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Semi-relativistic Equation Solutions for Bound States of the Heaviest Nuclei

A. J. Janavičius[1], D. Jurgaitis[2], K. Žilinskas[3]
**(Šiauliai University, Lithuania)*
*** (Šiauliai University, Lithuania)*
**** (Šiauliai University, Lithuania)*
Corresponding Author: A. J. Janavičius

ABSTRACT: The impossibility to reach the center of the nuclei island of stability by fusion reactions with stable projectiles requires theoretical analysis. We used the multiplicative perturbation theory for the discrete energies, which was developed by using the modified Lagrange's method with the model harmonic oscillator potential. Using this original method of perturbation theory for semi-relativistic equation, we have calculated single-nucleon energy levels for shells of neutrons and protons, where the relativistic corrections for the mass and potential were included. We have found that the heaviest atomic nuclei ${}^{298}_{114}X$, ${}^{328}_{114}X$, ${}^{334}_{120}X$, and ${}^{340}_{126}X$ from island of stability are more stable than many isotopes of uranium Z=92 and plutonium Z=94 because more neutrons and relativistic corrections can compensate the proton-proton repulsion.

The closed proton-neutron shells with magic pairs of proton-neutron numbers Z=114, N=184; Z=114, N=214; Z=120, N=214; Z=126, N=214 were obtained. The highly accurate method for solution of semi-relativistic equation for calculation of one-nucleon energy levels was used. The corrections for mass are comparable with the energies of protons and neutrons in the external shells and they are important for consideration of interaction potentials and stability of shells in the nuclei.
Keywords: integral equations, perturbation theory, stability of super heavy nuclei.

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I. INTRODUCTION

The interest in the problem of the synthesis of superheavy atomic nuclei increased significantly over the past years. It is connected with Dubna'ssuccessful experiments [1] on the synthesis of the 114 isotopes with A=287, 288, 289 where the last nucleus with N=175 neutrons has a last half-life of 30 seconds. Neutrons act as glue to hold protons together. Optimal relation N/Z corresponds approximately to 1.54. The announcement at Berkeley [2] was about detection of nuclei with Z=118 in the ${}^{86}Kr$ + ${}^{208}Pb$ fusion reaction with large cross section. Due to the strong Coulomb repulsion among protons, according to the liquid drop model, the nuclei would split immediately for Z>104 at less values of N/Z. Only the quantum shell effects [3] allow very heavy elements to exist for a longer time. For consideration of stability of the external shell of the heaviest nuclei, the highly accurate mathematical methods must be used. In the external shells of superheavy elements, the rest interaction is small [4] and the shells' energies can be calculated using the Woods-Saxon single-particle potential [5] where relativistic corrections for the nucleons' mass or kinetic energy must be included [6],[7]. The relativistic corrections for mass of nucleons in external shells, depending on state, can achieve –1.1 MeV and significantly increase the binding energies of nucleons in the external shells [7] by increasing the stability of heavy nuclei. In this case we must solve the semi-relativistic differential equation of the fourth order with sufficient accuracy. For this aim, the semi-relativistic equation has been reduced to the integral-differential equation with the kernel, which is proportional to the Green's function. It can be expressed by unperturbed wave functions and nonphysical solutions of the Schrodinger equation for model potential. This method allows us to solve the semi - relativistic equation where the relativistic corrections for mass and potential are included [7]. The corrections for mass are comparable with the energies of excited states and they are significantly increasing the external shells' stability of heavy nuclei. The corrections for potential are positive and small [7], except for some light nuclei, and do not play any role in the stability of nuclei. The energies of the one-nucleon levels E_{nlj} with the relativistic corrections for the mass E_m and the potential E_v for heavy nucleus [3] $^{285}_{114}X$, hypothetical nucleus[4] $^{310}_{126}X$ and proposed in the paper[8] $^{298}_{114}X$, $^{328}_{114}X$, $^{334}_{120}X$, $^{340}_{126}X$ like candidates to the more stable have been calculated for the spherically symmetric Woods-Saxon potential [5], [8]

$$V(r) = -V^{n,p}\,[1+\exp[\alpha^{n,p}(r-R)]]^{-1}, \quad (1)$$

and the spin-orbit potential

$$V_{sl}(r) = -\kappa\,\frac{1}{r}\frac{d}{dr}V(r)(\boldsymbol{\sigma} * \boldsymbol{l}), \quad (2)$$

with the following parameters [5]:

$$\alpha^{n,p} = 1.5873\ \text{fm}^{-1},\ R = 1.24\ A^{1/3},\ V^{n,p} = V_m\left(1 \mp \gamma\frac{N-Z}{A}\right), \quad (3)$$

$$\gamma = 0.63,\ V_m = 53.3\ \text{MeV},\ \kappa = 0.263\left(1 + 2\frac{N-Z}{A}\right)\text{fm}^2. \quad (4)$$

These parameters were fitted to the one-nucleon levels [5], [7], [8] in the region $15 \le A \le 340$. The Coulomb potential has been introduced in the usual form [9], [10]

$$V_c(r) = \frac{(Z-1)e^2}{4\pi\varepsilon_0 r}P,\ P = \frac{3r}{2R} - \frac{1}{2}\left(\frac{r}{R}\right)^3,\ r \le R, \quad (5)$$

$$P = 1,\ r > R.$$

II. THE INTEGRAL-DIFFERENTIAL SEMI-RELATIVISTIC EQUATIONS

If we consider relativistic corrections for mass and potential as perturbation, the semi-relativistic equation [7] can be expressed in the form

$$\frac{d^2}{dr^2}U_\alpha - \frac{l(l+1)}{r^2}U_\alpha + C\left[E_a - V_D - V_1(r)\right]U_\alpha = 0. \quad (6)$$

Here we introduced the differential operator of the fourth order representing relativistic corrections for the mass:

$$V_D = V(r) + V_{sl}(r) - V_1(r) + \frac{C_1}{C}D(r) + C_1 r\left(\frac{d}{dr}V(r)\right)\frac{d}{dr}\frac{1}{r}, \quad (7)$$

$$C = \frac{2m}{\hbar^2},\ C_1 = \left(\frac{\hbar}{2mc}\right)^2,$$

$$D(r) = \frac{d^4}{dr^4} - \frac{2L_0}{r^2}\frac{d^2}{dr^2} + \frac{4L_0}{r^3}\frac{d}{dr} + \frac{(L_0)^2 - 6L_0}{r^4},\quad L_0 = l(l+1). \quad (8)$$

The last term in (7) represents relativistic corrections of energy levels of the nucleons in the model potential V [6], [11]

$$V_1(r) = \frac{m\omega^2 r^2}{2} \quad (9)$$

for average field of the nucleus. The radial wave functions [11]

$$U_{nl} = e^{-0.5\rho}\rho^{0.5(l+1)}\sum_{k=0}^{n-1}a_k\rho^k,\ \rho = \frac{m\omega r^2}{\hbar},\ n=1, 2, 3 \quad (10)$$

$$a_{k+1} = \frac{k - 0.5(\varepsilon_{nl} - l - 1.5)}{(k+1)(k+l+1.5)}a_k,\ a_0 = 1 \quad (11)$$

and linearly independent nonphysical solutions [11] for model potential (9)

$$F_{nl} = e^{-0.5\rho}\ \rho^{-0.5l}\ \omega(\rho),\ \omega = \sum_{k=0}^{\infty} b_k \rho^k \ , \quad (12)$$

$$b_{k+1} = \frac{k - 0.5(\varepsilon_{nl} + l - 0.5)}{(k+1)(k - l + 0.5)} b_k \ , \ b_0 = 1 \quad (13)$$

have the following eigenvalues

$$E_{nl} = \varepsilon_{nl}\hbar\omega \ , \ \ \varepsilon_{nl} = 2n + l - 0.5 \quad (14)$$

and Wronskian

$$W_0 = (2l+1)\left(\frac{m\omega}{\hbar}\right)^{\frac{1}{2}} \quad (15)$$

The eigenfunctions of Eq. (6), in the case of multiplicative perturbation theory [11], must be expressed by multiplying the eigenfunction (10) U_{nl} of model potential (11)$V_1(r)$by the factor function [7] $\Phi_{2,nlj}$ which depends on the potential operator $V_D(r)$ for relativistic corrections, i. e.

$$U_\alpha = \Phi_{2,nlj}\ U_{nl} \ . \quad (16)$$

Substituting (16) into Eq. (6) we obtain this equation in the potential representation [7]

$$U_{nl}\ \frac{d^2}{dr^2}\ \Phi_2 + 2\left(\frac{d}{dr}\Phi_2\right)\left(\frac{d}{dr}U_{nl}\right) - CV_\delta\ U_{nl}\ \Phi_2 = 0, \quad (17)$$

$$V_\delta = V_D(r) - \Delta E_{nlj} \ , \qquad E_\alpha = E_{nl} + \Delta E_{nlj} \ . \quad (18)$$

Using the modified method of Lagrange [7],a very handy integral equation was obtained:

$$\Phi_2 U_{nl} = U_{nl} + \frac{U_{nl}}{W_0}\int_0^r F_{nl} CV_\delta \Phi_2 U_{nl} dr_1 - \frac{F_{nl}}{W_0}\int_0^r U_{nl}\ CV_\delta \Phi_2 U_{nl} dr_1, \quad (19)$$

$$\Delta E_{nlj} = \frac{\int_0^\infty U_{nl} V_D \Phi_{2,nlj} U_{nl} dr_1}{\int_0^\infty U_{nl} \Phi_{2,nlj} U_{nl} dr_1} \ . \quad (20)$$

The obtained integral equation (19) was solved by the iteration method. For the zero approximation at the right- hand side of the integral equations, we must take Φ_2 =1 and then find the first approach for ΔE_{nlj} from (20) .We can freely choose the model potential (9), but it is better when unperturbed wave functions are close to perturbed wave functions $\Phi_2 U_{nl}$. Then a small number of the iterations provides highly accurate results. In our method, the frequency $\omega = d \cdot \omega_0$ for the model harmonic potential can be determined by the r.m.s. radius of the nuclei [12]

$$\omega_0 = 41 A^{-\frac{1}{3}} \frac{MeV}{\hbar} . \quad (21)$$

The constant d was found by variation in the interval $0.8 \le d \le 1.2$ demanding the minimum of the energy.

We verified the parameters for Woods-Saxon potential (3), (4) calculating one-nucleon energies' levels of the double magic nucleus $^{208}_{82}$Pb taking in the care relativistic corrections for the mass and the potential (7). We used the results of calculations, where relativistic corrections for mass are significant;using some less interaction potential (1), (3) for neutrons $0.98V^n$ is presented in TABLE 1. The experimental meanings [9] of one-nucleon energies' levels for protons E^p_{nlj} and neutrons E^n_{nlj} in TABLE 1are presented as blacker and represent good coincidence with calculations taking $0.98V^n$ in (1) for neutrons. The double magic nucleus $^{208}_{82}$Pb is one of the best cases for shell-model calculations for definition [10] and optimization [7] of Woods-Saxon potentials. The included relativistic corrections for masses E^p_m and E^n_m are significant and improve the accuracy of energy levels of protons and neutrons and energy levels of excited states compared with experimental results [13], [14]. The obtained good coincidences in TABLE 1 propose possibility for application of Woods-Saxon potentials with some decreased 0.98 V_m parameter (3) of A. Chepurnov potentials [5] for considering stability of one $^{298}_{114}$X of the heaviest nuclei [8], [15]. The radial dependence of spin-orbit interaction (2) has maximum at the surface of nucleus. Excited states presented in TABLE 1 also must have strong dependence on significant relativistic corrections for mass.

Table 1. The protons E^p_{nlj} and neutrons E^n_{nlj} of one-nucleon levels and relativistic corrections for mass E^p_m, E^n_m for nuclei $^{208}_{82}$Pb. Z and N are numbers of protons and neutrons.

nlj, Z	E^p_{nlj}, MeV	E^p_m, MeV	nlj, N	E^n_{nlj}, MeV	E^n_m, MeV
$3p_{\frac{1}{2}}$ **126**	-0.198	-0.550	$3d_{\frac{3}{2}}$	-0.272	-0.490
$3p_{\frac{3}{2}}$	-0.717 **-0.620**	-1.02	$2g_{\frac{7}{2}}$	-0.310	-0.601
$2f_{\frac{5}{2}}$	-0.974 **-0.920**	-0.803	$4s_{\frac{1}{2}}$ **184**	-1.21	-0.333
$1i_{\frac{13}{2}}$ **114**	-2.46 **-2.15**	-0.587	$3d_{\frac{5}{2}}$	-1.72 **-2.30**	-0.500
$2f_{\frac{7}{2}}$	-3.24 **-2.86**	-0.662	$1j_{\frac{15}{2}}$	-1.97 **-2.45**	-0.846
$1h_{\frac{9}{2}}$	-3.76 **-3.76**	-0.381	$1i_{\frac{11}{2}}$	-2.72 **-3.09**	-0.551
$3s_{\frac{1}{2}}$ **82**	-8.74 **-8.97**	-0.387	$2g_{\frac{9}{2}}$	-3.73 **-3.86**	-0.557
$2d_{\frac{3}{2}}$	-9.36 **-9.32**	-0.912	$3p_{\frac{1}{2}}$ **126**	-7.36 **-7.36**	-0.485
$1h_{\frac{11}{2}}$	-10.0 **-10.3**	-0.368	$2f_{\frac{5}{2}}$	-7.94 **-7.93**	-0.417
$2d_{\frac{5}{2}}$	-10.8 -10.6	-0.321	$3p_{\frac{3}{2}}$	-8.27 **-8.25**	-0.480
$1g_{\frac{7}{2}}$	-12.2 **-12.4**	-0.281	$1i_{\frac{13}{2}}$	-8.74 **-9.00**	-0.530
$1g_{\frac{9}{2}}$ **50**	-15.7	-0.234	$2f_{\frac{7}{2}}$	-10.4	-0.369
			$1h_{\frac{9}{2}}$	-10.8 **-10.8**	-0.591
			$3s_{\frac{1}{2}}$ **82**	-15.4	-0.315

Table 2. The protons E^p_{nlj} and neutrons E^n_{nlj} of one-nucleon levels and relativistic corrections for mass E^p_m, E^n_m for nucleus $^{298}_{114}$X. Z and N are numbers of protons and neutrons.

nlj Z	E^p_{nlj}, MeV	E, ^p_m MeV	nlj N	E^n_{nlj}, MeV	E^n_m, MeV

$1i_{\frac{13}{2}}$ **114**	-5.89	-0.389	$4s_{\frac{1}{2}}$ **184**	-5.98	-0.486
$2f_{\frac{7}{2}}$	-6.08	-0.345	$3d_{\frac{3}{2}}$	-6.11	-0.951
$1h_{\frac{9}{2}}$	-7.01	-0.290	$3d_{\frac{5}{2}}$	-7.20	-0.497
$3s_{\frac{1}{2}}$ **82**	-9.64	-0.290	$2g_{\frac{7}{2}}$	-7.26	-0.323
$2d_{\frac{3}{2}}$	-10.2	-0.234	$1j_{\frac{15}{2}}$	-8.45	-0.681
$1h_{\frac{11}{2}}$	-11.7	-0.259	$2g_{\frac{9}{2}}$	-9.49	-0.426
$2d_{\frac{5}{2}}$	-12.7	-0.292	$1i_{\frac{11}{2}}$	-10.3	-1.26
$1g_{\frac{7}{2}}$	-13.8	-0.180	$3p_{\frac{1}{2}}$ **126**	-12.7	-0.399
$1g_{\frac{9}{2}}$ **50**	-16.4	-0.160	$3p_{\frac{3}{2}}$	-13.4	-0.391
$2p_{\frac{1}{2}}$	-17.6	-0.149	$2f_{\frac{5}{2}}$	-13.9	-0.354
$2p_{\frac{3}{2}}$	-18.3	-0.143	$1i_{\frac{13}{2}}$	-14.1	-0.421
$1f_{\frac{5}{2}}$	-11.8	-0.250	$2f_{\frac{7}{2}}$	-15.8	-0.352
$1f_{\frac{7}{2}}$ **28**	-20.9	-0.087	$1h_{\frac{9}{2}}$	-16.5	-0.240
$2s_{\frac{1}{2}}$ 20	-23.5	-0.166	**$3s_{\frac{1}{2}}$ 82**	-19.8	-0.234
$1d_{\frac{3}{2}}$	-24.1	-0.05	$1h_{\frac{11}{2}}$	-20.1	-0.318
$1d_{\frac{5}{2}}$	-24.9	-0.05	$2d_{\frac{3}{2}}$	-20.5	-0.230
$1p_{\frac{1}{2}}$ **8**	-27.6	-0.028	$2d_{\frac{5}{2}}$	-21.6	-0.222
$1p_{\frac{3}{2}}$	-27.8	-0.028	$1g_{\frac{7}{2}}$	-22.7	-0.200
$1s_{\frac{1}{2}}$ **2**	-29.9	-0.055	$1g_{\frac{9}{2}}$ **50**	-24.6	-0.178
			$2p^{\frac{1}{2}}$	-26.7	-0.131
			$2p^{\frac{3}{2}}$	-27.2	-0.134
			$1f^{\frac{5}{2}}$	-27.8	-0.214
			$1f^{\frac{7}{2}}$ 28	-28.7	-0.194

			$2s_{\frac{1}{2}}$ 20	-32.8	-0.260
			$1d_{\frac{3}{2}}$	-32.8	-0.260
			$1d_{\frac{5}{2}}$	-33.3	-0.061
			$1p_{\frac{1}{2}}$ 8	-37.0	-0.021
			$1p_{\frac{3}{2}}$	-37.9	-0.021
			$1s_{\frac{1}{2}}$ 2	-41.0	-0.007

Table 3. The protons E^{p}_{nlj} and neutrons E^{n}_{nlj} of one-nucleon levels and relativistic corrections for mass E^{p}_{m}, E^{n}_{m} for nucleus $^{294}_{112}X$. Z and N are numbers of protons and neutrons.

nlj Z	E^{p}_{nlj}, MeV	E^{p}_{m}, MeV	nlj N	E^{n}_{nlj}, MeV	E^{n}_{m}, MeV
$1i_{\frac{13}{2}}$ **112**	-6.09	-0.392			
$2f_{\frac{7}{2}}$	-6.14	-0.351	$3d_{\frac{3}{2}}$ **182**	-5.43	-0.493
$1h_{\frac{9}{2}}$	-7.15	-0.510	$3d_{\frac{5}{2}}$	-6.69	-0.499
$3s_{\frac{1}{2}}$ **82**	-9.56	-0.271	$2g_{\frac{7}{2}}$	-6.87	-0.724
$2d_{\frac{3}{2}}$	-10.4	-0.261	$1j_{\frac{15}{2}}$	-8.45	-0.681
$1h_{\frac{11}{2}}$	-11.7	-0.241	$2g_{\frac{9}{2}}$	-9.47	-0.571
$2d_{\frac{5}{2}}$	-12.7	-0.292	$1i_{\frac{11}{2}}$	-10.2	-0.950
$1g_{\frac{7}{2}}$	-13.8	-0.180	$3p_{\frac{1}{2}}$ **126**	-12.3	-0.404
$1g_{\frac{9}{2}}$ **50**	-16.4	-0.160	$3p_{\frac{3}{2}}$	-13.1	-0.392
$2p_{\frac{1}{2}}$	-17.6	-0.149	$2f_{\frac{5}{2}}$	-13.6	-0.360
$2p_{\frac{3}{2}}$	-18.3	-0.143	$1i_{\frac{13}{2}}$	-14.7	-0.375
$1f_{\frac{5}{2}}$	-11.8	-0.250	$2f_{\frac{7}{2}}$	-15.5	-0.354
$1f_{\frac{7}{2}}$ **28**	-20.9	-0.087	$1h_{\frac{9}{2}}$	-16.9	-0.319
$2s_{\frac{1}{2}}$ 20	-23.5	-0.166	**$3s_{\frac{1}{2}}$ 82**	-19.6	-0.246

1d $_{\frac{3}{2}}$	-24.1	-0.05	1h $_{\frac{11}{2}}$	-19.8	-0.113
1d $_{\frac{5}{2}}$	-24.9	-0.05	2d $_{\frac{3}{2}}$	-20.2	-0.241
1p $_{\frac{1}{2}}$ **8**	-27.6	-0.028	2d $_{\frac{5}{2}}$	-21.3	-0.232
1p $_{\frac{3}{2}}$	-27.8	-0.028	1g $_{\frac{7}{2}}$	-22.7	-0.176
1s $_{\frac{1}{2}}$ **2**	-29.9	-0.055	1g $_{\frac{9}{2}}$ **50**	-24.6	-0.163
			2p $_{\frac{1}{2}}$	-26.8	-0.131
			2p $_{\frac{3}{2}}$	-27.4	-0.471
			1f $_{\frac{5}{2}}$	-27.7	-0.144
			1f $_{\frac{7}{2}}$ **28**	-28.7	-0.194
			2s $_{\frac{1}{2}}$ **20**	-32.6	-0.005
			1d $_{\frac{3}{2}}$	-32.7	-0.163
			1d $_{\frac{5}{2}}$	-33.2	-0.061
			1p $_{\frac{1}{2}}$ **8**	-37.1	-0.034
			1p $_{\frac{3}{2}}$	-37.3	-0.023
			1s $_{\frac{1}{2}}$ **2**	-40.9	-0.007

III. RESULTS AND CONCLUSIONS

At first, we calculated one-nucleon levels for the hypothetical nucleus $^{340}_{126}$X and found closed proton-neutron shells with magic pairs Z=114, N=184; Z=114, N=214; Z=120, N=214; Z=126, N=214 of proton-neutron numbers. It is interesting to investigate the stability of the nucleus $^{298}_{114}$X with closed proton-neutrons shells in semi-relativistic approach using the potentials (1), (2), (5). We calculated the single-nucleon energy levels of protons and neutrons of the nuclei $^{298}_{114}$X, $^{294}_{112}$X. Results are presented in the TABLE 2.

We used an expression of the kinetic energy for α particle $E_k = E_{A-4,Z-2} + E_\alpha - E_{A,Z}$ for the binding energies $E_{A,Z}$ of decaying and daughter $E_{A-4,Z-2}$ nuclei and α particle $E_\alpha = 28.3\ MeV$. Taking in the care that deeper, beginning from 50 nucleons, one-particle energy levels $1g_{9/2}$ of nucleons presented in TABLE 2 andTABLE 3 for decaying $^{298}_{114}$X and daughter $^{294}_{112}$X nucleus practically coincide, we obtained $E_k = (2403.24 - 2404.86)MeV = -1.62 MeV$ (disintegration energy $Q_k = -1.62\ MeV$) and that nucleus $^{298}_{114}$X is stable [17] with respect to the α - decay. The similar but less exact calculations including less energy levels of nucleons and without correction of neutrons' potential $0.98V^n$ for the nucleus ${}^{298}_{114}X$ give stability [8] but with less disintegration energy [15] $Q_k = -0.68\ MeV$. The obtained results coincide with the prediction of possibility of an island of relatively stable super heavy elements ${}^{298}_{114}X$, ${}^{328}_{114}X$, ${}^{334}_{120}X$, ${}^{340}_{126}X$ [13], [14] with the near magic proton numbers $Z = 114$, 120, 126 and magic neutron numbers $N = 184, 214$. The reason of stability of heavy nuclei lies inthe balance between Coulomb forces and nuclear forces for double magic protons and neutron shells [15]. In this case, heavy nuclei can have very long half-lives, maybe the order of millions of ears. From the TABLE 1 we see that relativistic corrections for mass significantly increase the stability of the nucleus ${}^{298}_{114}X$. The same calculations [13] for the nucleus ${}^{328}_{114}X$ approximately coincide with previously results [8] (disintegration energy $Q_k = -0.701\ MeV$). The beta decay is forbidden for the proton and neutron shells' energies presented in [8] in TABLE 2 for the nucleus ${}^{298}_{114}X$. Both nuclei ${}^{298}_{114}X$ and ${}^{328}_{114}X$ are stable according to the beta decay. We obtained the nucleus ${}^{298}_{114}X$ with the following upper one-particle level $4s_{1/2}$ for neutrons –6.30 MeV and the second excited state$2f_{5/2}$ -3.35 MeV for protons. Then beta decay is forbidden for this case by the energy conservation law. The increasing ratio [16] N/Z increases stability of nuclei, and the optimum of stability corresponds to approximately 1.54. For the nucleus ${}^{298}_{114}X$ we have the relation 1.61. For the proton state $1i_{13/2}$ of nuclei ${}^{298}_{114}X$ and ${}^{328}_{114}X$ we have decreasing Coulomb energies from 22.31 MeV to 21.81 MeV consequently [18]. We have a similar situation for other proton states. Taking in the care this fact and relativistic corrections to the mass of nucleons, we can suppose that nuclei ${}^{298}_{114}X$, ${}^{328}_{114}X$ ${}^{334}_{120}X$, ${}^{340}_{126}X$ can be stable [8]. All results can be obtained only using the presented integral equations which can be solved with high accuracy for a mathematically complicated semi-relativistic task. The total nuclear energy evaluated by semi-empirical shell model calculations [18**]**does not coincide with stability calculations using the shell model.

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