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Some Properties of Determinant of Trapezoidal Fuzzy Number Matrices

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ABSTRACT: The fuzzy set theory has been applied in many fields such as management, engineering, matrices and so on. In this paper, some elementary operations on proposed trapezoidal fuzzy numbers (TrFNs) are defined. We also defined some operations on trapezoidal fuzzy matrices (TrFMs). The notion of Determinant of trapezoidal fuzzy matrices are introduced and discussed. Some of their relevant properties have also been verified.

Keywords: Fuzzy Arithmetic, Fuzzy number, Trapezoidal fuzzy number (TrFN), Trapezoidal fuzzy matrix(TrFM), Determinant of Trapezoidal fuzzy matrix(DTrFM).

I. INTRODUCTION

Fuzzy sets have been introduced by Lofti.A.Zadeh[8] Fuzzy set theory permits the gradual assessments of the membership of elements in a set which is described in the interval [0,1]. It can be used in a wide range of domains where information is incomplete and imprecise. Interval arithmetic was first suggested by Dwyer [13] in 1951, by means of Zadeh's extension principle [6,9], the usual Arithmetic operations on real numbers can be extended to the ones defined on Fuzzy numbers. Dubosis and Prade [5] has defined any of the fuzzy numbers. A fuzzy number is a quantity whose values are imprecise, rather than exact as is the case with single – valued numbers.

Trapezoidal fuzzy number's (TrFNs) are frequently used in application. It is well known that the matrix formulation of a mathematical formula gives extra facility to study the problem. Due to the presence of uncertainty in many mathematical formulations in different branches of science and technology. We introduce trapezoidal fuzzy matrices (TrFMs). To the best of our knowledge, through a lot of work on fuzzy matrices is available in literature. A brief review on fuzzy matrices is given below.

Fuzzy matrices were introduced for the first time by Thomason [12] who discussed the convergence of power of fuzzy matrix. Fuzzy matrices play an important role in scientific development. Two new operations and some applications of fuzzy matrices are given in [1,2,3]. Ragab.et.al [11] presented some properties on determinant and adjoint of square matrix. Kim [7] presented some important results on determinant of a square fuzzy matrices. Pal [10] introduced intuitionistic fuzzy determinant . Jaisankar et.al [4] proposed the Hessenberg of Triangular fuzzy number matrices.

The paper organized as follows, Firstly in section 2, we recall the definition of Trapezoidal fuzzy number and some operations on trapezoidal fuzzy numbers (TrFNs). In section 3, we have reviewed the definition of trapezoidal fuzzy matrix (TrFM) and some operations on Trapezoidal fuzzy matrices (TrFMs). In section 4, we defined the notion of Determinant of trapezoidal fuzzy matrix (DTrFM). In section 5, we have presented some properties of Determinant of trapezoidal fuzzy matrix (DTrFM). Finally in section 6, conclusion is included.

II. PRELIMINARIES

In this section, We recapitulate some underlying definitions and basic results of fuzzy numbers.

Definition 2.1 Fuzzy set

A fuzzy set is characterized by a membership function mapping the element of a domain, space or universe of discourse X to the unit interval [0,1]. A fuzzy set A in a universe of discourse X is defined as the following set of pairs

$$A=\{(x,\mu_A(x))\ ;\ x\in X\}$$

Here μ_A : X → [0,1] is a mapping called the degree of membership function of the fuzzy set A and μ_A(x) is called the membership value of x∈X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

Definition 2.2 Normal fuzzy set

A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exists at least one x∈X such that $\mu_A(x)$ =1.

Definition 2.3 Convex fuzzy set

A fuzzy set A={(x,$\mu_A(x)$)}⊆X is called Convex fuzzy set if all A_α are Convex set (i.e.,) for every element $x_1 \in A_\alpha$ and $x_2 \in A_\alpha$ for every α∈[0,1], $\lambda x_1+(1-\lambda) x_2 \in A_\alpha$ for all λ∈[0,1] otherwise the fuzzy set is called non-convex fuzzy set.

Definition 2.4 Fuzzy number

A fuzzy set $\tilde{A}$ defined on the set of real number R is said to be fuzzy number if its membership function has the following characteristics

i. $\tilde{A}$ is normal

ii. $\tilde{A}$ is convex

iii. The support of $\tilde{A}$ is closed and bounded then $\tilde{A}$ is called fuzzy number.

Definition 2.5 Trapezoidal fuzzy number

A fuzzy number $\tilde{A}^{TzL} = (a_1, a_2, a_3, a_4)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}^{TzL}}(x) = \begin{cases} 0 & , \quad x \le a_1 \\ \dfrac{x-a}{b-a} & , a_1 < x \le a_2 \\ 1 & , a_2 \le x \le a_3 \\ \dfrac{c-x}{d-c} & , a_3 < x \le a_4 \\ 0 & , \quad x \ge a_4 \end{cases}$$

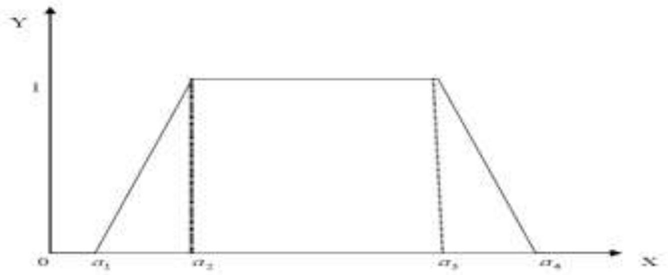

Fig 1: Trapezoidal Fuzzy Number

Definition 2.6 Ranking function

We defined a ranking function ℜ: F(R)→R which maps each fuzzy numbers to real line F(R) represent the set of all trapezoidal fuzzy number. If R be any linear ranking function

$$\Re(\tilde{A}^{TzL}) = \left(\frac{a_1+a_2+a_3+a_4}{4}\right)$$

Also we defined orders on F(R) by

$\Re(\tilde{A}^{TzL}) \ge \Re(\tilde{B}^{TzL})$ if and only if $\tilde{A}^{TzL} \ge_R \tilde{B}^{TzL}$

$\Re(\tilde{A}^{TzL}) \le \Re(\tilde{B}^{TzL})$ if and only if $\tilde{A}^{TzL} \le_R \tilde{B}^{TzL}$

$\Re\,(\tilde{A}^{TzL}) = \Re\,(\tilde{B}^{TzL})$ if and only if $\tilde{A}^{TzL} =_{D} \tilde{B}^{TzL}$

Definition 2.7 Arithmetic operations on trapezoidal fuzzy numbers (TrFNs)

Let $\tilde{A}^{TzL} = (a_1, a_2, a_3, a_4)$ and $\tilde{B}^{TzL} = (b_1, b_2, b_3, b_4)$ be trapezoidal fuzzy numbers (TrFNs) then we defined,

Addition

$$\tilde{A}^{TzL} + \tilde{B}^{TzL} = (a_1+b_1,\ a_2+b_2, a_3+b_3, a_4+b_4)$$

Subtraction

$$\tilde{A}^{TzL} - \tilde{B}^{TzL} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$$

Multiplication

$$\tilde{A}^{TzL} \times \tilde{B}^{TzL} = (a_1\Re(\tilde{B}^{TzL}),\ a_2\Re(\tilde{B}^{TzL}),\ a_3\Re(\tilde{B}^{TzL}),\ a_4\,\Re(\tilde{B}^{TzL}))$$

where $\Re(\tilde{B}^{TzL}) = \left(\dfrac{b_1+b_2+b_3+b_4}{4}\right)$ or $\Re(\tilde{b}^{TzL}) = \left(\dfrac{b_1+b_2+b_3+b_4}{4}\right)$

Division

$$\tilde{A}^{TzL} / \tilde{B}^{TzL} = \left(\frac{a_1}{\Re(\tilde{B}^{TzL})}, \frac{a_2}{\Re(\tilde{B}^{TzL})}, \frac{a_3}{\Re(\tilde{B}^{TzL})}, \frac{a_4}{\Re(\tilde{B}^{Tzl})}\right)$$

Where $\Re(\tilde{B}^{TzL}) = \left(\dfrac{b_1+b_2+b_3+b_4}{4}\right)$ or $\Re(\tilde{b}^{TzL}) = \left(\dfrac{b_1+b_2+b_3+b_4}{4}\right)$

Scalar multiplication

$$K\,\tilde{A}^{TzL} = \begin{cases} (ka_1, ka_2, ka_3, ka_4)\ if\ K \geq 0 \\ (ka_4, ka_3, ka_2, ka_1)\, if\, k < 0 \end{cases}$$

Definition 2.8 Zero trapezoidal fuzzy number

If $\tilde{A}^{TzL} = (0,0,0,0)$ then $\tilde{A}^{TzL}$ is said to be zero trapezoidal fuzzy number. It is defined by 0.

Definition 2.9 Zero equivalent trapezoidal fuzzy number

A trapezoidal fuzzy number $\tilde{A}^{TzL}$ is said to be a zero equivalent trapezoidal fuzzy number if $\Re\,(\tilde{A}^{TzL})=0$. It is defined by $\tilde{0}^{TzL}$.

Definition 2.10 Unit trapezoidal fuzzy number

If $\tilde{A}^{TzL} = (1,1,1,1)$ then $\tilde{A}^{TzL}$ is said to be a unit trapezoidal fuzzy number. It is denoted by 1.

Definition 2.11 Unit equivalent trapezoidal fuzzy number

A trapezoidal fuzzy number $\tilde{A}^{TzL}$ is said to be unit equivalent triangular fuzzy number. If $\Re\,(\tilde{A}^{TzL}) = 1$. It is denoted by $\tilde{1}^{TzL}$.

Definition 2.12 Inverse of trapezoidal fuzzy number

If $\tilde{a}^{TzL}$ is trapezoidal fuzzy number and $\tilde{a}^{TzL} \neq \tilde{0}^{TzL}$ then we define.

$$\tilde{a}^{Tzl^{-1}} = \frac{\tilde{1}^{Tzl}}{\tilde{a}^{TzL}}$$

III. TRAPEZOIDAL FUZZY MATRICES (TRFMS)

In this section, we introduced the trapezoidal fuzzy matrix and the operations of the matrices are presented.

Definition 3.1 Trapezoidal fuzzy matrix (TrFM)

A trapezoidal fuzzy matrix of order m×n is defined as A= $(\tilde{a}_{ij}^{TzL})_{m\times n}$, where $\tilde{a}_{ij}^{TzL}=\left(a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4}\right)$ is the ij^{th} element of A.

Definition 3.2 Operations on Trapezoidal Fuzzy Matrices (TrFMs)

As for classical matrices. We define the following operations on trapezoidal fuzzy matrices. Let A = ($\tilde{a}_{ij}^{TzL}$) and B =($\tilde{b}_{ij}^{TzL}$) be two trapezoidal fuzzy matrices (TrFMs) of same order. Then, we have the following

i. **Addition**
A+B = $\left(\tilde{a}_{ij}^{TzL} + \tilde{b}_{ij}^{TzL}\right)$

ii. **Subtraction**
A-B = $\left(\tilde{a}_{ij}^{TzL} - \tilde{b}_{ij}^{TzL}\right)$

iii. For A = $\left(\tilde{a}_{ij}^{TzL}\right)_{m\times n}$ and B = $\left(\tilde{b}_{ij}^{TzL}\right)_{n\times k}$ then AB = $\left(\tilde{c}_{ij}^{TzL}\right)_{m\times k}$ where $\tilde{c}_{ij}^{TzL}$ $=\sum_{p=1}^{n} \tilde{a}_{ip}^{TzL} . \tilde{b}_{pj}^{TzL}$, i=1,2,…,m and j=1,2,…,k.

iv. A^T or A^1 = $\left(\tilde{a}_{ji}^{TzL}\right)$

v. $KA = \left(K\tilde{a}_{ij}^{TzL}\right)$ where K is scalar.

IV. DETERMINANT OF TRAPEZOIDAL FUZZY MATRIX (DTRFM)

In this section, we introduced and discuss the Determinant matrix in the fuzzy nature.

Definition 4.1 Determinant of trapezoidal fuzzy matrix

The determinant of a square trapezoidal fuzzy matrix $A = (\tilde{a}_{ij}^{TzL})$ is denoted by (A) or det(A) and is defined as follows:

$$|A| = \sum_{h\in S_n} sign\ h \prod_{i=1}^{n} \tilde{a}_{ih(i)}^{TzL}$$
$$= \sum_{h\in S_n} sign\ h\ \tilde{a}_{1h(1)}^{TzL}\ \tilde{a}_{2h(2)}^{TzL} \dots \tilde{a}_{nh(n)}^{TzL}$$

Where $\tilde{a}_{ih(i)}^{TzL}$ are TrFNs and S_n denotes the symmetric group of all permutations of the indices {1,2,3,…,n} and sign h=1 or -1 according as the permutation $h = \begin{pmatrix} 1 & 2 & \dots & \dots & n \\ h(1) & h(2) & \dots & \dots & h(n) \end{pmatrix}$ is even or odd respectively.

Definition 4.2 Minor

Let $A = (\tilde{a}_{ij}^{TzL})$ be a square Trapezoidal fuzzy matrix of order n. The minor of an element $\tilde{a}_{ij}^{TzL}$ in A is a determinant of order (n$-$1) $\times$ (n$-$1) which is obtained by deleting the i^{th} row and the j^{th} column from A and is denoted by $\tilde{M}_{ij}^{TzL}$.

Definition 4.3 Cofactor

Let $A = \left(\tilde{a}_{ij}^{TzL}\right)$ be a square Trapezoidal fuzzy matrix of order n. The cofactor of an element $\tilde{a}_{ij}^{TzL}$ in A is denoted by $\tilde{A}_{ij}^{TzL}$ and is defined as $\tilde{A}_{ij}^{TzL} = (-1)^{i+j}\ \tilde{M}_{ij}^{TzL}$.

Definition 4.4 Aliter definition for determinant

Alternatively, the determinant of a square Trapezoidal fuzzy matrix $A = \left(\tilde{a}_{ij}^{TzL}\right)$ of order n may be expanded in the form

$$|A| = \sum_{j=1}^{n} \tilde{a}_{ij}^{TzL}\ \tilde{A}_{ij}^{TzL}\ ,\ \ i \in \{1,2,\dots,n\}$$

Where $\tilde{A}_{ij}^{TzL}$ is the cofactor of $\tilde{a}_{ij}^{TzL}$.

Thus the determinant is the sum of the products of the elements of any row (or column) and the cofactor of the corresponding elements of the same row (or column).

Definition 4.5 Adjoint

Let $A = \left(\tilde{a}_{ij}^{TzL}\right)$ be a square Trapezoidal fuzzy matrix of order n. Find the cofactor $\tilde{A}_{ij}^{TzL}$ of every elements $\tilde{a}_{ij}^{TzL}$ in A and replace every $\tilde{a}_{ij}^{TzL}$ by its cofactor $\tilde{A}_{ij}^{TzL}$ in A and let it be B. i.e, B = $(\tilde{A}_{ij}^{TzL})$. Then the transpose of B is called the adjoint or adjugate of A and is denoted by adjA. ie, $B^{'} = \left(\tilde{A}_{ji}^{TzL}\right) = adjA.$

V. PROPERTIES OF DETERMINANT OF TRAPEZOIDAL FUZZY NUMBER MATRICES

In this section, we introduced the properties of DTrFM.

5.1 Properties of DTrFM (Determinant of Trapezoidal Fuzzy matrix)

Property 5.1.1:

Let $A = \left(\tilde{a}_{ij}^{TzL}\right)$ be a square TrFM of order n. If all the elements of a row (or column) of A are 0 then $|A|$ is also 0.

Proof:

Let $A = \left(\tilde{a}_{ij}^{TzL}\right)$ be a square TrFM of order n and let all elements of r^{th} row, where $1 \le r \le n$ be 0.

ie, $\tilde{a}_{rj}^{TzL} = (0,0,0,0,)\ ,\ j = 1,2,\dots,n.$

Then $|A| = \sum_{j=1}^{n} \tilde{a}_{ij}^{TzL}\ \tilde{A}_{ij}^{TzL}\ \ ,i \in \{1,2,\dots,n\}$

Where $\tilde{A}_{ij}^{TzL}$ is the cofactor of $\tilde{a}_{ij}^{TzL}$.

Particularly if we expand through r^{th} row then we have $|A| = \sum_{j=1}^{n} \tilde{a}_{rj}^{TzL}\ \tilde{A}_{ij}^{TzL}$, which is the sum of the products of the elements of the r^{th} row and the cofactors of the corresponding elements of the r^{th} row. Since all $\tilde{a}_{rj}^{TzL}$ are 0, each term in this summation is 0 and hence $|A|$ is also 0.

Property 5.1.2:

Let $A = \left(\tilde{a}_{ij}^{TzL}\right)$ be a square TrFM of order n.If all the elements of a row(or column) of A are $\tilde{0}^{TzL}$ then $|A|$ is either 0 or $\tilde{0}^{TzL}$.

Proof:

Let $A = \left(\tilde{a}_{ij}^{TzL}\right)$ be a square TrFM of order n and let all the elements of r^{th} row be $\tilde{0}^{TzL}$. Since for any trapezoidal fuzzy number $\tilde{a}^{TzL} \neq 0,\ \tilde{a}^{TzL}.\tilde{0}^{TzL}$ = 0 and $\tilde{0}^{TzL}$. $\tilde{a}^{TzL}$ = $\tilde{0}^{TzL}$, if we expand through r^{th} row then in this case $|A|$ = $\tilde{0}^{TzL}$ and if we expand through other than r^{th} row then we have $|A| = 0.$

Property 5.1.3:

Let $A = \left(\tilde{a}_{ij}^{TzL}\right)$ be a square TrFM of order n where $\tilde{a}_{ij}^{TzL} = \left(a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4}\right)$. If a row (or column) is multiplied by a scalar k, then $|A|$ is multiplied by k.

Proof:

Case (i): K=0

If k=0, then the result is obvious since $|A| = 0$ when A has a 0 row.

Case (ii): K$\neq 0$

Let $B = (\tilde{b}_{ij}^{TzL})_{n\times n}$ where $\tilde{b}_{ij}^{TzL} = \left(b_{ij1}, b_{ij2}, b_{ij3}, b_{ij4}\right)$ is obtained from $A = (\tilde{a}_{ij}^{TzL})_{n\times n}$ by multiplying its r^{th} row by a scalar $k \neq 0$.

$\left(b_{ij1}, b_{ij2}, b_{ij3}, b_{ij4}\right) = \left(ka_{ij1}, ka_{ij2}, ka_{ij3}, ka_{ij4}\right)$ for all $i \neq r$ and

$\left(b_{rj1}, b_{rj2}, b_{rj3}, b_{rj4}\right) = \left(ka_{rj1}, ka_{rj2}, ka_{rj3}, ka_{rj4}\right)$ when k>0

$\left(b_{rj1}, b_{rj2}, b_{rj3}, b_{rj4}\right) = \left(ka_{rj4}, ka_{rj3}, ka_{rj2}, ka_{rj1}\right)$ when k<0.

Then by definition

$$|B| = \sum_{h\in S_n} sign\ h \prod_{i=1}^{n} \tilde{b}_{ih(i)}^{TzL}$$

$$= \sum_{h\in S_n} sign\ h \left[\tilde{b}_{1h(1)}^{TzL}\ \tilde{b}_{2h(2)}^{TzL} \dots \tilde{b}_{rh(r)}^{TzL} \dots \tilde{b}_{nh(n)}^{TzL}\right]$$

$$= \sum_{h\in S_n} sign\ h \left[\left(b_{1h(1)1}, b_{1h(1)2}, b_{1h(1)3}, b_{1h(1)4}\right) . \left(b_{2h(2)1}, b_{2h(2)2}, b_{2h(2)3}, b_{2h(2)4}\right) \dots \left(b_{rh(r)1}, b_{rh(r)2}, b_{rh(r)3}, b_{rh(r)4}\right) \dots \left(b_{nh(n)1}, b_{nh(n)2}, b_{nh(n)3}, b_{nh(n)4}\right)\right]$$

When k>0,

$$|B| = \sum_{h\in S_n} sign\ h \left[\left(a_{1h(1)1}, a_{1h(1)2}, a_{1h(1)3}, a_{1h(1)4}\right) . \left(a_{2h(2)1}, a_{2h(2)2}, a_{2h(2)3}, a_{2h(2)4}\right) \dots \left(ka_{rh(r)1}, ka_{rh(r)2}, ka_{rh(r)3}, ka_{rh(r)4}\right) \dots \left(a_{nh(n)1}, a_{nh(n)2}, a_{nh(n)3}, a_{nh(n)4}\right)\right]$$

$$= k \sum_{h\in S_n} sign\ h \left[\left(a_{1h(1)1}, a_{1h(1)2}, a_{1h(1)3}, a_{1h(1)4}\right) . \left(a_{2h(2)1}, a_{2h(2)2}, a_{2h(2)3}, a_{2h(2)4}\right) \dots \left(a_{rh(r)1}, a_{rh(r)2}, a_{rh(r)3}, a_{rh(r)4}\right) \dots \left(a_{nh(n)1}, a_{nh(n)2}, a_{nh(n)3}, a_{nh(n)4}\right)\right]$$

$$|B| = k \sum_{h\in S_n} sign\ h \prod_{i=1}^{n} \tilde{a}_{ih(i)}^{TzL}$$

$$|B| = k\ |A|.$$

Similarly when k<0,

$$|B| = \sum_{h\in S_n} sign\ h \left[\left(a_{1h(1)1}, a_{1h(1)2}, a_{1h(1)3}, a_{1h(1)4}\right) . \left(a_{2h(2)1}, a_{2h(2)2}, a_{2h(2)3}, a_{2h(2)4}\right) \dots \left(ka_{rh(r)4}, ka_{rh(r)3}, ka_{rh(r)2}, ka_{rh(r)1}\right) \dots \left(a_{nh(n)1}, a_{nh(n)2}, a_{nh(n)3}, a_{nh(n)4}\right)\right]$$

$$= k \sum_{h\in S_n} sign\ h \left[\left(a_{1h(1)1}, a_{1h(1)2}, a_{1h(1)3}, a_{1h(1)4}\right) . \left(a_{2h(2)1}, a_{2h(2)2}, a_{2h(2)3}, a_{2h(2)4}\right) \dots \left(a_{rh(r)1}, a_{rh(r)2}, a_{rh(r)3}, a_{rh(r)4}\right) \dots \left(a_{nh(n)1}, a_{nh(n)2}, a_{nh(n)3}, a_{nh(n)4}\right)\right]$$

$$|B| = k \sum_{h\in S_n} sign\ h \prod_{i=1}^{n} \tilde{a}_{ih(i)}^{TzL}$$

$$|B| = k\ |A|.$$

Hence the result follows.

Property 5.1.4:

If any two rows (or column) of a square TrFM A are interchanged then its determinant changes the sign of $|\boldsymbol{A}|$.

Proof:

Let $A = (\tilde{a}_{ij}^{TzL})_{n\times n}$ be a square TrFM. If $B = (\tilde{b}_{ij}^{TzL})_{n\times n}$ is obtained from A by interchanging the r^{th} and s^{th} row (r<s) of A , then $\tilde{b}_{ij}^{TzL} = \tilde{a}_{ij}^{TzL}$ $, i \neq r, i \neq s,$ and $\tilde{b}_{rj}^{TzL} = \tilde{a}_{rj}^{TzL}$.

Now $|B| = \sum_{h\in S_n} sign\ h\ \tilde{b}_{1h(1)}^{TzL} . \tilde{b}_{2h(2)}^{TzL} \dots \tilde{b}_{rh(r)}^{TzL} \dots \tilde{b}_{sh(s)}^{TzL} \dots \tilde{b}_{nh(n)}^{TzL}$

$= \sum_{h\in S_n} sign\ h\ \tilde{a}_{1h(1)}^{TzL} . \tilde{a}_{2h(2)}^{TzL} \dots \tilde{a}_{sh(r)}^{TzL} \dots \tilde{a}_{rh(s)}^{TzL} \dots \tilde{a}_{nh(n)}^{TzL}$

$$|B| = \sum_{h\in S_n} sign\ h\ [(a_{1h(1)1}, a_{1h(1)2}, a_{1h(1)3}, a_{1h(1)4}).(a_{2h(2)1}, a_{2h(2)2}, a_{2h(2)3}, a_{2h(2)4}) \dots (a_{sh(r)1}, a_{sh(r)2}, a_{sh(r)3}, a_{sh(r)4}) \dots (a_{rh(s)1}, a_{rh(s)2}, a_{rh(s)3}, a_{rh(s)4}) \dots (a_{nh(n)1}, a_{nh(n)2}, a_{nh(n)3}, a_{nh(n)4})]$$

Let $\lambda = \begin{pmatrix} 1 & 2 \dots & r \dots & s \dots & n \\ 1 & 2 \dots & s \dots & r \dots & n \end{pmatrix}$. Then λ is a transposition interchanging r and s and hence $\lambda = -1.$

Let $\varphi = \lambda h$. As h runs through all permutations on {1,2,…,n}, φ also runs over the same permutations because $\lambda h_1 = \lambda h_2$ or $h_1 = h_2$.

Now $\varphi = \lambda h$

$$\varphi = \begin{pmatrix} 1 & 2 \dots & r \dots & s \dots & n \\ 1 & 2 \dots & s \dots & r \dots & n \end{pmatrix} \begin{pmatrix} 1 & 2 & \dots & r \dots & s \dots & n \\ h(1) & h(2) & \dots & h(r) \dots & h(s) \dots & h(n) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & \dots & r \dots & s \dots & n \\ h(1) & h(2) & \dots & h(s) \dots & h(r) \dots & h(n) \end{pmatrix}$$

Therefore $\varphi(i) = h(i), i \neq r, i \neq s$ and $\varphi(r) = h(s),\ \varphi(s) = h(r)$. Since λ is odd, φ is even or odd according as h is odd or even. Therefore $sign\ \varphi = -sign\ h$

$$|B| = \sum_{h\in S_n} sign\ h\ [(a_{1h(1)1}, a_{1h(1)2}, a_{1h(1)3}, a_{1h(1)4}).(a_{2h(2)1}, a_{2h(2)2}, a_{2h(2)3}, a_{2h(2)4}) \dots (a_{sh(r)1}, a_{sh(r)2}, a_{sh(r)3}, a_{sh(r)4}) \dots (a_{rh(s)1}, a_{rh(s)2}, a_{rh(s)3}, a_{rh(s)4}) \dots (a_{nh(n)1}, a_{nh(n)2}, a_{nh(n)3}, a_{nh(n)4})]$$

$$|B| = -\sum_{h\in S_n} sign\ \varphi\ [(a_{1\varphi(1)1}, a_{1\varphi(1)2}, a_{1\varphi(1)3}, a_{1\varphi(1)4}).(a_{2\varphi(2)1}, a_{2\varphi(2)2}, a_{2\varphi(2)3}, a_{2\varphi(2)4}) \dots (a_{s\varphi(s)1}, a_{s\varphi(s)2}, a_{s\varphi(s)3}, a_{s\varphi(s)4}) \dots (a_{r\varphi(r)1}, a_{r\varphi(r)2}, a_{r\varphi(r)3}, a_{r\varphi(r)4}) \dots (a_{n\varphi(n)1}, a_{n\varphi(n)2}, a_{n\varphi(n)3}, a_{n\varphi(n)4})]$$

$|B| = -|A|.$

Hence proved.

Property 5.1.5:

If A is a square TrFM then $|\boldsymbol{A}| = |\boldsymbol{A}'|$

Proof:

Let $A = (\tilde{a}_{ij}^{TzL})_{n\times n}$ be a square TrFM and $A' = B = (\tilde{b}_{ij}^{TzL})_{n\times n}$. Then $\tilde{b}_{ij}^{TzL} = \tilde{a}_{ji}^{Tzl}$.

Now $|B| = \sum_{h\in S_n} sign\ h\ \ \tilde{b}_{1h(1)}^{TzL} .\ \tilde{b}_{2h(2)}^{TzL} \dots\ \tilde{b}_{nh(n)}^{TzL}$

$= \sum_{h \in S_n} sign\ h\ \tilde{a}^{TzL}_{h(1)1} \cdot \tilde{a}^{TzL}_{h(2)2} \cdots \tilde{a}^{TzL}_{h(n)n}$

Let φ be the permutation of {1, 2, …, n} such that $\varphi h = I$, the identity permutation. Then $\varphi = h^{-1}$. Since h runs over the whole set of permutations, φ also runs over the same set of permutations. Let h(i) = j then $i = h^{-1}(j) = \varphi(j)$ and $a_{h(i)i} = a_{j\varphi(j)}$ for all i, j. Therefore

$$|B| = \sum_{h \in S_n} sign\ h\ [(a_{h(1)11}, a_{h(1)12}, a_{h(1)13}, a_{h(1)14}).(a_{h(2)21}, a_{h(2)22}, a_{h(2)23}, a_{h(2)24}) \cdots (a_{h(n)n1}, a_{h(n)n2}, a_{h(n)n3}, a_{h(n)n4})]$$

$$|B| = \sum_{h \in S_n} sign\ \varphi\ [(a_{1\varphi(1)1}, a_{1\varphi(1)2}, a_{1\varphi(1)3}, a_{1\varphi(1)4}).(a_{2\varphi(2)1}, a_{2\varphi(2)2}, a_{2\varphi(2)3}, a_{2\varphi(2)4}) \cdots (a_{n\varphi(n)1}, a_{n\varphi(n)2}, a_{n\varphi(n)3}, a_{n\varphi(n)4})]$$

$|B| = |A|$.

Hence $|\mathrm{A}| = |\mathrm{A}'|$.

Property 5.1.6:

The determinant of a triangular TrFM is given by the product of its diagonal elements.

Proof:

Let $A = (\tilde{a}^{TzL}_{ij})_{n \times n}$ be a square triangular TrFM. Without loss of generality, let us assume that A is a lower triangular TrFM.

ie, $\tilde{a}^{TzL}_{ij} = 0$ for i < j. Take a term t of $|A|$.

t = $\tilde{a}^{TzL}_{h(1)1} \cdot \tilde{a}^{TzL}_{h(2)2} \cdots \tilde{a}^{TzL}_{h(n)n}$

Let $h(1) \neq 1$. So that 1 < h(1) and so $\tilde{a}^{TzL}_{h(1)1} = 0$ and thus t = 0. This means that each term is 0 if $h(1) \neq 1$.

Let now h(1) = 1 but $h(2) \neq 2$. Then 2 < h(2) and so $\tilde{a}^{TzL}_{h(2)2} = 0$ and thus t = 0. This means that each term is 0 if $h(1) \neq 1$ or $h(2) \neq 2$.

However in a similar manner we can see that each term must be 0 if $h(1) \neq 1$,or $h(2) \neq 2$,or …, $h(n) \neq n$. Consequently

$|A| = \tilde{a}^{TzL}_{11} . \tilde{a}^{TzL}_{22} \ldots \tilde{a}^{TzL}_{nn}$

$= \prod_{i=1}^{n} \tilde{a}^{TzL}_{ii}$

= product of its diagonal elements.

Similarly when A is an upper triangular, the result follows.

Property 5.1.7:

The determinant of a diagonal TrFM is the product of its diagonal elements.

Proof:

Let $A = (\tilde{a}^{TzL}_{ij})_{n \times n}$ be a diagonal TrFM.

ie, $\tilde{a}^{TzL}_{ij} = 0$ for i ≠ j. Take a term t of $|A|$.

t = $\tilde{a}^{TzL}_{h(1)1} \cdot \tilde{a}^{TzL}_{h(2)2} \cdots \tilde{a}^{TzL}_{h(n)n}$

Let $h(1) \neq 1$. then $\tilde{a}^{TzL}_{h(1)1} = 0$ and thus t = 0. This means that each term is 0 if $h(1) \neq 1$.

Let now h(1) = 1 but $h(2) \neq 2$. Then $\tilde{a}^{TzL}_{h(2)2} = 0$ and thus t = 0. This means that each term is 0 if $h(1) \neq 1$ or $h(2) \neq 2$.

However in the similar way, we can see that each term for which $h(1) \neq 1$,or $h(2) \neq 2$,or …, $h(n) \neq n$ must be 0. Consequently

$$|A| = \tilde{a}_{11}^{TzL} . \tilde{a}_{22}^{TzL} \dots \tilde{a}_{nn}^{TzL}$$
$$= \prod_{i=1}^{n} \tilde{a}_{ii}^{TzL}$$

= product of its diagonal elements.

Hence the result follows.

VI. CONCLUSION

In this article, We have concentrated the notion of Determinant of matrix using trapezoidal fuzzy number is discussed. Using the Determinant of Trapezoidal fuzzy Matrix work can be extended to the another domain of adjoint matrices is discuss in future.

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