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Heat Transfer in the flow of a Non-Newtonian second-order fluid over an enclosed torsionally oscillating discs in the presence of the magnetic field

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ABSTRACT : *The problem of the heat transfer in the flow of an incompressible non-Newtonian second-order fluid over an enclosed torsionally oscillating discs in the presence of the magnetic field has been discussed. The obtained differential equations are highly non-linear and contain upto fifth order derivatives of the flow and energy functions. Hence exact or numerical solutions of the differential equations are not possible subject to the given natural boundary conditions; therefore the regular perturbation technique is applied. The flow functions* H, G, L *and* M *are expanded in the powers of the amplitude* ε *(taken small) of the oscillations. The behaviour of the temperature distribution at different values of Reynolds number, phase difference, magnetic field and second-order parameters has been studied and shown graphically. The results obtained are compared with those for the infinite torsionally oscillating discs by taking the Reynolds number of out-flow* R_m *and circulatory flow* R_L *equals to zero. Nusselt number at oscillating and stator disc has also been calculated and its behaviour is represented graphically.*

Keywords: *Heat Transfer; Second-Order Fluid; Enclosed Torsionally Oscillating Disc, Magnetic Field.*

I. INTRODUCTION

The phenomenon of flow of the fluid over an enclosed torsionally oscillating disc (enclosed in a cylindrical casing) has important engineering applications. The most common practical application of it is the domestic washing machine and blower of curd etc. Soo [1] has considered first the problem of laminar flow over an enclosed rotating disc in case of Newtonian fluid. Sharma and Agarwal [2] have discussed the heat transfer from an enclosed rotating disc in case of newtonian fluid. Thereafter Singh K. R. and H. G. Sharma [3] have discussed the heat transfer in the flow of a second-order fluid between two enclosed rotating discs. The torsional oscillations of newtonian fluids have been discussed by Rosenblat [4]. He has also discussed the case when the Newtonian fluid is confined between two infinite torsionally oscillating discs [5]. Sharma & Gupta [6] have considered a general case of flow of a second-order fluid between two infinite torsionally oscillating discs. Thereafter Sharma & K. R. Singh [7]have solved the problem of heat transfer in the flow of a non-Newtonian second-order fluid between torsionally oscillating discs. Hayat [8] has considered non-Newtonian flows over an oscillating plate with variable suction. KR Singh, VK Agrawal & A Singh [9] have discussed Heat transfer in the flow of a non-Newtonian second-order fluid between two enclosed counter torsionally oscillating disc. Chawla [10] has considered flow past of a torsionally oscillating plane Riley &Wybrow [11] have considered the flow induced by the torsional oscillations of an elliptic cylinder. KR Singh, VK Agarwal [12] have solved heat Ttansfer in the flow of a non-newtonian second-order fluid between two enclosed counter torsionally oscillating discs with uniform suction and injection. Bluckburn[13] has considered a study of two-dimensional flow past of an oscillating cylinder. SadhnaKahre [14] studied the steady flow between a rotating and porous stationary disc in the presence of transverse magnetic field.Singh&Singhal [15] have discussed flow of a non-newtonianreiner-rivlin fluid between two enclosed torsionally oscillating porous discs. Agarwal & Agarwal [16] have solved flow of a non-newtoniansecond-order fluid over an enclosedtorsionally oscillating disc. Agarwal & Agarwal [17] have also discussed flow of a non-newtonian second-order fluid over an enclosed torsionally oscillating disc in the presence of magnetic field.

Due to complexity of the differential equations and tedious calculations of the solutions, no one has tried to solve the most practical problems of enclosed torsionally oscillating discs so far. The authors have considered the present problem of heat transfer in the flow of a non-Newtonian second-order fluid over an enclosed torsionally oscillating discs in the presence of the magnetic field and calculated successfully the steady and unsteady part both of the flow and energy functions. The flow and energy functions are expanded in the powers of the amplitude ε (assumed to be small) of the oscillations of the disc. The non-Newtonian effects are exhibited through two dimensionless parameters$\tau_1 \left(= \frac{n\mu_2}{\mu_1}\right)$ and$\tau_2 \left(= \frac{n\mu_3}{\mu_1}\right)$, where μ_1, μ_2, μ_3 are coefficient of Newtonian viscosity, elastico-viscosity and cross-viscosity respectively, n being the uniform frequency of the oscillation. The variation of temperature distribution with elastico-viscous parameterτ_1, cross-viscous parameter τ_2(based on the relation $\tau_1 = \alpha\tau_2$where $\alpha = -0.2$ as for 5.46% poly-iso-butylene type solution in cetane at 30℃ (Markowitz [18]), Reynolds number R, magnetic field *'m'* at different phase differenceτ is shown graphically.

II. FORMULATION OF THE PROBLEM

The constitutive equation of an incompressible second-order fluid as suggested by Colemann and Noll [19] can be written as:

$$\tau_{ij} = -p\delta_{ij} + 2\mu_1 d_{ij} + 2\mu_2 e_{ij} + 4\mu_3 c_{ij} \tag{1}$$

where

$$d_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), e_{ij} = \frac{1}{2}(a_{i,j} + a_{j,i}) + u^{\alpha}_{,i} u_{\alpha,j} c_{ij} = d_{i\alpha} d^{\alpha}_{j} \tag{2}$$

p is the hydrostatic pressure, τ_{ij} is the stress-tensor, d_{ij} is rate of deformation (rate of strain or flow) tensor, u_i and a_i are the velocity and acceleration vector.
The equation (1) together with the momentum equation for no extraneous force

$$\rho\left(\frac{\partial u_i}{\partial x} + u^{\alpha} u_{i,\alpha}\right) = \tau^{\alpha}_{i,\alpha} \tag{3}$$

and the equation of continuity for incompressible fluid$u^{i}_{,i} = 0$ (4)

whereρ is the density of the fluid and comma (,) represents covariant differentiation, form the set of governing equations.

In the three dimensional cylindrical set of co-ordinates (r, θ, z) the system consists of a finite oscillating disc of radius r_s (coinciding with the plane $z = 0$) performing rotatory oscillations of the type $r\Omega\cos\tau$ of small amplitudeϵ , about the perpendicular axis $r = 0$ with a constant angular velocity Ω in an incompressible second-order fluid forming the part of a cylindrical casing or housing. The top of the casing (coinciding with the plane $z = z_o < r_s$) may be considered as a stationary disc (stator) placed parallel to and at a distance equal to gap length $z_0$from the oscillating disc. The symmetrical radial steady outflow has a small mass rate '**m**' of radial outflow (*'$-\boldsymbol{m}$'* for net radial inflow). The inlet condition is taken as a simple radial source flow along z-axis starting from radiusr_0. A constant magnetic field B_0 is applied normal to the plane of the oscillating disc. The induced magnetic field is neglected.

Assuming (u, v, w) as the velocity components along the cylindrical system of axes (r, θ, z) the relevant boundary conditions of the problem are:

$$z = 0, \; u = 0, \quad v = r\Omega e^{i\tau}(\text{Real part}), w = 0, \qquad T = T_a$$

$$z = z_0, \; u = 0, \; v = 0, \qquad w = 0, \qquad T = T_b \tag{5}$$

where the gap $z_0$is assumed small in comparison with the disc radius r_s. The velocity components for the axisymmetric flow compatible with the continuity criterion can be taken as[1],[2],[3].

$$U = -\xi H'(\zeta,\tau) + \left(\frac{R_m}{R_z}\right)\frac{M'(\zeta,\tau)}{\xi},\; V = \xi G(\zeta,\tau) + \left(\frac{R_L}{R_z}\right)\frac{L(\zeta,\tau)}{\xi}, W = 2H(\zeta,\tau) \tag{6}$$

and for the temperature, we take

$$T = T_b + \left(\frac{v_1\Omega}{C_v}\right)\{\phi(\zeta,\tau) + \xi^2\psi(\zeta,\tau)\} \tag{7}$$

Where$U = \frac{u}{\Omega z_0}, V = \frac{v}{\Omega z_0}, W = \frac{w}{\Omega z_0}, \xi = \frac{r}{z_0}, \zeta, \tau$are dimensionless quantities and $H(\zeta,\tau), G(\zeta,\tau), L(\zeta,\tau), M'(\zeta,\tau)$, $\phi(\zeta,\tau), \Psi(\zeta,\tau)$ are dimensionless function of the dimensionless variables $\zeta = \frac{z}{z_0}$and $\tau = nt$. $R_m = \frac{m}{2\pi\rho z_0 v_1}, R_m = \frac{L}{2\pi\rho z_0 v_1}$are dimensionless number to be called the Reynolds number of net radial outflow and circulatory flow respectively. $R_z \left(= \frac{\Omega z_0^2}{v_1}\right)$be the flow Reynolds number. The small mass rate *'$\boldsymbol{m}$'* of the radial outflow is represented by$\boldsymbol{m} = 2\pi\rho\int_0^{z_0} rudz$ (8)

Using expression (6) and (7), the boundary condition (5) transform for $G, L\&H$ into the following form:

$$G(0,\tau) = Real(e^{i\tau}), G(1,\tau) = 0, L(0,\tau) = 0, \; L(1,\tau) = 0, H(0,\tau) = 0,$$

$$H(1,\tau) = 0, H'(0,\tau) = 0, \quad H'(1,\tau) = 0, \phi(0,\tau) = \frac{1}{E} = S, \; \phi(1,\tau) = 0, \Psi(0,\tau) = 0, \; \Psi(1,\tau) = 0 \tag{9}$$

where$E\left\{= \frac{\Omega \nu_1}{C_v(T_a - T_b)}\right\}$ is the Eckert number.
The conditions on M on the boundaries are obtainable form the expression (8) for m as follows:

$$M(1,\tau) - M(0,\tau) = 1 \qquad (10)$$

which on choosing the discs as streamlines reduces to

$$M(1,\tau) = 1, M(0,\tau) = 0 \qquad (11)$$

Using eqs.(1) and expression (6)in equation (3) and neglecting the squares & higher powers of R_m/R_z (assumed small), we have the following equations in dimensionless form:

$$-\frac{1}{\rho z_0}\frac{\partial p}{\partial \xi} = -n\Omega z_0\left\{\xi H' - \left(\frac{R_m}{R_z}\right)\frac{\partial M'}{\xi}\right\} + \Omega^2 z_0 \xi\left(H'^2 - 2HH'' - G^2\right) + \Omega^2 z_0\left(\frac{R_m}{R_z}\right)\frac{2HM''}{\xi} - \Omega^2 z_0\left(\frac{R_L}{R_z}\right)\frac{2LG}{\xi} + \frac{\nu_1\Omega}{z_0}\left\{H'''\xi - \left(\frac{R_m}{R_z}\right)\frac{M'''}{\xi}\right\} - \frac{2\nu_2}{z_0}\left[\left\{\frac{n\Omega}{2}\left(\frac{R_m}{R_z}\right)\frac{\partial M'''}{\xi} - \xi H'''\right\} + \Omega^2\xi\left(H''^2 - HH^{iv}\right) + \left(\frac{R_m}{R_z}\right)\frac{\Omega^2}{\xi}\left(H'''M' + H''M'' + H'M''' +\right.\right.$$

HMiv−RLRZ2Ω2ξL′G′+LG′′−4ν3Ω2z0RmRz12ξH′′′M′+H′′M′′+H′M′′′−RLRz12ξ2L′G′+LG′′+ξ4H′′2−G′2−2H′H′′′+σB02Ωz0ρ−ξH′+RmRzM′ξ (12)

$$0 = -n\Omega z_0\left\{\xi\partial G + \left(\frac{R_L}{R_z}\right)\frac{\partial L}{\xi}\right\} - 2\Omega^2 z_0\xi(HG' - H'G) - \Omega^2 z_0\left(\frac{R_m}{R_z}\right)\frac{2M'G}{\xi} - \Omega^2 z_0\left(\frac{R_L}{R_z}\right)\frac{2HL'}{\xi} + \frac{\nu_1\Omega}{z_0}\left\{\xi G'' +\right.$$

RLRzL′′ξ+2ν2z0nΩ2ξG′′+RLRz∂L′′ξ+RLRzΩ2ξH′′L′+H′′′L+HL′′′+H′L′′+Ω2ξHG′′′−H′′G′+RMRz2Ω2ξM′G′′+M′′G′+2ν3Ω2z0ξH′G′′−H′′G′+RLRz1ξH′′L′+H′′′L+H′L′′+RMRz1ξ2M′′G′+M′G′′−σB02Ωz0ρξG+RLRzLξ
(13)

$$-\left(\frac{1}{\rho z_0}\right)\left(\frac{\partial p}{\partial \zeta}\right) = 2n\Omega z_0\partial H + 4\Omega^2 z_0 HH' - 2\nu_1\frac{\Omega H''}{z_0} - \frac{2\nu_2}{z_0}\left\{n\Omega\partial H'' + 2\Omega^2\xi^2(H''H''' + G'G'') + \Omega^2(22H'H'' + 2HH''') - \left(\frac{R_M}{R_z}\right)2\Omega^2(H''M''' + H'''M'') + \left(\frac{R_L}{R_z}\right)2\Omega^2(L'G'' + L''G')\right\} - \frac{2\nu_3\Omega^2}{z_0}\left\{\xi^2(H''H''' + G'G'') + 14H'H'' - \left(\frac{R_M}{R_z}\right)(H''M''' + H'''M'') + \left(\frac{R_L}{R_z}\right)(L'G'' + L''G')\right\}$$

(14)

$$\rho C_v\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}\right) = K\left\{\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\left(\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right\} + \Phi \qquad (15)$$

Where$\Phi = \tau_j^i d_i^j$ (16)

$C_v$is the specific heat at constant volume, Φ be the viscous-dissipation function,τ_j^i is the mixed deviatoric stress tensor, K is the thermal conductivity, ρ is the density of the fluid; B_0 and σ are intensity of the magnetic field and conductivity of the fluid considered.
Differentiating (12) w.r.t. ζ and (14) w.r.t. ξ and then eliminating $\partial^2 p/\partial\zeta.\partial\xi$from the equation thus obtained. We get

$$-n\Omega z_0\{\xi\partial H'' - (R_m/R_z)\partial M''/\xi\} - 2\Omega^2 z_0\xi(HH''' + GG') + (R_m/R_z)(2\Omega^2 z_0/\xi)(H'M'' + HM''') - (R_L/R_z)(2\Omega^2 z_0/\xi)(LG' + L'G) - (\nu_1\Omega/z_0)\{(R_m/R_z)(M^{iv}/\xi) - \xi H^{iv}\} - (2\nu_2/z_0)\left[(n\Omega/2)\{(R_m/R_z)(\partial M^{iv}/\xi) - \xi\partial H^{iv}\} - \Omega^2\xi(2H''H''' + H'H^{iv} + HH^{v} + 4G'G'') + (R_m/R_z)(\Omega^2/\xi)(2H'''M'' + H^{iv}M' + 2H''M''' + 2H'M^{iv} + HM^{v}) - (R_L/R_z)(2\Omega^2/\xi)(2L'G'' + L''G' + LG''')\right] - (2\nu_3\Omega^2/z_0)\{(R_m/R_z)(1/\xi)(H^{iv}M' + 2H''M'' + 2H''M''' + H'M^{iv}) - (R_L/R_z)(1/\xi)(3L'G'' + 2L''G' + LG''') - \xi(H'H^{iv} + 3G'G'' + 2H''H''')\} + (\sigma B_0^2\Omega z_0/\rho)\{-\xi H'' + (R_m/R_z)(M''/\xi)\} = 0$$

(17)

On equating the coefficients of ξ and 1/ξ from the equation (13) & (17), we get the following equations:

$$G'' = R\partial G + 2\in R(HG' - H'G) - \tau_1\partial G'' - 2\in\tau_1(HG''' - H''G') - 2\in\tau_2(H'G'' - H''G') + m^2 G \qquad (18)$$

$$L'' = R\partial L + 2\in R(M'G + HL') - \tau_1\partial L'' - 2\in\tau_1(H''L' + H'''L + HL''' + H'L'' + 2M'G'' + 2M''G') - 2\in\tau_2(H''L' + H'''L + H'L'' + 2M''G' + M'G'') + m^2 L \qquad (19)$$

$$H^{iv} = R\partial H'' + 2\in R(HH''' + GG') - \tau_1\partial H^{iv} - 2\in\tau_1(H'H^{iv} + HH^{V} + 2H''H''' + 4G'G'') - 2\in\tau_2(H'H^{iv} + 2H''H''' + 3G'G'') + m^2 H'' \qquad (20)$$

$$M^{iv} = R\partial M'' + 2\in R(H'M'' + HM''' - LG' - L'G) - \tau_1\partial M^{iv} - 2\in\tau_1(2H'''M'' + H^{iv}M' + 2H''M''' + 2H'M^{iv} + HM^{v} - 4L'G'' - 2L''G' - 2LG''') - 2\in\tau_2(H^{iv}M' + 2H'''M'' + 2H''M''' + H'M^{iv} - 3L'G'' - 2L''G' - LG''')$$

$+m^2M''$ (21)
where R ($=nz_0^2/\nu_1$) is the Reynolds number, $\tau_1(=n\nu_2/\nu_1)$, $\tau_2(=n\nu_3/\nu_1)$ and $\in(=\Omega/n)$ are the dimensionless parameter, $m^2 = \sigma B_0^2 z_0^2/\mu_1$ is the dimensionless magnetic field and $R_{\mathbf{m}}/R_{\mathbf{L}} = \mathbf{m/L} \approx 1$.
Using (7), (16) in (15) and equating the cofficient of ξ^2 and independent term of ξ^2, we get

$$\Psi'' = \in RP_r[\partial\Psi/\in - 2H'\Psi + 2H\Psi' - H''^2 - G'^2 - \tau_1(H''\partial H'' + G'\partial G') - 2\in\tau_1(H'H''^2 + H'G'^2 + HH''H''' + HG'G'') - 3\in\tau_2(H'H''^2 + H'G'^2)], \quad (22)$$

$$4\Psi + \phi'' = \in RPr\,[\partial\phi/\in + (R_{\mathbf{m}}/R_z)2M'\Psi + 2H\phi' - 12H'^2 + (R_{\mathbf{m}}/R_z)2H''M'' - (R_{\mathbf{L}}/R_z)\,2L'G' - \tau_1\{12H'\partial H' - (R_{\mathbf{m}}/R_z)(H''\partial M'' + M''\partial H'') + (R_{\mathbf{L}}/R_z)(G'\partial L')\}] \quad (23)$$

where $P_r\,(=\mu_1 C_v/K)$ is the Prandtl number.

III. SOLUTION OF THE PROBLEM

Substituting the expressions

$$G(\zeta,\tau) = \Sigma\in^N G_N(\zeta,\tau);\ L(\zeta,\tau) = \Sigma\in^N L_N(\zeta,\tau);\ H(\zeta,\tau) = \Sigma\in^N H_N(\zeta,\tau);\ M(\zeta,\tau) = \Sigma\in^N M_N(\zeta,\tau) \quad (24)$$

into (18) to (23) neglecting the terms with coefficient of $\in^2$ (assumed negligible small) and equating the terms independent of $\in$ and coefficient of $\in$, we get the following equations:

$$G_0'' = R\,\partial G_0/\partial\tau - \tau_1\partial G_0''/\partial\tau + m^2G_0 \quad (25)$$

$$G_1'' = R\,\partial G_1/\partial\tau - 2R(H_0'G_0' - H_0G_0') - \tau_1\partial G_1''/\partial\tau - 2\tau_1(H_0G_0''' - H_0''G_0') - 2\tau_2(H_0'G_0'' - H_0''G_0') + m^2G_1 \quad (26)$$

$$L_0'' = R\,\partial L_0/\partial\tau - \tau_1\partial L_0''/\partial\tau + m^2L_0 \quad (27)$$

$$L_1'' = R\partial L_1/\partial\tau - 2R(M_0'G_0 + H_0L_0') - \tau_1\partial L_1''/\partial\tau - 2\tau_1(H_0'''L_0 + H_0''L_0' + H_0'L_0'' + H_0L_0''' + 2M_0''G_0' + 2M_0'G_0'') - 2\tau_2(H_0'''L_0 + H_0''L_0' + H_0'L_0'' + 2M_0''G_0' + M_0'G_0'') + m^2L_1 \quad (28)$$

$$H_0^{iv} = R\partial H_0''/\partial\tau - \tau_1\partial H_0^{iv}/\partial\tau + m^2H_0'' \quad (29)$$

$$H_1^{iv} = R\partial H_1''/\partial\tau + 2R(H_0H_0''' + G_0G_0') - \tau_1\partial H_1^{iv}/\partial\tau - 2\tau_1(H_0'H_0^{iv} + H_0H_0^{v} + 2H_0''H_0''' + 4G_0'G_0'') - 2\tau_2(3G_0'G_0'' + H_0'H_0^{iv} + 2H_0''H_0''') + m^2H_1'' \quad (30)$$

$$M_0^{iv} = R\,\partial M_0''/\partial\tau - \tau_1\partial M_0^{iv}/\partial\tau + m^2M_0'' \quad (31)$$

$$M_1^{iv} = R\partial M_1''/\partial\tau + 2R(H_0'M_0'' + H_0M_0''' - L_0'G_0 - L_0G_0') - \tau_1\partial M_1^{iv}/\partial\tau - 2\tau_1(2H_0'''M_0'' + H_0^{iv}M_0' + 2H_0''M_0''' - 4L_0'G_0'' - 2L_0''G_0' - 2L_0G_0''' + H_0M_0^{v} + 2H_0'M_0^{iv}) - 2\tau_2(2H_0'''M_0'' + H_0^{iv}M_0' + 2H_0''M_0''' - 3L_0'G_0'' - 2L_0''G_0' - L_0G_0''' + H_0'M_0^{iv}) + m^2M_1'' \quad (32)$$

$$\Psi_0'' = RP_r\partial\Psi_0, \quad (33)$$

$$\Psi_1'' = RP_r[\partial\Psi_1 - 2H_0'\Psi_0 + 2H_0\Psi_0' - H_0''^2 - G_0'^2 - \tau_1(H_0''\partial H_0'' + G_0'\partial G_0'), \quad (34)$$

$$4\Psi_0 + \phi_0'' = RP_r\partial\phi_0, \quad (35)$$

$$4\Psi_1 + \phi_1'' = RP_r\,[\partial\phi_1 + (R_m/R_z)2M_0'\Psi_0 + 2H_0\phi_0' - 12H_0'^2 + (R_m/R_z)2H_0''M_0'' - (R_L/R_z)2L_0'G_0' - \tau_1\{12H_0'\partial H_0' - (R_m/R_z)(H_0''\partial M_0'' + M_0''\partial H_0'') + (R_L/R_z)(G_0'\partial L_0')\}\,]. \quad (36)$$

Taking $G_n(\zeta,\tau) = G_{ns}(\zeta) + e^{i\tau}G_{nt}(\zeta)$; $L_n(\zeta,\tau) = L_{ns}(\zeta) + e^{i\tau}L_{nt}(\zeta)$
$H_n(\zeta,\tau) = H_{ns}(\zeta) + e^{2i\tau}H_{nt}(\zeta)$; $M_n(\zeta,\tau) = M_{ns}(\zeta) + e^{2i\tau}M_{nt}(\zeta)$ (37)

Complex notation has been adopted here with the convention that only real parts of the complex quantities have the physical meaning.
Using (19) and (28), the boundary conditions (8) & (10) for n = 0, 1 transforms to

$G_{0s}(0) = 0$, $G_{0t}(0) = 1$, $G_{1s}(0) = 0$, $G_{1t}(0) = 0$, $G_{0s}(1) = 0$, $G_{0t}(1) = 0$, $G_{1s}(1) = 0$, $G_{1t}(1) = 0$, $H_{0s}(0) = 0$ $H_{0t}(0) = 0$, $H_{1s}(0) = 0$, $H_{1t}(0) = 0$,
$H_{0s}(1) = 0$, $H_{0t}(1) = 0$, $H_{1s}(1) = 0$, $H_{1t}(1) = 0$, $H'_{0s}(0) = 0$, $H'_{0t}(0) = 0$, $H'_{1s}(0) = 0$, $H'_{1t}(0) = 0$, $H'_{0s}(1) = 0$, $H'_{0t}(1) = 0$, $H'_{1s}(1) = 0$, $H'_{1t}(1) = 0$,
$L_{0s}(0) = 0$, $L_{0t}(0) = 0$, $L_{1s}(0) = 0$, $L_{1t}(0) = 0$, $L_{0s}(1) = 0$, $L_{0t}(1) = 0$, $L_{1s}(1) = 0$, $L_{1t}(1) = 0$, $M'_{0s}(0) = 0$, $M'_{0t}(0) = 0$, $M'_{1s}(0) = 0$, $M'_{1t}(0) = 0$,
$M'_{0s}(1) = 0$, $M'_{0t}(1) = 0$, $M'_{1s}(1) = 0$, $M'_{1t}(1) = 0$, $M_{0s}(0) = 0$, $M_{0t}(0) = 0$, $M_{1s}(0) = 0$, $M_{1t}(0) = 0$, $M_{0s}(1) = 1$, $M_{0t}(1) = 0$, $M_{1s}(1) = 0$, $M_{1t}(1) = 0$.
$\Psi_{0s}(0) = 0$, $\Psi_{0t}(0) = 0$, $\Psi_{1s}(0) = 0$, $\Psi_{1t}(0) = 0$, $\Psi_{0s}(1) = 0$, $\Psi_{0t}(1) = 0$, $\Psi_{1s}(1) = 0$, $\Psi_{1t}(1) = 0$, $\phi_{0s}(0) = S$, $\phi_{0t}(0) = 0$, $\phi_{1s}(0) = 0$, $\phi_{1t}(0) = 0$,
$\phi_{0s}(1) = 0$, $\phi_{0t}(1) = 0$, $\phi_{1s}(1) = 0$, $\phi_{1t}(1) = 0$, (38)

$G_{0s}(\zeta) = G_{1s}(\zeta) = G_{1t}(\zeta) = 0$, $G_{0t}(\zeta) = \{1 - (e^f/2\text{Sinh } f)\}e^{f\zeta} + (e^f/2\text{Sinh } f)\}e^{-f\zeta}$,
where $f = \{(iR + m^2)/(1 + I\tau_1)\}^{1/2} = A + iB$,
where $A = [[(m^2 + R\tau_1) + \{(m^2 + R\tau_1)^2 + (R - m^2\tau_1)^2\}]/\{2(1 + \tau_1^2)\}]^{1/2}$,
$B = [[\{(m^2 + R\tau_1)^2 + (R - m^2\tau_1)^2\} - (m^2 + R\tau_1)]/\{2(1 + \tau_1^2)\}]^{1/2}$,

$$G_0(\zeta,\tau) = \text{Real}\{e^{i\tau}G_{0t}(\zeta)\} = (A_3 + A_5)\text{Cos}\tau - (A_4 + A_6)\text{Sin}\tau,$$

where $A_1 = e^A(\text{Cos}^2B\,\text{Sinh}A + \text{Cosh}A\,\text{Sin}^2B)/\{2(\text{Sinh}^2A + \text{Sin}^2B)\}$,
$A_2 = e^A(\text{Sin}B\,\text{Cos}B\,\text{Sinh}A - \text{Cos}B\,\text{Cosh}A\,\text{Sin}B)/\{2(\text{Sinh}^2A + \text{Sin}^2B)\}$,
$A_3 = e^{A\zeta}\{(1 - A_1)\text{Cos}B\zeta + A_2\text{Sin}B\zeta\}$, $A_4 = e^{A\zeta}\{(1 - A_1)\text{Sin}B\zeta - A_2\text{Cos}B\zeta\}$,

$A_5 = e^{-A\zeta}\{A_1 CosB\zeta + A_2 SinB\zeta\}$, $A_6 = e^{-A\zeta}\{A_2 CosB\zeta - A_1 SinB\zeta\}$,
$G(\zeta,\tau) = G_0(\zeta,\tau) + \in G_1(\zeta,\tau)$.
$M_{0s}(\zeta) = C_1 e^{m\zeta}/m^2 + C_2 e^{-m\zeta}/m^2 + C_3\zeta + C_4, M'_{0s}(\zeta) = C_1 e^{m\zeta}/m - C_2 e^{-m\zeta}/m + C_3$,
Where $C_1 = C_2(e^{-m}-1)/(e^m-1)$, $C_2 = m^2(e^m-1)/\{4+e^m(m-2)-e^{-m}(m+2)\}, C_3 = -(C_1-C_2)/m, C_4 = (C_1+C_2)/m^2$.
$M_{0t}(\zeta) = M_{1s}(\zeta) = M_{1t}(\zeta) = 0$, $M(\zeta,\tau) = M_0(\zeta,\tau) + \in M_1(\zeta,\tau) = M_0(\zeta,\tau)$.

$H_{0s}(\zeta) = H_{0t}(\zeta) = H_{1s}(\zeta) = 0$,
$H_{1t}(\zeta) = C_1 e^{d\zeta}/d2 + C_2 e^{-d\zeta}/d2 + C_3\zeta + C_4 + (Z_1 - f^2 Z_2)[\{1-(e^f/2Sinhf)\}2e^{f\zeta}/4f - e^{2f}(1-\zeta)/\ (16fSinh^2 f)]$,
where $d = \{(2iR+m^2)/(1+2i\tau_1)\}^{1/2} = C + iD$,
where $C = [[(m^2+4R\tau_1)+\{(m^2+4R\tau_1)^2+(2R-2m^2\tau_1)^2\}]/\{2(1+4\tau_1^2)\}]^{1/2}$,
$D = [[\{(m^2+4R\tau_1)^2+(R-4m^2\tau_1)^2\} - (2m^2+2R\tau_1)]/\{2(1+4\tau_1^2)\}]^{1/2}$,
$Z_1 = 2R/\{(1+I\tau_1)(4f^2-d^2)\}$, $Z_2 = (8\tau_1+6\tau_2)/\{(1+I\tau_1)(4f^2-d^2)\}$, $C_1 = \{d(Z_{10}-Z_9-Z_6+Z_5+C_2(e^{-d}-1)\}$,
$C_2 = Z_{11}/\{4-e^{-d}(d+2)+e^d(d-2)\}$, $C_3 = -\{(C_1/d)-(C_2/d)+Z_5-Z_6\}$, $C_4 = -\{(C_1/d^2)+(C_2/d^2)+Z_3-Z_4\}$,
$Z_3 = Z_1[\{1-(e^f/2Sinhf)\}^2/4f - fe^{2f}/16fSinh^2 f]$, $Z_4 = Z_2[f\{1-(e^f/2Sinhf)\}^2/4 - fe^{2f}/16Sinh^2 f]$,
$Z_5 = Z_1[\{1-(e^f/2Sinhf)\}^2/2 + e^{2f}/8Sinh^2 f]$, $Z_6 = Z_2[f^2\{1-(e^f/2Sinhf)\}^2/2 + f^2 e^{2f}/8Sinh^2 f]$,
$Z_7 = Z_1[e^{2f}\{1-(ef/2Sinhf)\}^2/4f - 1/16fSinh^2 f]$, $Z_8 = Z_2[fe^{2f}\{1-(e^f/2Sinhf)\}^2/4 - f/16Sinh^2 f]$,
$Z_9 = Z_1[e^{2f}\{1-(e^f/2Sinhf)\}^2/2 + 1/8Sinh^2 f]$, $Z_{10} = Z_2[f^2 e^{2f}\{1-(e^f/2Sinhf)\}^2/2 + f^2/8Sinh^2 f]$,
$H_1(\zeta,\tau) = Real\{e^{2I\tau} H_{1t}(\zeta)\} = (A_{81}+A_{89})Cos2\tau - (A_{82}+A_{90})Sin2\tau$,
where $A_{13} = 4A^2-4B^2-C^2+D^2, A_{14} = 8AB-2CD$, $A_{15} = 2R(A_{13}-\tau_1 A_{14})/\{(A_{13}-\tau_1 A_{14})^2+(\tau_1 A_{13}+A_{14})^2\}$,
$A_{16} = -2R(A_{13}\tau_1+A_{14})/\{(A_{13}-\tau_1 A_{14})^2+(\tau_1 A_{13}+A_{14})^2\}, A_{17} = (8\tau_1+6\tau_2)(A_{13}-2\tau_1 A_{14})/\{(A_{13}-2\tau_1 A_{14})^2+(2\tau_1 A_{13}+A_{14})^2\}$,
$A_{18} = -(8\tau_1+6\tau_2)(2\tau_1 A_{13}+A_{14})/\{(A_{13}-2\tau_1 A_{14})^2+(2\tau_1 A_{13}+A_{14})^2\}, A_{19} = [A\{(1-A_1)^2-A_2^2\}+2BA_2(1-A_1)]/\{4(A^2+B^2)\}$,
$A_{20} = -[B\{(1-A_1)^2-A_2^2\}+2AA_2(1-A_1)]/\{4(A^2+B^2)\}, A_{21} = Sinh^2 ACos^2 B - Cosh^2 ASin^2 B$,
$A_{22} = 2SinhACosBCoshASinB, A_{23} = e^{2A}(A_{21}Cos2B+A_{22}Sin2B)/(A_{21}^2+A_{22}^2)$,
$A_{24} = e^{2A}(A_{21}Sin2B - A_{22}Cos2B)/(A_{21}^2+A_{22}^2), A_{25} = (AA_{23}+BA_{24})/\{16(A^2+B^2)\}, A_{26} = (AA_{24}-BA_{23})/\{16(A^2+B^2)\}$,
$A_{27} = A_{15}(A_{19}-A_{25})-A_{16}(A_{20}-A_{26})$, $A_{28} = A_{16}(A_{19}-A_{25})+A_{15}(A_{20}-A_{26})$, $B_1 = (1-A_1)^2-A_2^2$, $B_2 = -2(1-A_1)A_2$,
$X_1 = (A^2-B^2)(A_{17}A_{27}-A_{18}A_{28})-2AB(A_{18}A_{27}+A_{17}A_{28})$,
$X_2 = 2AB(A_{17}A_{27}-A_{18}A_{28})+(A^2-B^2)(A_{18}A_{27}+A_{17}A_{28})$,
$A_{29} = (X_1A_{15}+X_2A_{16})/(A_{15}^2+A_{16}^2)$, $A_{30} = (X_2A_{15}-X_1A_{16})/(A_{15}^2+A_{16}^2)$,
$A_{31} = A_{15}\{(B_1/2)+(A_{23}/8)\} - A_{16}\{(B_2/2)+(A_{24}/8)\}$, $A_{32} = A_{16}\{(B_1/2)+(A_{23}/8)\} + A_{15}\{(B_2/2)+(A_{24}/8)\}$,
$X_3 = (A^2-B^2)(A_{17}A_{31}-A_{18}A_{32})-2AB(A_{18}A_{31}+A_{17}A_{32}), X_4 = 2AB(A_{17}A_{31}-A_{18}A_{32}) + (A^2-B^2)(A_{18}A_{31}+A_{17}A_{32})$,
$A_{33} = (X_3A_{15}+X_4A_{16})/(A_{15}^2+A_{16}^2), A_{34} = (X_4A_{15}-X_3A_{16})/(A_{15}^2+A_{16}^2), A_{35} = e^{2A}(A_{19}Cos2B-A_{20}Sin2B)$,
$A_{36} = e^{2A}(A_{20}Cos2B+A_{19}Sin2B)$, $A_{37} = (AA_{21}-BA_{22})/[16\{(AA_{21}-BA_{22})^2+(BA_{21}+AA_{22})^2\}]$,
$A_{38} = -(BA_{21}+AA_{22})/[16\{(AA_{21}-BA_{22})^2+(BA_{21}+AA_{22})^2\}]$, $A_{39} = A_{15}(A_{35}-A_{37})-A_{16}(A_{36}-A_{38})$,
$A_{40} = A_{16}(A_{35}-A_{37})+A_{15}(A_{36}-A_{38})$, $X_5 = (A^2-B^2)(A_{17}A_{39}-A_{18}A_{40})-2AB(A_{18}A_{39}+A_{17}A_{40})$,
$X_6 = 2AB(A_{17}A_{39}-A_{18}A_{40})+(A^2-B^2)(A_{18}A_{39}+A_{17}A_{40})$, $A_{41} = (X_5A_{15}+X_6A_{16})/(A_{15}^2+A_{16}^2)$,
$A_{42} = (X_6A_{15}-X_5A_{16})/(A_{15}^2+A_{16}^2)$, $A_{43} = e^{2A}(B_1Cos2B-B_2Sin2B)/2, A_{44} = e^{2A}(B_2Cos2B+B_1Sin2B)/2$,
$A_{45} = A_{21}/\{8(A_{21}^2+A_{22}^2)\}$, $A_{46} = -A_{22}/\{8(A_{21}^2+A_{22}^2)\}$, $A_{47} = A_{15}(A_{43}+A_{45})-A_{16}(A_{44}+A_{46})$,
$A_{48} = A_{16}(A_{43}+A_{45})+A_{15}(A_{44}+A_{46}), X_7 = (A^2-B^2)(A_{17}A_{47}-A_{18}A_{48})-2AB(A_{18}A_{47}+A_{17}A_{48})$,
$X_8 = 2AB(A_{17}A_{47}-A_{18}A_{48}) + (A^2-B^2)(A_{18}A_{47}+A_{17}A_{48}), A_{49} = (X_7A_{15}+X_8A_{16})/(A_{15}^2+A_{16}^2)$,
$A_{50} = (X_8A_{15}-X_7A_{16})/(A_{15}^2+A_{16}^2)$, $A_{51} = (C^2-D^2)(e^C CosD-1)-2CDe^C SinD$,
$A_{52} = 2CD(e^C CosD-1) + (C^2-D^2)e^C SinD$,
$A_{53} = C(e^C CosD-C-1)-D(e^C SinD-D), A_{54} = D(e^C CosD-C-1)+C(e^C SinD-D), A_{55} = A_{27}-A_{29}+A_{31}-A_{33}-A_{39}+A_{41}$,
$A_{56} = A_{28}-A_{30}+A_{32}-A_{34}-A_{40}+A_{42}, A_{57} = A_{49}-A_{47}-A_{33}+A_{31}, A_{58} = A_{50}-A_{48}-A_{34}+A_{32}$,
$A_{59} = A_{51}A_{55}-A_{52}A_{56}-A_{53}A_{57}+A_{54}A_{58}, A_{60} = A_{52}A_{55}+A_{51}A_{56}-A_{54}A_{57}-A_{53}A_{58}$,
$A_{65} = 4-e^{-C}\{(C+2)CosD+DsinD\}+e^C\{(C-2)CosD-DsinD\}$,
$A_{66} = -e^{-C}\{DcosD-(C+2)CosD\}+e^C\{(C-2)SinD+DcosD\}$,
$A_{67} = (A_{59}A_{65}+A_{60}A_{66})/(A_{65}^2+A_{66}^2), A_{68} = (A_{60}A_{65}-A_{59}A_{66})/(A_{65}^2+A_{66}^2)$,
$A_{69} = CA_{57}-DA_{58}+A_{67}(e^{-C}cosD-1)+A_{68}e^{-C}SinD, A_{70} = DA_{57}+CA_{58}+A_{68}(e^{-C}cosD-1)-A_{67}e^{-C}SinD$,
$A_{71} = \{A_{69}(e^C CosD-1)+A_{70}e^C SinD\}/\{(e^C CosD-1)^2+e^{2C}Sin^2 D\}$,
$A_{72} = \{A_{70}(e^C CosD-1)-A_{69}e^C SinD\}/\{(e^C CosD-1)2+e^{2C}Sin^2 D\}$,
$A_{73} = -[\{(CA_{71}+DA_{72}-CA_{67}-DA_{68})/(C^2+D^2)\}+A_{31}-A_{33}], A_{74} = -[\{(CA_{72}-DA_{71}-CA_{68}+DA_{67})/(C^2+D^2)\}+A_{32}-A_{34}]$,
$A_{75} = \{A_{71}(C^2-D^2)+2CDA_{72}\}/\{(C^2-D^2)^2+4C^2D^2\}, A_{76} = \{A_{72}(C^2-D^2)-2CDA_{71}\}/\{(C^2-D^2)^2+4C^2D^2\}$,
$A_{77} = \{A_{67}(C^2-D^2)+2CDA_{68}\}/\{(C^2-D^2)^2+4C^2D^2\}, A_{78} = \{A_{68}(C^2-D^2)-2CDA_{67}\}/\{(C^2-D^2)^2+4C^2D^2\}$,
$A_{79} = -(A_{75}+A_{77}+A_{27}-A_{29}), A_{80} = -(A_{76}+A_{78}+A_{28}-A_{30})$,
$A_{81} = e^{C\zeta}(A_{75}CosD\zeta-A_{76}SinD\zeta)+e^{-C\zeta}(A_{77}CosD\zeta+A_{78}SinD\zeta)+A_{73}\zeta+A_{79}$,
$A_{82} = e^{C\zeta}(A_{76}CosD\zeta+A_{75}SinD\zeta)+e^{-C\zeta}(A_{78}CosD\zeta-A_{77}SinD\zeta)+A_{74}\zeta+A_{80}$,
$A_{83} = A_{15}-(A^2-B^2)A_{17}+2ABA_{18}, A_{84} = A_{16}-(A^2-B^2)A_{18}-2ABA_{17}, A_{85} = e^{2A\zeta}(A_{19}Cos2B\zeta-A_{20}Sin2B\zeta)$,
$A_{86} = e^{2A\zeta}(A_{20}Cos2B\zeta+A_{19}Sin2B\zeta), A_{87} = e^{2A(1-\zeta)}\{A_{37}Cos2B(1-\zeta)-A_{38}Sin2B(1-\zeta)\}$,

$A_{88} = e^{2A(1-\zeta)}\{A_{38}Cos2B(1-\zeta)+A_{37}Sin2B(1-\zeta)\}$,
$A_{89} = A_{83}(A_{85}-A_{87})-A_{84}(A_{86}-A_{88}), A_{90} = A_{84}(A_{85}-A_{87})+A_{83}(A_{86}-A_{88})$,
$H(\zeta,\tau) = H_0(\zeta,\tau) + \in H_1(\zeta,\tau) = \in H_1(\zeta,\tau)$

$L_{0s}(\zeta) = L_{0t}(\zeta) = L_{1s}(\zeta) = 0$
$L_{1t}(\zeta) = C_5e^{f\zeta}+C_6e^{-f\zeta}+\{\alpha_3e^{(m+f)\zeta}+\alpha_7e^{-(m+f)\zeta}\}/(m^2+2mf)+ \{\alpha_4e^{-(m-f)\zeta}+\alpha_6e^{(m-f)\zeta}\}/ (m^2-2mf)+(\zeta/2f)(\alpha_5e^{f\zeta}-\alpha_8e^{-f\zeta})-(\alpha_5e^{f\zeta}+\alpha_8e^{-f\zeta})/4f^2$,
where $C_5 = -[C_6+\{(\alpha_3+\alpha_7)/(m^2+2mf)\}+ \{(\alpha_4+\alpha_6)/(m^2-2mf)\}-\{(\alpha_5+\alpha_8)/(4f^2)\}]$,
$C_6 = [[\alpha_3\{e^{(m+f)}-e^f\}+\{\alpha_7e^{-(m+f)}-e^f\}]/(m^2+2mf)+[\alpha_4\{e^{-(m-f)}-e^f\}+\{\alpha_6e^{(m-f)}-e^f\}]/(m^2-2mf)+(\alpha_5e^f-\alpha_8e^{-f})/2f+\alpha_8Sinhf/2f^2]/(2Sinhf)$,
$\alpha_1 = 1-(e^f/2Sinhf)$, $\alpha_2 = e^f/2Sinhf$,
$\alpha_3 = \{2RC_1\alpha_1/m(1+I\tau_1)\}-\{4(\tau_1+\tau_2)C_1\alpha_1f/(1+I\tau_1)\}-\{(4\tau_1+2\tau_2)C_1\alpha_1f^2/m(1+I\tau_1)\}$,
$\alpha_4 = -\{2RC_2\alpha_1/m(1+I\tau_1)\}-\{4(\tau_1+\tau_2)C_2\alpha_1f/(1+I\tau_1)\}+\{(4\tau_1+2\tau_2)C_2\alpha_1f^2/m(1+I\tau_1)\}$,
$\alpha_5 = \{2RC_3\alpha_1/(1+I\tau_1)\}-(4\tau_1+2\tau_2)C_2\alpha_1f^2/(1+I\tau_1)$,
$\alpha_6 = \{2RC_1\alpha_2/m(1+I\tau_1)\}+\{4(\tau_1+\tau_2)C_1\alpha_2f/(1+I\tau_1)\}-\{(4\tau_1+2\tau_2)C_1\alpha_2f^2/m(1+I\tau_1)\}$,
$\alpha_7 = -\{2RC_2\alpha_2/m(1+I\tau_1)\}+\{4(\tau_1+\tau_2)C_2\alpha_2f/(1+I\tau_1)\}+\{(4\tau_1+2\tau_2)C_2\alpha_2f^2/m(1+I\tau_1)\}$,
$\alpha_8 = \{2RC_3\alpha_2/(1+I\tau_1)\}-(4\tau_1+2\tau_2)C_3\alpha_2f^2/(1+I\tau_1)$,
$L_1(\zeta,\tau) = Real\{e^{i\tau} L_{1t}(\zeta)\} = (B_{47}+B_{51}+B_{55}+B_{59}-B_{63})Cos\tau - (B_{48}+B_{52}+B_{56}+B_{60}-B_{64})Sin2\tau$,
where $B_3 = 1/(1+\tau_1^2)$, $B_4 = -\tau_1/(1+\tau_1^2)$,
$B_5 = (2RC_1/m)\{B_3(1-A_1)+B_4A_2\}-4(\tau_1+\tau_2)C_1\{(1-A_1)(AB_3-BB_4)+A_2(AB_4+BB_3)\}-\{(4\tau_1+2\tau_2)C_1/m\}[(A^2-B^2)\{B_3(1-A_1)+B_4A_2\}-2AB\{B_4(1-A_1)-B_3A_2\}]$,
$B_6 = (2RC_1/m)\{B_4(1-A_1)-B_3A_2\}-4(\tau_1+\tau_2)C_1\{(1-A_1)(AB_4+BB_3)+A_2(AB_3-BB_4)\}-\{(4\tau_1+2\tau_2)C_1/m\}[2AB\{B_3(1-A_1)+B_4A_2\}+(A^2-B^2)\{B_4(1-A_1)- B_3A_2\}]$,
$B_7 = -(2RC_2/m)\{B_3(1-A_1)+B_4A_2\}-4(\tau_1+\tau_2)C_2\{(1-A_1)(AB_3-BB_4)+A_2(AB_4+BB_3)\}+\{(4\tau_1+2\tau_2)C_2/m\}[(A^2-B^2)\{B_3(1-A_1)+B_4A_2\}-2AB\{B_4(1-A_1)-B_3A_2\}]$,
$B_8 = -(2RC_2/m)\{B_4(1-A_1)-B_3A_2\}-4(\tau_1+\tau_2)C_2\{(1-A_1)(AB_4+BB_3)+A_2(AB_3-BB_4)\} +\{(4\tau_1+2\tau_2)C_2/m\}[2AB\{B_3(1-A_1)+B_4A_2\}+(A^2-B^2)\{B_4(1-A_1)- B_3A_2\}]$,
$B_9 = 2RC_3\{B_3(1-A_1)+B_4A_2\}-(4\tau_1+2\tau_2)C_3[(A^2-B^2)\{B_3(1-A_1)+B_4A_2\}- 2AB\{B_4(1-A_1)-B_3A_2\}]$,
$B_{10} = 2RC_3\{B_4(1-A_1)-B_3A_2\}-(4\tau_1+2\tau_2)C_3[2AB\{B_3(1-A_1)+B_4A_2\}+(A^2-B^2)\{B_4(1-A_1)-B_3A_2\}]$,
$B_{11} = (2RC_1/m)(B_3A_1-B_4A_2)+4(\tau_1+\tau_2)C_1\{A(B_3A_1-B_4A_2)-B(B_4A_1+B_3A_2)\} -\{(4\tau_1+2\tau_2)C_1/m\}\{(B_3A_1-B_4A_2)(A^2-B^2)-2AB(B_4A_1+B_3A_2)\}$,
$B_{12} = (2RC_1/m)(B_4A_1+B_3A_2)+4(\tau_1+\tau_2)C_1\{B(B_3A_1-B_4A_2)+A(B_4A_1+B_3A_2)\} -\{(4\tau_1+2\tau_2)C_1/m\}\{(B_3A_1-B_4A_2)2AB+(A^2-B^2)(B_4A_1+B_3A_2)\}$,
$B_{13} = -(2RC_2/m)(B_3A_1-B_4A_2)+4(\tau_1+\tau_2)C_2\{A(B_3A_1-B_4A_2)-B(B_4A_1+B_3A_2)\}+\{(4\tau_1+2\tau_2)C_2/m\}\{(B_3A_1-B_4A_2)(A^2-B^2)-2AB(B_4A_1+B_3A_2)\}$,
$B_{14} = -(2RC_2/m)(B_4A_1+B_3A_2)+4(\tau_1+\tau_2)C_2\{B(B_3A_1-B_4A_2)+A(B_4A_1+ B_3A_2)\}+\{(4\tau_1+2\tau_2)C_2/m\}\{(B_3A_1-B_4A_2)2AB+(A^2-B^2)(B_4A_1+B_3A_2)\}$,
$B_{15} = 2RC_3\{B_3A_1-B_4A_2\}-(4\tau_1+2\tau_2)C_3\{(A^2-B^2)(B_3A_1-B_4A_2)-2AB(B_4A_1+B_3A_2)]$,
$B_{16} = 2RC_3\{B_4A_1+B_3A_2\}-(4\tau_1+2\tau_2)C_3\{(B_3A_1-B_4A_2)2AB+(A^2-B^2)(B_4A_1+B_3A_2)]$,
$Y_1 = B_5CosB(e^{(m+A)}-e^A)- B_6SinB(e^{(m+A)}-e^A)$, $Y_2 = B_6CosB(e^{(m+A)}-e^A)+ B_5SinB(e^{(m+A)}-e^A)$,
$Y_3 = B_{13}CosB(e^{-(m+A)}-e^A)+ B_{14}SinB(e^{-(m+A)}+e^A)$, $Y_4 = B_{14}CosB(e^{-(m+A)}-e^A)- B_{13}SinB(e^{-(m+A)}+e^A)$,
$B_{17} = (Y_1+Y_3)$, $B_{18} = (Y_2+Y_4)$, $Y_5 = (e^{-(m-A)}-e^A)(B_7CosB-B_8SinB), Y_6 = (e^{-(m-A)}-e^A)(B_8CosB+B_7SinB)$,
$Y_7 = B_{11}CosB(e^{(m-A)}-e^A)+B_{12}SinB(e^{(m-A)}+e^A), Y_8 = B_{12}CosB(e^{(m-A)}-e^A)-B_{11}SinB(e^{(m-A)}+e^A)$,
$B_{19} = (Y_5+Y_7)$, $B_{20} = (Y_6+Y_8)$, $B_{21} = (m^2+2mA)/\{(m^2+2mA)^2+4m^2B^2\}, B_{22} = -2mB/\{(m^2+2mA)^2+4m^2B^2\}$,
$B_{23} = (m^2-2mA)/\{(m^2-2mA)^2+4m^2B^2\}$, $B_{24} = 2mB/\{(m^2+2mA)^2+4m^2B^2\}$,
$B_{25} = e^A(B_9CosB-B_{10}SinB)- e^{-A}(B_{15}CosB+B_{16}SinB)$, $B_{26} = e^A(B_{10}CosB+B_9SinB)- e^{-A}(B_{16}CosB-B_{15}SinB)$,
$B_{27} = (AB_{25}+BB_{26})/\{2(A^2+B^2)\}$, $B_{28} = (AB_{26}-BB_{25})/\{2(A^2+B^2)\}$, $B_{29} = B_{15}SinhACosB-B_{16}CoshASinB$,
$B_{30} = B_{16}SinhACosB+B_{15}CoshASinB$, $B_{31} = \{B_{29}(A^2-B^2)+2B_{30}AB\}/[2\{(A^2-B^2)^2+4A^2B^2\}]$,
$B_{32} = \{B_{30}(A^2-B^2)-2B_{29}AB\}/[2\{(A^2-B^2)^2+4A^2B^2\}]$, $B_{33} = B_{17}B_{21}-B_{18}B_{22}$, $B_{34} = B_{18}B_{21}+B_{17}B_{22}$,
$B_{35} = B_{19}B_{23}-B_{20}B_{24}$, $B_{36} = B_{20}B_{23}+B_{19}B_{24}$,
$B_{37} = \{(B_{33}+B_{35}+B_{27}+B_{31})SinhACosB+(B_{34}+B_{36}+B_{28}+B_{32})CoshASinB\}/2(Sinh^2ACos^2B+ Cosh^2ASin^2B)\}$,
$B_{38} = \{(B_{34}+B_{36}+B_{28}+B_{32})SinhACosB-(B_{33}+B_{35}+B_{27}+B_{31})CoshASinB\}/\{2(Sinh^2ACos^2B+ Cosh^2ASin^2B)\}$,
$B_{39} = B_{21}(B_5+B_{13})-B_{22}(B_6+B_{14})$, $B_{40} = B_{22}(B_5+B_{13})+B_{21}(B_6+B_{14})$, $B_{41} = B_{23}(B_7+B_{11})-B_{24}(B_8+B_{12})$,
$B_{42} = B_{24}(B_7+B_{11})+B_{23}(B_8+B_{12})$, $B_{43} = \{(A^2-B^2)(B_9+B_{15})+2AB(B_{10}+B_{16})\}/[4\{(A^2-B^2)^2+4A^2B^2\}]$,
$B_{44} = \{(A^2-B^2)(B_{10}+B_{16})-2AB(B_9+B_{15})\}/[4\{(A^2-B^2)^2+4A^2B^2\}], B_{45} = -(B_{37}+B_{39}+B_{41}-B_{43})$,
$B_{46} = -(B_{38}+B_{40}+B_{42}-B_{44})$, $B_{47} = e^{A\zeta}(B_{45}CosB\zeta-B_{46}SinB\zeta)+e^{-A\zeta}(B_{37}CosB\zeta+B_{38}SinB\zeta)$,
$B_{48} = e^{A\zeta}(B_{46}CosB\zeta+B_{45}SinB\zeta)+e^{-A\zeta}(B_{38}CosB\zeta-B_{37}SinB\zeta)$,

$B_{49} = e^{(m+A)\zeta}(B_5 CosB\zeta - B_6 SinB\zeta) + e^{-(m+A)\zeta}(B_{13} CosB\zeta + B_{14} SinB\zeta)$,
$B_{50} = e^{(m+A)\zeta}(B_6 CosB\zeta + B_5 SinB\zeta) + e^{-(m+A)\zeta}(B_{14} CosB\zeta - B_{13} SinB\zeta)$,
$B_{51} = B_{49}B_{21} - B_{50}B_{22}, B_{52} = B_{50}B_{21} + B_{49}B_{22}, B_{53} = e^{-(m-A)\zeta}(B_7 CosB\zeta - B_8 SinB\zeta) + e^{(m-A)\zeta}(B_{11} CosB\zeta + B_{12} SinB\zeta)$,
$B_{54} = e^{-(m-A)\zeta}(B_8 CosB\zeta + B_7 SinB\zeta) + e^{(m-A)\zeta}(B_{12} CosB\zeta - B_{11} SinB\zeta), B_{55} = B_{53}B_{23} - B_{54}B_{24}$,
$B_{56} = B_{54}B_{23} + B_{53}B_{24}, B_{57} = e^{A\zeta}(B_9 CosB\zeta - B_{10} SinB\zeta) - e^{-A\zeta}(B_{15} CosB\zeta + B_{16} SinB\zeta)$,
$B_{58} = e^{A\zeta}(B_{10} CosB\zeta + B_9 SinB\zeta) - e^{-A\zeta}(B_{16} CosB\zeta - B_{15} SinB\zeta), B_{59} = \zeta(AB_{57} + BB_{58})/\{2(A^2+B^2)\}$,
$B_{60} = \zeta(AB_{58} - BB_{57})/\{2(A^2+B^2)\}, B_{61} = e^{A\zeta}(B_9 CosB\zeta - B_{10} SinB\zeta) + e^{-A\zeta}(B_{15} CosB\zeta + B_{16} SinB\zeta)$,
$B_{62} = e^{A\zeta}(B_{10} CosB\zeta + B_9 SinB\zeta) + e^{-A\zeta}(B_{16} CosB\zeta - B_{15} SinB\zeta), B_{63} = \{B_{61}(A^2-B^2) + 2ABB_{62}\}/[4\{(A^2-B^2)^2 + 4A^2B^2\}]$,
$B_{64} = \{B_{62}(A^2-B^2) - 2ABB_{61}\}/[4\{(A^2-B^2)^2 + 4A^2B^2\}]$,
$L(\zeta,\tau) = L_0(\zeta,\tau) + \in L_1(\zeta,\tau) = \in L_1(\zeta,\tau)$

$\Psi_{0s}(\zeta) = \Psi_{0t}(\zeta) = \Psi_{1s}(\zeta) = 0$,
$\Psi_{1t}(\zeta) = C_1 e^{q\zeta} + C_2 e^{-q\zeta} - RP_r f^2(1+I\tau_1)[\{(\alpha_1^2 e^{2f\zeta} + \alpha_2^2 e^{-2f\zeta})/(4f^2-q^2)\} + (2\alpha_1\alpha_2/q^2)]$,
where $C_1 = RP_r f^2(1+I\tau_1)[\{(\alpha_1^2+\alpha_2^2)/(4f^2-q^2)\} + 2\alpha_1\alpha_2/q^2] - C_2$,
$C_2 = \{RP_r f^2(1+I\tau_1)/2Sinhq\}[\ [\{e^q(\alpha_1^2+\alpha_2^2) - (\alpha_1^2 e^{2f} + \alpha_2^2 e^{-2f})\}/(4f^2-q^2)] + \ 2\alpha_1\alpha_2(e^q-1)/q^2]$,
q = (2iRPr)1/2 = Q(1+I),
$\Psi_1(\zeta,\tau) = Real\{e^{2i\tau}\Psi_{1t}(\zeta)\}\ = (B_{27}-B_{31})Cos2\tau - (B_{28}-B_{32})Sin2\tau$,
where $X_1 = e^A(SinhACos^2B + CoshASin^2B)/\{2(Sinh^2A + Sin^2B)\}$,
$X_2 = e^A(SinhACosBSinB - CoshASinBCosB)/\{2(Sinh^2A + Sin^2B)\}$, $B_1 = (1-X_1)^2 - X_2^2$, $B_2 = -2X_2(1-X_1)$,
$B_3 = (X_1^2 - X_2^2)$, $B_4 = 2X_1X_2$, $B_5 = A^2 - B^2 - 2AB\tau_1$, $B_6 = 2AB + \tau_1(A^2-B^2)$,
$B_7 = (B_5 SinhQCosQ + B_6 CoshQSinQ)/(Sinh^2Qcos^2Q + Cosh^2QSin^2Q)$,
$B_8 = (B_6 SinhQCosQ - B_5 CoshQSinQ)/(Sinh^2Qcos^2Q + Cosh^2QSin^2Q)$,
$B_9 = e^Q\{(B_1+B_3)CosQ - (B_2+B_4)SinQ\}$, $B_{10} = e^Q\{(B_1+B_3)SinQ + (B_2+B_4)CosQ\}$,
$B_{11} = \{e^{2A}(B_1 Cos2B - B_2 Sin2B) + e^{-2A}(B_3 Cos2B + B_4 Sin2B)\}$,
$B_{12} = \{e^{2A}(B_2 Cos2B + B_1 Sin2B) + e^{-2A}(B_4 Cos2B - B_3 Sin2B)\}$, $B_{13} = (4A^2-4B^2)/\{\ (4A^2-4B^2)^2 + (8AB-2RP_r)^2\}$,
$B_{14} = -(8AB-2RP_r)\ /\{\ (4A^2-4B^2)^2 + (8AB-2RP_r)^2\}$, $B_{15} = B_{13}(B_9-B_{11}) - B_{14}(B_{10}-B_{12})$,
$B_{16} = B_{14}(B_9B_{11}) + B_{13}(B_{10}-B_{12})$, $B_{17} = \{X_2(1-X_1) - X_1X_2\}/(2RP_r)$, $B_{18} = -\{X_1(1-X_1) + X_2^2\}/(2RP_r)$,
$B_{19} = B_{17}(e^Q CosQ-1) - B_{18}e^Q SinQ$, $B_{20} = B_{18}(e^Q CosQ-1) + B_{17}e^Q SinQ$, $B_{21} = RP_r\{B_7(B_{15}+2B_{19}) - B_8(B_{16}+2B_{20})\}/2$, $B_{22} = RP_r\{B_8(B_{15}+2B_{19}) + B_7(B_{16}+2B_{20})\}/2$, $B_{23} = B_{13}(B_1+B_3) - B_{14}(B_2+B_4)$,
$B_{24} = B_{14}(B_1+B_3) + B_{13}(B_2+B_4)$,
$B_{25} = RP_r\{B_5(B_{23}+2B_{17}) - B_6(B_{24}+2B_{18})\} - B_{21}$, $B_{26} = RP_r\{B_6(B_{23}+2B_{17}) + B_5(B_{24}+2B_{18})\} - B_{22}$,
$B_{27} = e^{Q\zeta}(B_{25} CosQ\zeta - B_{26} SinQ\zeta) + e^{-Q\zeta}(B_{21} CosQ\zeta + B_{22} SinQ\zeta)$,
$B_{28} = e^{Q\zeta}(B_{26} CosQ\zeta + B_{25} SinQ\zeta) + e^{-Q\zeta}(B_{22} CosQ\zeta - B_{21} SinQ\zeta)$,
$B_{29} = e^{2A\zeta}(B_1 Cos2B\zeta - B_2 Sin2B\zeta) + e^{-2A\zeta}(B_3 Cos2B\zeta + B_4 Sin2B\zeta)$,
$B_{30} = e^{2A\zeta}(B_2 Cos2B\zeta + B_1 Sin2B\zeta) + e^{-2A\zeta}(B_4 Cos2B\zeta - B_3 Sin2B\zeta)$,
$B_{31} = RP_r\{B_5(B_{29}B_{13} - B_{30}B_{14} + 2B_{17}) - B_6(B_{30}B_{13} + B_{14}B_{29} + 2B_{18})\}$,
$B_{32} = RP_r\{B_6(B_{29}B_{13} - B_{30}$
$B_{14} + 2B_{17}) + B_5(B_{30}B_{13} + B_{14}B_{29} + 2B_{18})\}$,
$\Psi(\zeta,\tau) = \Psi_0(\zeta,\tau) + \in\Psi_1(\zeta,\tau) = \in\Psi_1(\zeta,\tau)$.

$\phi_{0s}(\zeta) = S(1-\zeta), \phi_{0t}(\zeta) = \phi_{1s}(\zeta) = 0$,
$\phi_{1t}(\zeta) = C_3 e^{q\zeta} + C_4 e^{-q\zeta} - 2\zeta(C_1 e^{q\zeta} - C_2 e^{-q\zeta})/q + (C_1 e^{q\zeta} + C_2 e^{-q\zeta})/q^2 + 4RP_r f^2(1+I\tau_1)\ [\{(\alpha_1^2 e^{2f\zeta} + \alpha_2^2 e^{-2f\zeta})/(4f^2-q^2)^2\} - 2\alpha_1\alpha_2/q^4]$,
where $C_4 = -(C_1 e^q - C_2 e^{-q})/(qSinhq) - C_2/q^2 + \{2RP_r f^2(1+I\tau_1)/Sinhq\}[\{\alpha_1^2(e^{2f}-e^q) + \ \alpha_2^2(e^{-2f}-e^q)/(4f^2-q^2)^{\ 2}\} + 2\alpha_1\alpha_2(e^q-1)/q^4]$,
$C_3 = -[(C_1+C_2)/q^2 + C_4 + 4RP_r f^2(1+I\tau_1)[\{(\alpha_1^2+\alpha_2^2)/(4f^2-q^2)^{\ 2}\} - 2\alpha_1\alpha_2/q^4]]$,
$\phi_1(\zeta,\tau) = Real\{e^{2i\tau}\phi_{1t}(\zeta)\}\ \ = B_{67}Cos2\tau - B_{68}Sin2\tau$,
$B_{33} = SinhQCosQ/(Sinh^2Qcos^2Q + \ Sin^2Qcosh^2Q), B_{34} = -SinQCoshQ/(Sinh^2Qcos^2Q + \ Sin^2Qcosh^2Q)$,
$B_{35} = (B_{33}+B_{34})/(2Q)$, $B_{36} = (B_{34}-B_{33})/(2Q)$, $B_{37} = e^Q(B_{25}CosQ - B_{26}SinQ) - \ e^{-Q}(B_{21}CosQ + B_{22}SinQ)$,
$B_{38} = e^Q(B_{26}CosQ + B_{25}SinQ) - \ e^{-Q}(B_{22}CosQ - B_{21}SinQ)$, $B_{39} = B_{37}B_{35} - B_{38}B_{36}$, $B_{40} = B_{38}B_{35} + B_{37}B_{36}$,
$B_{41} = (B_{11}-B_9)(B_{13}^2 - B_{14}^2) - 2B_{13}B_{14}(B_{12}-B_{10})$, $B_{42} = (B_{12}-B_{10})(B_{13}^2 - B_{14}^2) + 2B_{13}B_{14}(B_{11}-B_9)$,
$B_{43} = 2RP_r[B_7\{B_{41} + (B_{20}/(RP_r)\} - B_8\{B_{42} - (B_{19}/(RP_r)\}]$, $B_{44} = 2RP_r[B_8\{B_{41} + (B_{20}/(RP_r)\} + B_7\{B_{42} - (B_{19}/(RP_r)\}]$,
$B_{45} = -B_{39} - \{B_{22}/(2RP_r)\} + B_{43}$, $B_{46} = -B_{40} + \{B_{21}/(2RP_r)\} + B_{44}$, $B_{47} = (B_{26}+B_{22})/(2RP_r)$, $B_{48} = -(B_{25}+B_{21})/(2RP_r)$,
$B_{49} = B_{23}B_{13} - B_{24}B_{14} - (B_{18}/RP_r)$, $B_{50} = B_{24}B_{13} + B_{23}B_{14} + (B_{17}/RP_r)$, $B_{51} = 4RP_r(B_5B_{49} - B_6B_{50})$,
$B_{52} = 4RP_r(B_6B_{49} + B_5B_{50})$, $B_{53} = -(B_{47}+B_{45}+B_{51})$, $B_{54} = -(B_{48}+B_{46}+B_{52})$,
$B_{55} = e^{Q\zeta}(B_{53}CosQ\zeta - B_{54}SinQ\zeta) + \ e^{-Q\zeta}(B_{45}CosQ\zeta + B_{46}SinQ\zeta)$,
$B_{56} = e^{Q\zeta}(B_{54}CosQ\zeta + B_{53}SinQ\zeta) + \ e^{-Q\zeta}(B_{46}CosQ\zeta - B_{45}SinQ\zeta)$,
$B_{57} = e^{Q\zeta}(B_{25}CosQ\zeta - B_{26}SinQ\zeta) - \ e^{-Q\zeta}(B_{21}CosQ\zeta + B_{22}SinQ\zeta)$,

$B_{58} = e^{Q\zeta}(B_{26}CosQ\zeta+B_{25}SinQ\zeta)- e^{-Q\zeta}(B_{22}CosQ\zeta-B_{21}SinQ\zeta)$,
$B_{59} = \zeta(B_{57}+B_{58})/Q, B_{60} = \zeta(B_{58}-B_{57})/Q, B_{61} = B_{29}(B_{13}^2-B_{14}^2)-2B_{13}B_{14}B_{30}, B_{62} = B_{30}(B_{13}^2-B_{14}^2)+2B_{13}B_{14}B_{29}$,
$B_{63} = B_{61}-(B_{18}/RP_r), B_{64} = B_{62}+(B_{17}/RP_r), B_{65} = 4RP_r(B_5B_{63}-B_6B_{64}), B_{66} = 4RP_r(B_6B_{63}+B_5B_{64})$,
$B_{67} = B_{55}-B_{59}+(B_{28}/2RP_r)+B_{65}, B_{68} = B_{56}-B_{60}-(B_{27}/2RP_r)+B_{66}$,
$\phi(\zeta,\tau) = \phi_0(\zeta,\tau) +\in\phi_1(\zeta,\tau)$.

Nusselt number at the oscillating and the stator disc are

$$Nu_b = Q_bz_0/\{K(T_a-T_b)\} = -E[N_4+\{N_2(\xi^2-\xi_0^2)/2\}],$$
$$Nu_a = Q_az_0/\{K(T_a-T_b)\} = -E[N_3+\{N_1(\xi^2-\xi_0^2)/2\}],$$

where $N_1 = Q(Y_{14}-Y_{15}-Y_{16}+Y_{17})+2A(Y_{18}-Y_{20})+2B(Y_{19}+Y_{21})$,
$N_2 = Qe^{Q}\{CosQ(Y_{14}-Y_{15})-SinQ(Y_{14}+Y_{15})\}- Qe^{-Q}\{CosQ(Y_{16}-Y_{17})+SinQ(Y_{16}+Y_{17})\}+2e^{2A}\{Cos2B(AY_{18}+BY_{19})$
$-Sin2B(BY_{18}-AY_{19})\}-2e^{-2A}\{Cos2B(AY_{20}-BY_{21})+Sin2B(BY_{20}+AY_{21})\}$,
$N_3 = -S+Q(Y_1+Y_2-Y_3+Y_4)-(Y_5-Y_7)/Q+2A(Y_9-Y_{11})+2B(Y_{10}+Y_{12})$,
$N_4 = -S+Qe^{Q}\{CosQ(Y_1+Y_2)+SinQ(Y_2-Y_1)\}-Qe^{-Q}\{CosQ(Y_3-Y_4)+SinQ(Y_3+Y_4)\}-(e^{Q}/Q)(Y_5CosQ+Y_6SinQ)$
$-e^{Q}\{CosQ(Y_5-Y_6)+SinQ(Y_5+Y_6)\}+(e^{-Q}/Q)(Y_7CosQ+Y_8SinQ)-e^{-Q}\{CosQ(Y_7-Y_8)+SinQ(Y_7+Y_8)\}+$
$2e^{2A}\{Cos2B(AY_9+BY_{10})+Sin2B(AY_{10}-BY_9)\}-2e^{-2A}\{Cos2B(AY_{11}-BY_{12})+Sin2B(BY_{11}+AY_{12})\}$,
$Y_1 = \{B_{53}+(B_{26})/(2RP_r)\}\in Cos2\tau-\{B_{54}-(B_{25})/(2RP_r)\}\in Sin2\tau$,
$Y_2 = \{-B_{54}+(B_{25})/(2RP_r)\}\in Cos2\tau-\{B_{53}+(B_{26})/(2RP_r)\}\in Sin2\tau$,
$Y_3 = \{B_{45}+(B_{22})/(2RP_r)\}\in Cos2\tau-\{B_{46}-(B_{22})/(2RP_r)\}\in Sin2\tau$,
$Y_4 = \{B_{46}-(B_{21})/(2RP_r)\}\in Cos2\tau+\{B_{45}+(B_{21})/(2RP_r)\}\in Sin2\tau$,
$Y_5 = (B_{25}+B_{26})\in Cos2\tau-(B_{26}-B_{25})\in Sin2\tau, Y_6 = (B_{25}-B_{26})\in Cos2\tau-(B_{26}+B_{25})\in Sin2\tau$,
$Y_7 = (B_{21}+B_{22})\in Cos2\tau-(B_{22}-B_{21})\in Sin2\tau$, $Y_8 = (B_{22}-B_{21})\in Cos2\tau+(B_{21}+B_{22})\in Sin2\tau$,
$Y_9 = [4RP_rB_5\{B_1(B_{13}^2-B_{14}^2)-2B_{13}B_{14}B_2\}-4RPrB_6\{B_2(B_{13}^2-B_{14}^2)- 2B_{13}B_{14}B_1\}]\in Cos2\tau-[4RP_rB_6\{B_1(B_{13}^2-B_{14}^2)$
$-2B_{13}B_{14}B_2\}-4RP_rB_5 \{B_2(B_{13}^2-B_{14}^2)+2B_{13}B_{14}B_1\}]\in Sin2\tau$,
$Y_{10} = [4RP_rB_5\{-B_2(B_{13}^2-B_{14}^2)-2B_{13}B_{14}B_1\}-4RPrB_6\{B_1(B_{13}^2-B_{14}^2)- 2B_{13}B_{14}B_2]\in Cos2\tau-[4RP_rB_6\{-B_2(B_{13}^2-B_{14}^2)$
$-2B_{13}B_{14}B_1\}+4RP_rB_5 \{B_1(B_{13}^2-B_{14}^2)-2B_{13}B_{14}B_2\}]\in Sin2\tau$,
$Y_{11} = [4RP_rB_5\{B_3(B_{13}^2-B_{14}^2)-2B_{13}B_{14}B_4\}-4RP_rB_6\{B_4(B_{13}^2-B_{14}^2)+ 2B_{13}B_{14}B_3\}]\in Cos2\tau-[4RP_rB_6\{B_3(B_{13}^2-B_{14}^2)$
$-2B_{13}B_{14}B_4\}+4RP_rB_5 \{B_4(B_{13}^2-B_{14}^2)+2B_{13}B_{14}B_3]\in Sin2\tau$,
$Y_{12} = [4RP_rB_5\{B_4(B_{13}^2-B_{14}^2)+2B_{13}B_{14}B_3\}-4RP_rB_6\{-B_3(B_{13}^2-B_{14}^2)+ 2B_{13}B_{14}B_4\}]\in Cos2\tau-[4RP_rB_6$
$\{B_4(B_{13}^2-B_{14}^2)+2B_{13}B_{14}B_3\}+4RP_rB_5\{-B_3(B_{13}^2-B_{14}^2)+2B_{13}B_{14}B_4]\in Sin2\tau$,

IV. RESULTS AND DISCUSSION

The variation of the dimensionless temperature $(T^*- T_b^*)/ (T_a^*- T_b^*)$ with ζ for different values of elastico-viscous parameter τ_1 = 1, 4, 11; when ξ = 5, $\in$ = 0.02, E = 5, P_r = 4, R = 4, m = 2 at phase difference τ = $\pi/3$ and $2\pi/3$ is shown in fig (1) and fig (2) respectively. It is evident from fig (1) that the temperature increases near the lower disc and start decreasing soon rapidly. It is also clear that the dimensionless temperature decreases with an increase in elastico-viscous parameter τ_1 throughout the gap-length. It can be seen from fig (2) that the temperature increases with an increase in elastico-viscous parameter τ_1 near the lower disc and decreases near the upper disc.

The behaviour of the dimensionless temperature $(T^*- T_b^*)/ (T_a^*- T_b^*)$ with ζ for different values of Reynolds number R = 1, 5, 9; when ξ = 5, $\in$ = 0.02, E = 5, P_r = 4, elastico-viscous parameter τ_1 =2, m = 2 at phase difference $\tau = \pi/3$ and $2\pi/3$ is shown in fig (3) and fig (4) respectively. It can be observed from fig (3) that the temperature is maximum near the lower disc and increases with an increase in Reynolds number whenever it decreases near the upper disc. It is evident from fig (4) that at R = 1, the temperature decreases continuously uptoζ = 0.7 approximately and start increasing slowly thereafter. At R = 5 and 9, the temperature decreases near the lower disc first then start increasing and attains its maximum value in the first half of the gap-length. It is also clear that the temperature increases with an increase with an increase in Reynolds number throughout the gap-length.

The variation of the dimensionless temperature $(T^*- T_b^*)/ (T_a^*- T_b^*)$ with ζ for different values of magnetic field parameter m = 1, 3, 5; when ξ = 5, $\in$ = 0.02, E = 5, P_r = 4, elastico-viscous parameter τ_1 =2, R = 4 at phase difference $\tau = \pi/3$ and $2\pi/3$ is shown in fig (5) and fig (6) respectively. It can be evident from fig (5) that the temperature attains its maximum value in the first half near the lower disc and increases with an increase in magnetic field parameter m. the behaviour of the temperature in fig (6) is reversed to that of fig (5).

The variation of the Nusselt Number Nu_a with ξ for different values of elastico-viscous parameter τ_1 = 2, 3, 4; when R = 1, E = 5, P_r = 4, ξ_0 = 5, $\in$ = 0.02, m = 2, $\tau = \pi/3$ is shown in fig (7). It is clear from this figure that Nusselt number decreases with ξ throughout the gap-length. It is also seen that Nusselt number increases with an increase in elastico-viscous parameter τ_1 throughout the gap-length.

The variation of the Nusselt Number Nu_b with ξ for different values of elastico-viscous parameter τ_1 = 2, 3, 4; when R = 1, E = 5, P_r = 4, ξ_0 = 5, $\in$ = 0.02, m = 2, $\tau = \pi/3$ is shown in fig (8). It is clear from this figure that Nusselt number increases with ξ throughout the gap-length. It is also seen that Nusselt number decreases with an increase in elastico-viscous parameter τ_1 throughout the gap-length.

The variation of the Nusselt Number Nu_a with ξ for different values of Reynolds number R = 2, 3, 4; when τ_1 = 2, E = 5, P_r = 4, ξ_0 = 5, $\in$ = 0.02, m = 2, $\tau = \pi/3$ is shown in fig (9). It is clear from this figure that Nusselt number decreases with ξ throughout the gap-length. It is also seen that Nusselt number decreases with an increase in Reynolds number R throughout the gap-length.

The variation of the Nusselt Number Nu_b with ξ for different values of Reynolds number R = 2, 3, 4; when τ_1 = 2, E = 5, P_r = 4, ξ_0 = 5, $\in$ = 0.02, m = 2, $\tau = \pi/3$ is shown in fig (10). It is clear from this figure that Nusselt number increases with ξ throughout the gap-length. It is also seen that Nusselt number decreases with an increase in Reynolds number R throughout the gap-length.

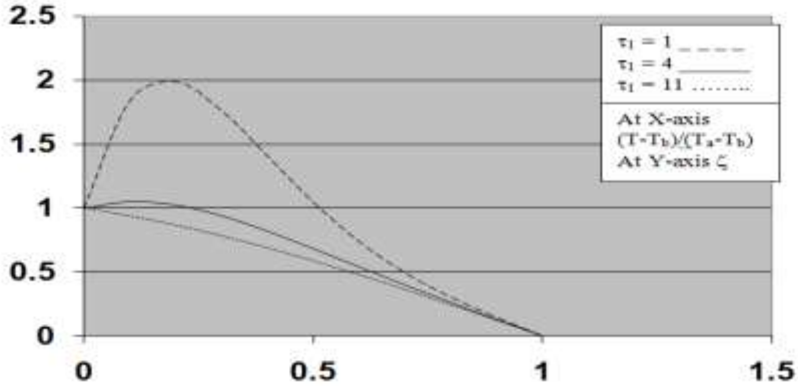
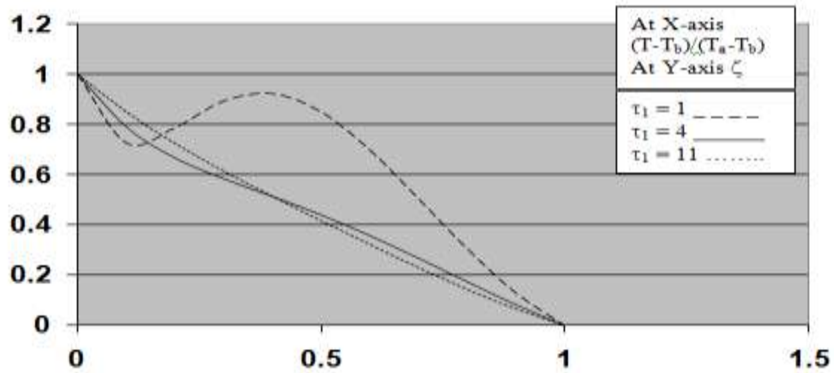
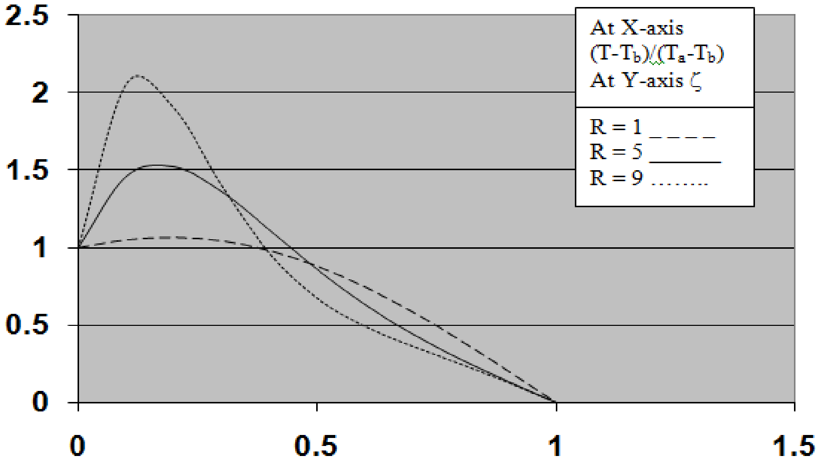


Fig Fig(1) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different elastico-viscous parameter τ_1 at $\tau = \pi/3$.

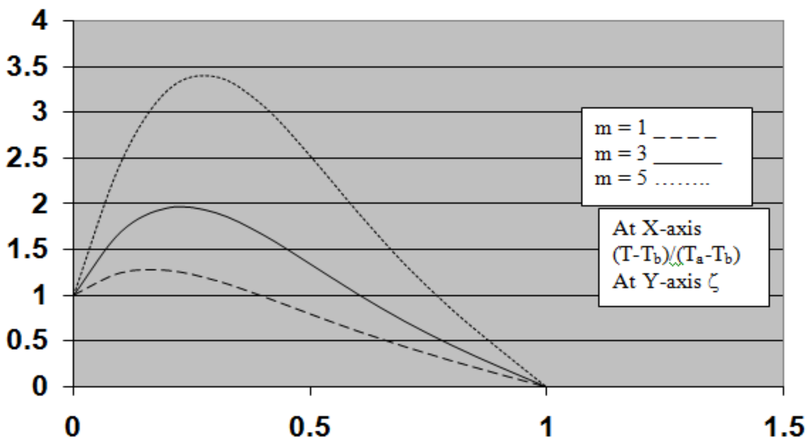


Fig(2) variation of temperature distribution T^* at different elastico-viscous parameter τ_1 at $\tau = 2\pi/3$.

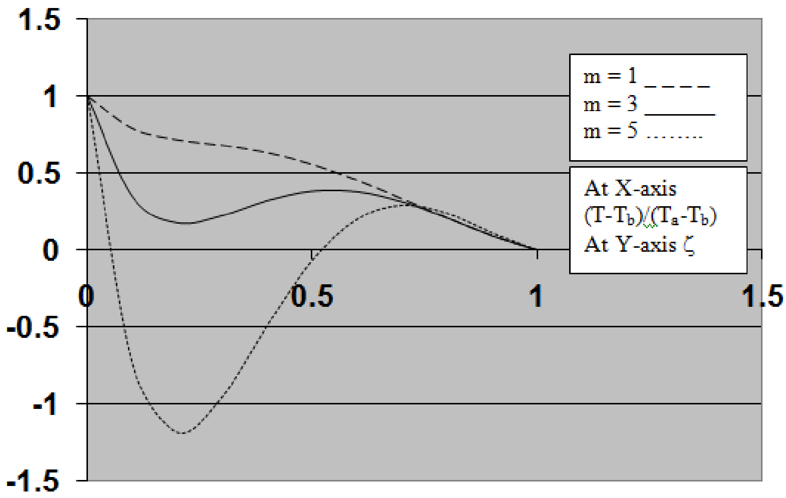
Fig(3) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different Reynolds number R at $\tau = \pi/3$.



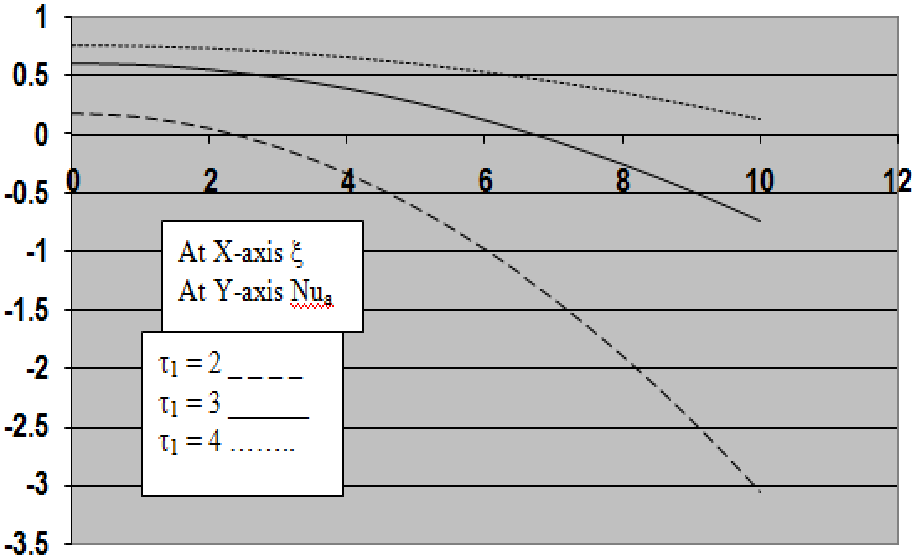
Fig(4) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different Reynolds number R at $\tau = 2\pi/3$.



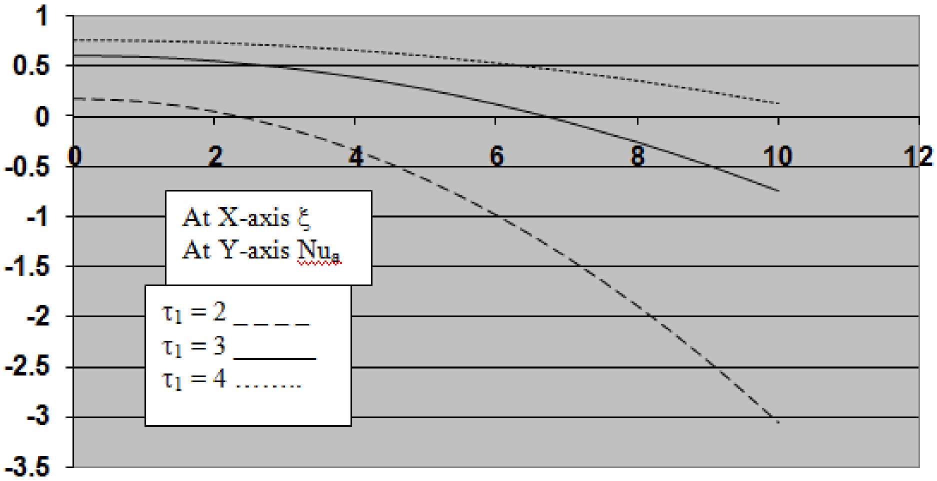
Fig(5) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different magnetic field m at $\tau = \pi/3$.



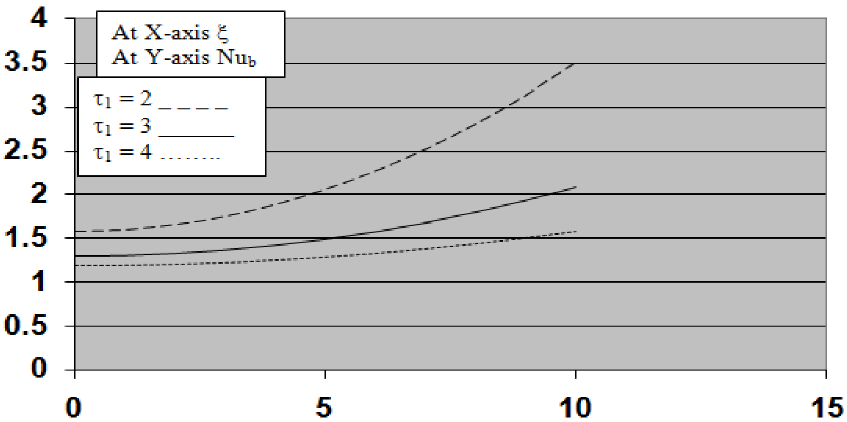
Fig(6) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different magnetic field m at $\tau = 2\pi/3$.



Fig(6) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different magnetic field m at $\tau = 2\pi/3$.



Fig(7) variation of Nusselt number Nu_a at different elastico-viscous parameter τ_1 at $\tau = \pi/3$.



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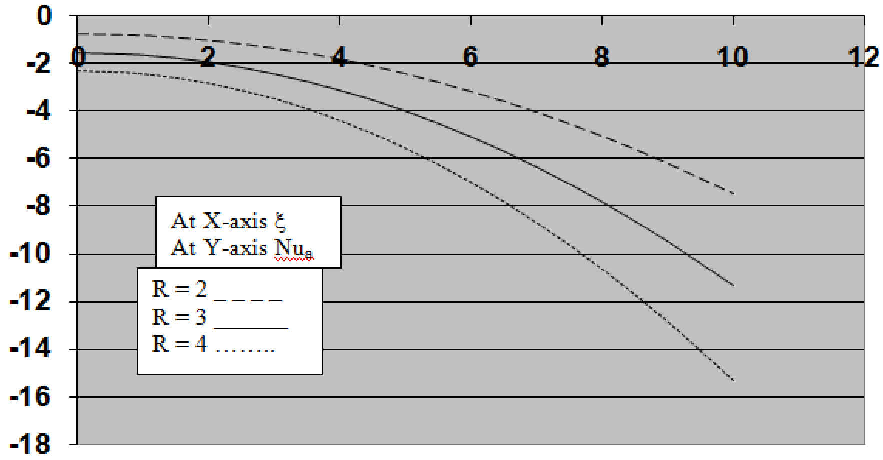
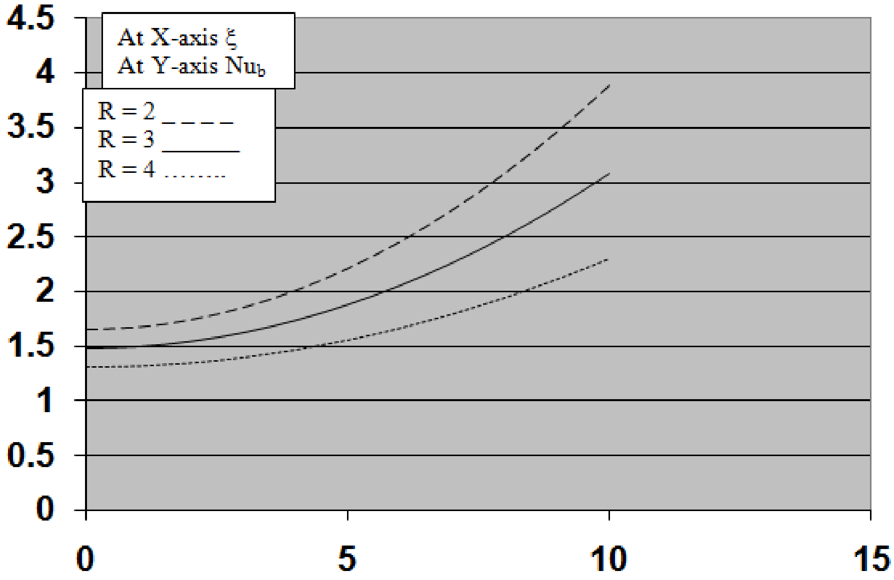
Fig(8) variation of Nusselt number Nu_b at different elastico-viscous parameter τ_1 at $\tau = \pi/3$.



Fig(9) variation of Nusselt number Nu_a at different Reynolds number R at $\tau = \pi/3$.



Fig(10) variation of Nusselt number Nu_b at different Reynolds number R at $\tau = \pi/3$.

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