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The Comprehensive Computation Model of Gas Permeability Based on Fuzzy Complementary Judgment Matrix

ChunfuWei

School Of Mathematics &Information Science, Henan Polytechnic University, Jiaozuo, 454003- China
Corresponding author. E-mail address: mathwcf@163.com (Chunfu Wei).

ABSTRACT: *In this paper, in order to reveal the gas migration law of loaded coal under multi-factor coupling, the researches on gas permeability were carried out under different influencing factors, namely effective stress, gas pressure, confining pressure and moisture content, with the self-developed experimental platform of gas permeability. Meanwhile, the function relationship of each influencing factor and permeability was established by use of the mathematical least squares principle. In this paper, the comprehensive expression of gas permeability was established, which is based on fuzzy complementary judgment matrix. And the comprehensive expression was drawn from the experimental conclusions of the loaded coal under multi-factor coupling.*

Keywords:*Coupling, Gas permeability, fuzzy complementary judgment matrix, Gas permeability experimental platform*

I. INTRODUCTION

Recently, with the depth and intensity of coal mining increasing, the progressively larger volume of gas pour out in coal seam has always been a serious threat to coal mining safety and productivity, especially in high gassy coal mines. And gas drainage is the important means to the prevention of coal and gas dynamic failure. Therefore, it is of significance to study the permeability law of gas drainage, and reasonably assign the boreholes.

Among the work, reviews on the controlled factors of permeability of coal containing gas were made by some researchers at early stage. Zhao [1] studied the coal-gas coupled mathematical model and presented the experimental relations among gas permeability of coal samples, pore gas pressure and in situ stress under the action of triaxial stress meter. Li and Zhao [2-3] conducted detailed researches on the controlled factors of permeability of coal containing gas, strain-stress curve of gas, and physical and mechanical properties of soft and hard coal.

Recently, considerable efforts have been made to study the evolution of gas permeability, the gas flow law in coal seam, and the coupled effect of coal and gas during coal seam mining through laboratory experiments, numerical simulations, and field tests. Yin [4-5] studied gas permeability in the process of strain-stress, suggesting that pour out quantity of gas is closely related to deformation of coal samples, that is to say, gas pour out quantity decreases as the coal samples compress and confining pressure increases. From the physical simulation of industrial production of biogas, the evaluation and identification method of methane permeability mechanism was studied in [6]. As discussed in [7], through three axis servo control permeability experimental platform, permeability and mechanical properties of coal samples was studied under unloading condition. Through the self-made permeability experimental platform, permeability of carbon dioxide gas to flow through the anthracite was studied and the gas permeability evolution model was built [8]. Different mathematical models of gas permeability were built under different conditions in [9-10]. For the permeability mechanism of gas in the coal seam extraction, Liang [11] carried out studies on the interaction law of coal and rock deformation and gas permeability.

Although great progress has been made, it is difficult to study the evolution of gas permeability. In this paper, with the self-developed experimental platform of coal containing gas multi- coupling at Henan Polytechnic University, the influencing law of gas permeability was studied under various conditions of gas pressure, confining pressure, axial stress and moisture content, and the fitting equation of each influencing factor and permeability was established by use of the mathematical least squares principle. The notion of coupling factor was introduced to build gas permeability model, which provides the theoretical guidance for revealing the law of gas flow in the loaded coal containing gas.

II. SCHEMATIC OF FLUID-SOLID COUPLING PERMEABILITY EXPERIMENTAL SYSTEM AND ITS PRINCIPLES

2.1 Experimental system

In this paper, the gas permeability experimental platform is designed and self-developed by Henan Polytechnic University, which consists of axial pressure, confining pressure load control system, high pressure cylinder pressure chamber, methane gas source control system, heated water systems and data acquisition system, as shown in Fig.1.

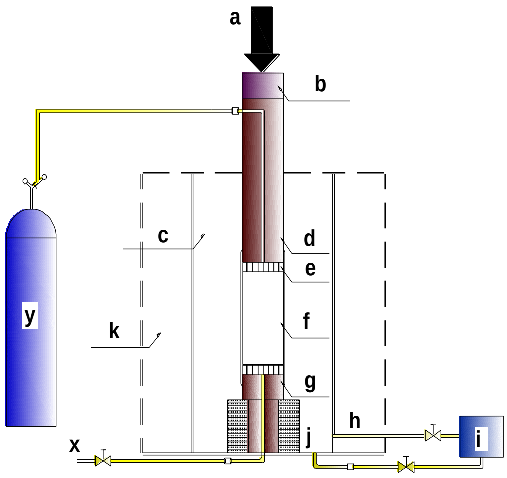


Fig.1 Schematic of fluid-solid coupling permeability experimental system. (a) axial loading system; (b) stress sensor; (c) triaxial cell; (d)upper pressure head; (e) breathable version; (f) coal sample; (g) under pressure head; (h) the oil inlet; (i) Hydraulic pump station; (j) The oil outlet; (k) Constant temperature water bath; (x) the gas outlet; (y) High pressure gas cylinders.

The main technical indicators of the permeability experiment system is as follows: ① axial displacement: 0 ~ 60mm; ② confining pressure control: 0 ~ 40MPa; ③ axial pressure control: 0 ~ 150MPa; ④ large water temperature control accuracy: ± 0.01 ℃; ⑤ gas pressure control range: 0 ~ 13.5MPa; ⑥ temperature control range: 0 ~ 100 ℃; ⑦ sample size: Φ30mm × 100mm, Φ50mm × 100mm, Φ80mm × 100mm; ⑧ stress, displacement, pore pressure, deformation, temperature and flow are automatically collected by a data acquisition instrument.

2.2 Principles of gas permeability

Gas permeability measurement is usually divided into two kinds: one is the scene direct determination, the other is the determination of physical laboratory experiment. The former is of high investment cost, long testing time, large deviations, and the disadvantages of on-site construction difficulties. In view of this, the latter, the determination of physical laboratory simulation, was adopted in this paper to measure the gas permeability of loaded coal containing gas, laying the experimental foundation for further exploration of the gas migration law inside the coal seam. The experiments are carried out based on the "routine core analysis method" (SY/T t5336-1996 standard) and American ASTM standard [12]. Gas permeability of coal samples is calculated through the gas pressure gradient of the samples at two ends and gas volume flow through. The calculation formula is as follows:

$$k = \frac{2Qp_0\mu L}{A(p_1^2 - p_2^2)} \quad (1)$$

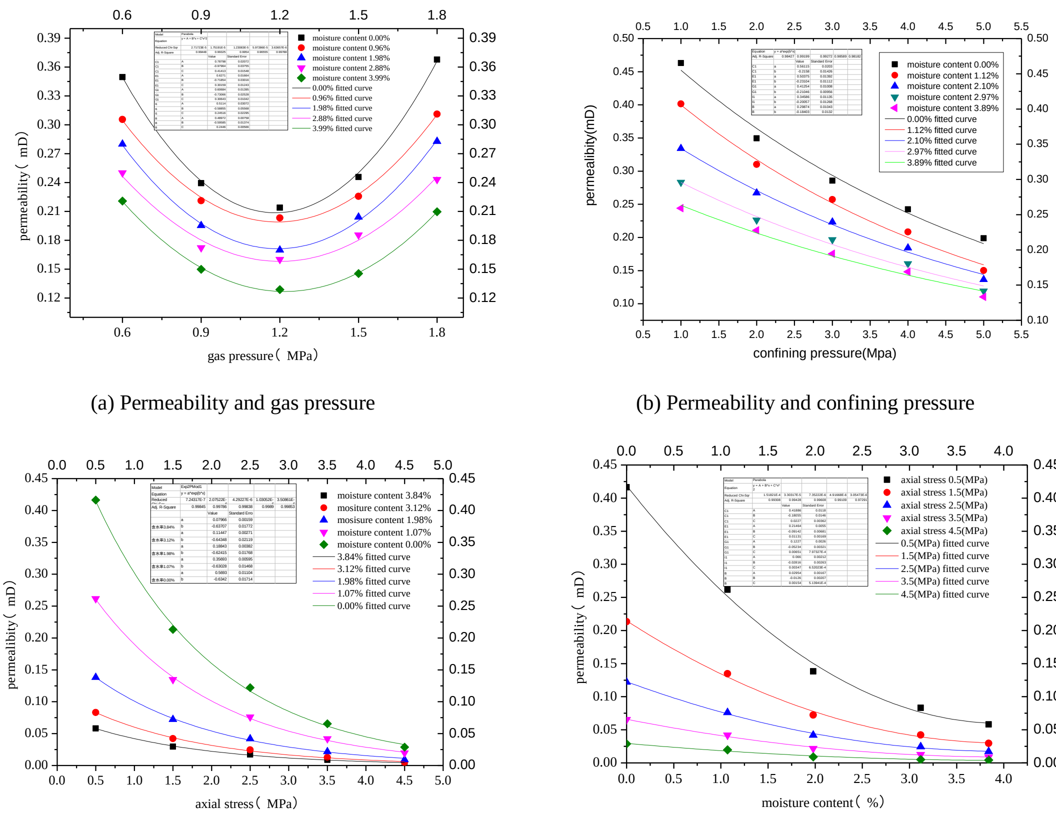
Where, Q denotes the gas flow through the unit of time in cm^3/s; K is gas permeability in mD; μ is for the gas viscosity coefficient in Pa·s, its value is （10.26+0.0305t）$\times 10^{-6}$Pa·s; t ranges from 0℃ to 100℃; L is the length of coal sample in mm; A is the cross section of the specimen in cm^2; and $p_0$is atmospheric pressure of 0.1MPa, p_1 is the inlet gas pressure in MPa, $p_2$is the outlet for gas pressure in Mpa，

III. ANALYSIS OF EXPERIMENTAL RESULTS

Gas molecules in coal medium have two states-adsorption and free, usually free-state gas rich in pore and fissure space of the coal and rock skeleton, showing that gas pressure affects the permeability. The rules of gas permeability withvarious gas pressure are studied by analyzing and regressively analyzing the permeability under various gas pressure, and constant axial stress and confining pressure. As shown in Fig.2(a), each curve represents the permeability curve of the same coal sample under different moisture content, the average correlation coefficient of 0.995, the general expression of the gas pressure p and permeability k is obtained by the least square method is:

$$k = \alpha_1 p^2 + \beta_1 p + \gamma_1 \quad (2)$$

Where, α_1， β_1， γ_1 arethe fitting coefficient.



(a) Permeability and gas pressure

(b) Permeability and confining pressure

(c) Permeability and axial pressure

(d) Permeability and moisture content

Fig. 2 The function relationship between permeability and each influencing factor

Based on the original experimental data and permeability calculation principle , the corresponding permeability K is obtained; and then regressively analyze the experimental data, shown in Fig.2b, the average correlation coefficient is 0.988, the function relationship between permeability and confining pressure is obtained by the principle of least square method:

$$k = \alpha_2 \exp\left(-\beta_2 \sigma_3\right) \qquad (3)$$

Where, α_2 , β_2 are the fitting coefficient.

The regression analysis of experimental data on permeability and axial stress is made, asshown in Fig.2c. The average correlation coefficient is 0.995, a function relationship between permeability and axial stress is:

$$k = \alpha_3 \exp\left(-\beta_3 \sigma_1\right) \qquad (4)$$

Where, α_3 , β_3 are the fitting coefficient.

The regression analysis of experimental data on permeability and moisture content is made, asshown in Fig.2d.The average correlation coefficient is 0.994, a function relationship between permeability and moisture content is:

$$k = \alpha_4 W^2 + \beta_4 W + \gamma_4 \qquad (5)$$

Where, α_4 , β_4 , γ_4 are the fitting coefficient.

IV. GAS PERMEABILITY MODELl

With different vertical stress, horizontal stress, gas pressure and moisture content and other factors suffered by coal containing gas, gas permeability will also be changed, and therefore the prediction of the effective permeability of coal containing gas should also take the impact of these factors into account. For the convenience and maneuverability, people often set other factors constant, study the function relation between the permeability and a certain influencing factor. In fact, the size of gas permeability in the coal seam at some stage is the combined effect of the moisture content, the vertical stress, gas pressure, the horizontal stress and temperature acting together; While most experiments are to set certain factor as variables, and other influencing factors in this procedure are assumed to be constant; At this point, no matter which experimental factors resulting equation uses to represent the scene, the experiments will have a certain one-sidedness. Therefore, by introducing the coupling factor, neglecting the influence of temperature and other factors, the author established a comprehensive expression of gas permeability under multi-factor coupling:

$$\begin{cases} k = \sum_{i=1}^{n} \varpi_i k_i = \varpi_1 f(\sigma_1) + \varpi_2 f(W) + \\ \varpi_3 f(\sigma_3) + \varpi_4 f(p) \\ \sum_{i=1}^{n} \varpi_i = 1 \quad (\mathrm{i} = 1, 2, 3, 4) \end{cases} \qquad (6)$$

Where, it is assumed that $f(\sigma_1)$, $f(p)$, $f(\sigma_3)$ and $f(W)$ respectively represent the functions between permeability and axial stress, gas pressure, containing pressure and moisture content. ϖ_i is coupling factor, which is the weight of a certain influencing factor under the multi-factor coupling effects.

Due to the uncertainty of the coupling system itself as well as our ambiguous understanding of the system, it is difficult for researchers to give concrete numerical coupling coefficient. But we can qualitatively compare pairwise relative to the sensitivity of the various factors affecting the permeability,building fuzzy complementary judgment matrix for solving the coupling coefficient. Then in this paper, based on fuzzy complementary judgment matrix, a general method for solving coupled vector $\varpi = (\varpi_1, \varpi_2 \cdots \varpi_n)^T$ is proposed.

First, as discussed in the reported works [13], fuzzy judgment matrix and the definition of fuzzy complementary judgment matrix are introduced: Set judgment matrix $A = (a_{ij})_{n \times n}$, if $0 \le a_{ij} \le 1$, the matrix

A is called fuzzy judgment matrix; Set judgment matrix $A = (a_{ij})_{n\times n}$, if $a_{ij} + a_{ji} = 1$, $a_{ii} = 1$, the matrix A is called fuzzy complementary judgment matrix.Tab.4-1 shows the three complementary scale value [14-15], according to the definition of fuzzy complementary judgment matrix, thus matrix composed of the previous three scale value is fuzzy complementary judgment matrix.

Tab.4-1 Complementary language scale value

0-1 scale	five scale	nine scale	meaning
0	0.1	0.1	element B is extremely sensitive to element A
		0.138	element B is highly sensitive to element A
	0.3	0.325	element B is more sensitive than element A
		0.439	element B is slightly sensitive to element A
0.5	0.5	0.5	element A is equally sensitive to element B
		0.561	element A is slightly sensitive to element B
	0.7	0.675	element A is more sensitive than element B
		0.862	element A is highly sensitive to element B
1	0.9	0.9	element A is extremely sensitive to element B

Then, assume that the target layer of permeability k is related to the factors $B_1, B_2, \cdots, B_N$, of the next level B (a collection of permeability influencing factors: gas pressure, confining pressure, axial stress, moisture content),so every influencing factor B_i has a coupling coefficient $\varpi_i = \varpi(B) \quad (i = 1,2,\cdots,n)$ in k, which can be expressed as:

$$\varpi = (\varpi_1, \varpi_2, \cdots, \varpi_n)^T, \quad (7)$$

called coupled vector

Set the numerical ratio ϖ_i / ϖ_j of the elements B_i and B_j of layer B as the elements, the judgment matrix M is as follows:

$$M = (\varpi_i / \varpi_j)_{n\times n}, \quad (8)$$

By left multiplying the judgment matrix M with coupling coefficient vector ϖ, we have

$$M\varpi = n\varpi. \quad (9)$$

By the matrix theory, the largest eigenvalue n of matrix M is denoted as $\lambda_{\max}$, the corresponding eigenvectors to eigenvalue n is ϖ, which is also the desired coupling vector.

The eigenvectors ϖ is obtained by the following procedures. By establishing the judgment Matrix to make pairwise comparison on the degree of permeability affected by certain factors, we can obtain the fuzzy language value b_{ij}, and then get the estimation matrix B of numerical ratio matrix M, called the judgment matrix of coupling factors. The steps are as follows:

① Determine the fuzzy judgment value $f(B_i / B_j)$ by pairwise comparing the influencing factorsin Tab.4-2.

Tab.4-2 Coupling factor sensitivity judgment value table

Sensitive degree of comparing B_i with B_j	$f(B_i / B_j)$	$f(B_j / B_i)$
B_i is equally sensitive to B_j	1	1
B_i is slightly sensitive to B_j	3	1
B_i is more sensitive than B_j	5	1
B_i is highly sensitive to B_j	7	1
B_i is absolutely sensitive to B_j	9	1

Note: the sensitive degree of comparing B_i with B_j is among 2,4,6,8.

Take any two elements B_i and B_j, comparing the sensitivity of B_i to B_j, make $f(B_i / B_j)$ as the discriminant value of the "sensitivity"; Conversely, $f(B_j / B_i)$ represents the discriminant value of the "sensitivity" of B_j to B_i. The discriminant value of $f(B_i / B_j)$ and $f(B_j / B_i)$ can be established by fuzzy language discriminant value, shown in Tab.2.For example,during the experiment, if the factor B_i affecting the permeability *k* ismore sensitive than the factor B_j, the discriminant value is $f(B_i / B_j) = 5$.

② Establish fuzzy complementary judgment matrix B ,

If

$$b_{ij} = \frac{f(B_i / B_j)}{f(B_j / B_i)} \quad (i, j = 1,2,\cdots,n), \tag{10}$$

we have the following complementary judgment matrix

$$B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{n1} & b_{12} & \cdots & b_{nn} \end{bmatrix}, \tag{11}$$

Where, $b_{ij} = 1$, $b_{ij} \cdot b_{ji} = 1$, $\forall i, j \in \{1,2,\cdots,n\}$.

The largest eigenvalue $\lambda_{\max}$ of matrix B andthe corresponding eigenvectors are obtained. The eigenvectors is also the corresponding coupling vector $B = \{B_1, B_2, \cdots, B_n\}$.

③Compute the eigenvectors. Common methods of computing the eigenvectors are, among others,the following: ANC, square root algorithm, eigenvalue method and the least squares method. Theoretically, it should solvethe exact values of the each coupling factor, that is, the eigenvalue of matrix B. In order to calculate conveniently, in the practical applications,approximate solution to the eigenvalue of matrix B usually is taken as the coupling coefficient. "Square root algorithm" is adopted in this paper to compute the eigenvalue of matrix B

Normalize matrix B by row, we have

$$c_{ij} = \frac{b_{ij}}{\sum_{i=1}^{n} b_{ij}} \; (i, j = 1,2,\cdots,n) \tag{12}$$

The normalized matrix is denoted as C, then sum the matrix by line, we have

$$m_i = \sum_{j=1}^{n} c_{ij} \quad (i, j = 1,2,\cdots,n) \tag{13}$$

Then normalize vector $m = (m_1, m_2, \cdots, m_n)^T$, the desired eigenvector is obtained. Each component of the eigenvector is the coupling factor corresponding to the factors affecting permeability. That is:

$$\varpi = (\varpi_1, \varpi_2, \cdots, \varpi_n)^T. \tag{14}$$

Where, $\varpi_i = m_i \Big/ \sum_{i=1}^{n} m_i \qquad (i \in \{1,2,\cdots,n\})$

Verification of the matrix for consistency.The fuzzy complementary judgment matrix needed to be verified for consistency,because the experts may have been subjective and biased in the process of establishing the complementary judgment matrix, given thecomplexity of being objective. The procedure for verification of consistency of the judgment matrix is as follows:

Step 1, compute the largest eigenvalue λ_{mzx}.

$$\lambda_{mzx} = \sum_{i=1}^{n} \frac{(B\varpi)_i}{n\varpi_i}, \tag{15}$$

Where, B is afuzzy complementary judgment matrix, ϖ is the eigenvector of matrix B, λ_{mzx} is the largest eigenvalue. The value of the *i*th element of the eigenvector ϖ is denoted as ϖ_i; n is the matrix order of matrix B.

Step 2, compute the consistency index CI :

$$CI = \frac{\lambda_{\max} - n}{n - 1}. \tag{16}$$

Step 3, compute the consistency proportion CR :

$$CR = CI / RI. \tag{17}$$

Tab.4-3 The average random consistency index value

n	1	2	3	4	5	6	7	8
RI	0	0	0.52	0.89	1.12	1.26	1.36	1.41
n	9	10	11	12	13	14	15	
RI	1.46	1.49	1.52	1.54	1.56	1.58	1.59	

Where, RI is the mean max consistency index, the data is shownin Tab.4-3[15]. When CR < 0.1, we can accept that the estimation matrix has satisfied consistency, otherwise we must adjustthe factors of the estimation matrix until it has satisfied consistency.

V. CONCLUSIONS

Experiment results showed that with gas pressure increasing, permeability of coal samples was in non-linear increase (phases of increase were rather obvious), and a function of permeability and gas pressure presented a "V"; That is, the gas pressure existed a turning point, smaller than this value, the permeability decreased with gas pressure increasing, larger than this value, the permeability increased with the increase of gas pressure. The experimental data were fitted by the function relationship between permeability and various influencing factors. The permeability and the axial stress, confining pressure showed negative exponential function relationship. And permeability and moisture content showed quadratic function. Finally, based on fuzzy complementary judgment matrix, model of coal gas seepage was established under the coupling effect of the influencing factors such as gas pressure, axial stress, confining pressure and moisture content, etc.

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