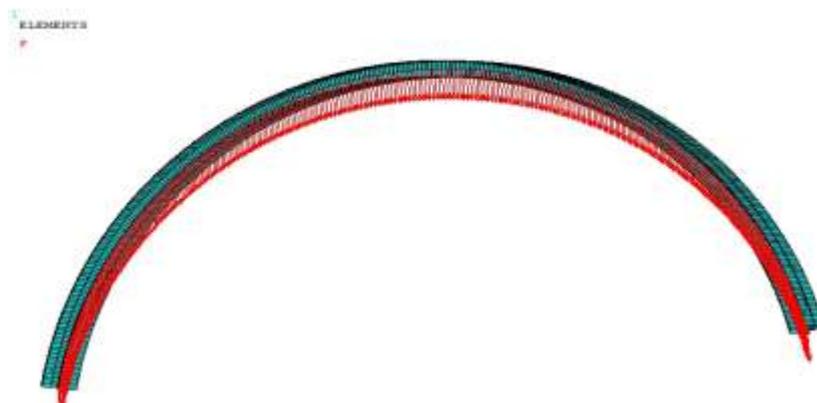
$q_u$  = Uniformly distributed vertical load subjected to ultimate flexural torsional buckling load

$R$  = Radius of arch

$f_y$  = Yield stress

$A$  = Cross sectional area

The uniformly distributed vertical loading diagram of ANSYS modeling in fig. 5



**Fig. 5** uniformly distributed vertical loading

Hollow Box sections are also a closed type sections and used widely to construct the steel circular arches. These types of sections having high load carrying capacity in the structure because of rectangular profile in the plane of loading. The curved profile and the thrust at the arch ends are the key characters which differentiate the arch from the straight beam columns, thus the rise-to-span ratio is an important factor to the buckling resistance of arches. Hollow Box sections of  $h \times b \times t_f \times t_w = 800 \times 400 \times 30 \times 30$  mm in which  $h$  = height of section,  $b$  = width of section,  $t_f$  = thickness of flange and  $t_w$  = thickness of web. Whereas the yield stress is  $f_y = 390$  MPa, and the end bending rigidity is  $k_a = 5EI_y/S$  here,  $I_y$  = moment of inertia of section. From these material properties the eigenvalue buckling analysis is carried out for deriving design method for flexural torsional buckling resistance of circular arches with uniformly distributed vertical loads for hollow circular cross section and for compressive actions induced on the structure. The results shows below in fig. 6 below,

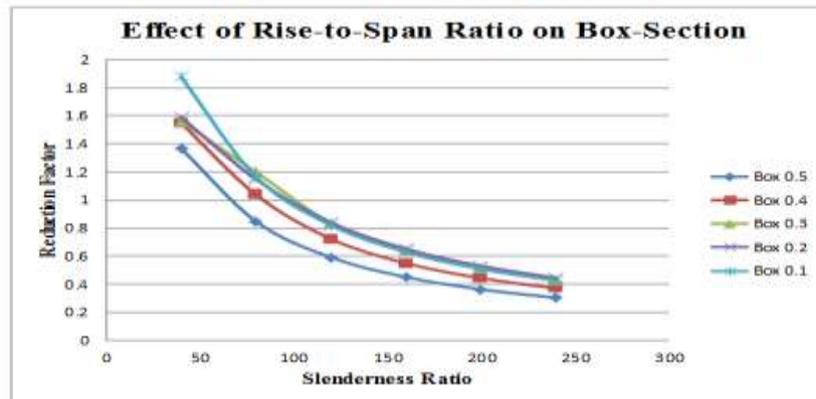


Fig. 6 Effect of rise-to-span ratio on Box-section

From above results shows that from the rise-to-span ratio affects the reduction factor and slenderness ratio of the section. Here reduction factor is different for different rise-to-span ratio and slenderness ratio, that's why we cannot predict the design method on the basis of slenderness ratio for Box-section type arches. Therefore, normalized slenderness ratio has to be introduced here to minimize these effects and obtained the single curve as determined in the Eurocode as a column curve. Rise-to-span, section type and different material properties which further discussed for this study.

## VI. NORMALIZED SLENDERNESS RATIO

Flexural buckling reduction factor for a given axially loaded straight column, can be determined according to the sectional type and effective length on the basis of the corresponding column curve provided by standard design codes like, Eurocode-3, AISC code etc. however the flexural-torsional buckling reduction factor of arches in compression depends on various mechanical and geometric parameters such as the slenderness, the rise-to-span ratio, the section type and its dimension and the end restraints provided, thus by introducing Normalized slenderness ratio in the analysis of flexural-torsional buckling can able to predict the reduction factor of arches with single unified curve which accounts all of these effects. Formula for normalized slenderness ratio as shown below,

$$\lambda_a = \sqrt{(N_y / N_{acr})}$$

Where,  $N_y$  = Squash load of cross-section =  $f_y \cdot A$

$N_{acr}$  = Horizontal thrust obtained from EBA analysis

$N_{acr}$  consider for the effects of parameters like the slenderness, the rise-to-span ratio, the arch end restraining conditions and the sectional dimensions on out-of-plane stability and the squash load  $N_y$  considers the material inelasticity, therefore the out-of-plane normalized slenderness  $\lambda_a$  contains influence of all the previously described parameters simultaneously. Thus using the dimensionless slenderness  $\lambda_a$  as the horizontal coordinate, reduction factor of arches can be quite different from those using the geometric slenderness  $\lambda$ .

## VII. EFFECT OF NORMALIZED SLENDERNESS RATIO ON BOX-SECTION

Normalized slenderness ratio is used for the eliminating the parameters which affect the reduction factor. Here for the box section the out-of-plane slenderness ratio gives the scatter effect on the reduction factor and from out-of-plane slenderness ratio design method cannot predict. In fig. 7 normalized slenderness ratio get introduced and obtained a single unified curve for arches with uniformly distributed vertical loading. The curve obtained here are similar to the curve derived by the Eurocode EN-1993-2:2006.

The reduction factor for lower rise-to-span ratio i.e. in between 0.1 to 0.3 and lower slenderness ratio gives the reduction factor above 1 that is; it shows the section is over safe. The reason of this effect is due to the horizontal thrust is minimum in the section i.e. compression actions are minimum in the section but due to uniformly distributed vertical loading the bending actions are predominant in the section. In this study the bending actions induced on the structure are not consider.

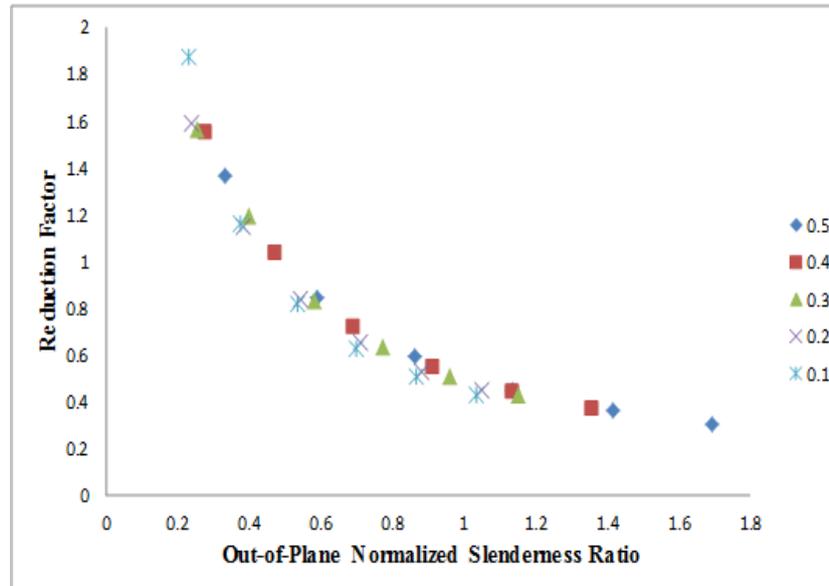


Fig. 7 Effect of Normalized slenderness on Box-section

### VIII. CONCLUSION

The investigation is carried out for circular arches with elastic end bending restraints and uniformly distributed vertical loads for hollow circular cross section having different rise to span ratio. From this study the following conclusions can be drawn.

- The experimental results 91 kN per point load and FE result 84.47 kN per point load are considerably match with 7.17% error, so for flexural-torsional buckling analysis of semicircular arches we can use this FE analysis method.
- The numerical result shows good agreement with experimental results. It shows that the in-plane deflections in numerical are pretty much closer, the error between numerical value is 12.5%. The out-of-plane deflection at point  $P_1$  are also shows the error of only 3.2% with the experimental results.
- BEAM188 and COMBIN39 elements are compatible with practical conditions and can be used in buckling analysis of arches with elastic end bending restraining condition.
- Design method for arches with hollow circular cross section and different rise-to-span ratio can be predicted by software analysis carried out in ANSYS software and by BEAM188 and COMBIN39 elements.
- Design method from normalized slenderness ratio for circular arches with uniformly distributed vertical loads can be predicted and the results obtained for the box section circular arches are showing good agreement with the column curves obtained by the Eurocode.

### REFERENCES

- [1]. Pi Yong Lin, and Trahair N. S. (1998) "Out-of-plane inelastic buckling and strength of steel arches". Journal of structural engineering, Vol. 124:No.2, 174-183.
- [2]. Pi Yong Lin, and Bradford M. A. (2005) "Out-of-plane strength design of fixed steel I-section arches". Journal of structural engineering, Vol. 131: No. 4, 560-568.
- [3]. Dou Chao and Guo Y. L. (2012) "Out-of-plane inelastic stability and strength design of circular arches in uniform compression". Journal of Building Structure, Vol. 33: No. 1, 104-110.
- [4]. Dou Chao, Guo Y. L., Pi Y. L., and Zhao S. Y. (2015). "Experimental investigation into flexural-torsional ultimate resistance of circular arches". Journal of Structural Engineering, ISSN 0733-9445 / 04015006.
- [5]. Dou Chao, Pi Y. L., (2015). "Flexural-Torsional Buckling Resistance Design of Circular Arches with Elastic End Restraints". ISSN 0733-9445/04015014
- [6]. Sakimoto T., and Kamatsu S. (1983). "Ultimate Strength formula for steel arches", Journal of Structural Engineering, Vol. 109: No. 3, 613-627.