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Exact Value of Pi Π (17 – 8√3)

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ABSTRACT:- *In this paper I have shown the proof of exact value of pi. The derivation of this value is supported by number of geometrical constructions, arithmetic calculations and use of some simple algebraic formulae.*
Introduction:-
We know that problem of exact value of pi is not new to this world. It has been discussed for more than a thousand years ago. Now days we can find exact value of pi more than 10 trillion digits with the help of computer to achieve more accuracy. Even then we can't reach up to exact value of pi which will give area of circle equal to area of square.
Numbers of mathematician have tried to divide circle into n-sides polygon in order to get more accurate value. As per this concept Higher the number of sides of polygon more will be the accuracy of value of pi. But this will never give exact results [all the results are approaching towards exactness]. Due to this method all the world assumed that pi is transcendental number not algebraic.

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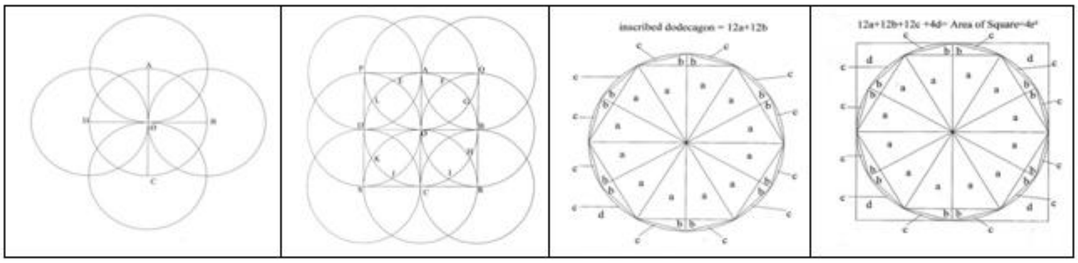
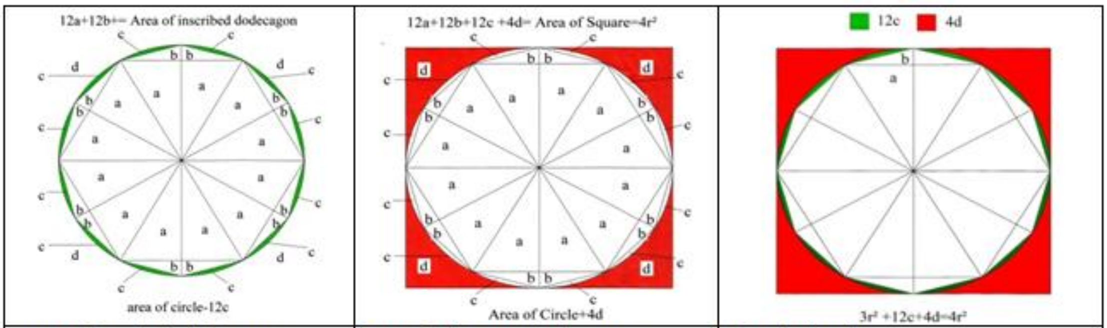
Let: C = circumference of circle, D = diameter of circle, A = area of circle, R = radius of circle,
we know that: $(C \div D) = \pi$, Area of circle $= \pi r^2$, Area of circle $\div r^2 = \pi$ I.e. $(C \div D) = (A \div R^2)$

If we calculate $(C \div D)$ we cannot measure end point of circumference. So it will give approximate results.
I started research to find exact area of circle using $(A \div R^2)$ method. This method gives me exact area of circle. $=(17 - 8\sqrt{3})$
If pi is transcendental I.e. $\pi = 3.1415926535897\ldots$ infinite digit old value
Then $(\pi - 3) = 0.1415926535897\ldots$ & $(4 - \pi) = 0.8584073464102\ldots$ infinite digit
$(\pi - 3) + (4 - \pi) = 1$ = area of (circumscribed square – inscribed dodecagon)
$1 = (0.1415926535897\ldots + 0.8584073464102\ldots) = 0.9999999999999\ldots$ infinite digit
this is not exactly equal to 1. If pi is algebraic i.e. $(17 - 8\sqrt{3})$
Then $[4 - (17 - 8\sqrt{3})] = (8\sqrt{3} - 13)$, $[\pi - 3 = (17 - 8\sqrt{3}) - 3] = (14 - 8\sqrt{3})$, $(4 - \pi) + (\pi - 3) = 1$
$1 = (8\sqrt{3} - 13) + (14 - 8\sqrt{3})$ I have prepared many proofs & here I am giving one of them.
Basic figures



Basic information: Note: let a, b, c & d each part shows area in following figures

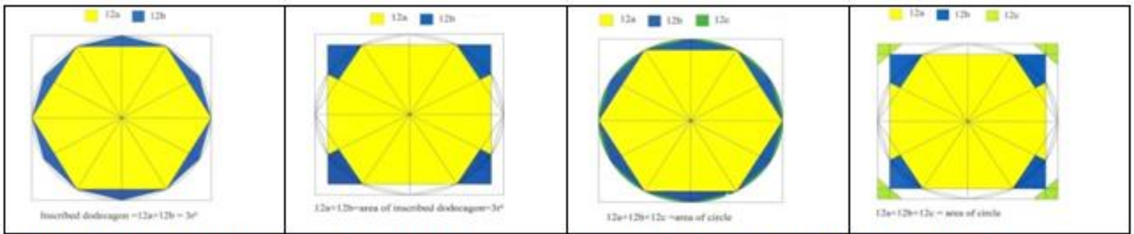


Area of inscribed dodecagon	**Area of circumscribed square**	**$(4-\pi)\,r^2 = 4d$**
$= (12a + 12b) \quad = 3r^2$	**$= (12a + 12b + 12c + 4d) \quad = 4r^2$**	**$(\pi - 3)\,r^2 = 12c$**
$= (\pi r^2 - 12c)$	**$= (\pi r^2 + 4d)$**	**$(4d + 12c) = [(4-\pi) + (\pi - 3)] = 1$**

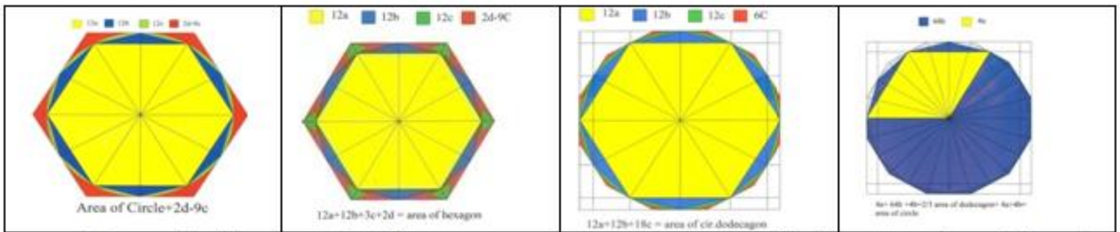
Area of inscribed hexagon	= 12a	$= (1.5\sqrt{3})\, r^2$	$a = (0.125\sqrt{3})\, r^2$
Area of inscribed dodecagon	= (12a + 12b)	$= 3r^2$	$= (\pi r^2 - 12c)$
(12a + 12b) – 12a	= 12b	$= (3 - 1.5\sqrt{3})\, r^2$	$b = (0.25 - 0.125\sqrt{3})\, r^2$
Area of circumscribed square	= (12a + 12b + 12c + 4d)	$= 4r^2$	$= (\pi r^2 + 4d)$
Area of circle	= (12a + 12b + 12c)	$= (3r^2 + 12c)$	$= (4r^2 - 4d)$

$(12c + 4d = 1r^2)/4 \quad = (3c + d = 0.25r^2 = a + b)$

As we know, the exact area of inscribed dodecagon = $3r^2$. In order to calculate exact area of circle, we have to calculate exact area of 12c. Hence there is no need to divide whole circle into infinite number of parts to calculate its accurate area. How to estimate the exact area of part 12c & part 4d?



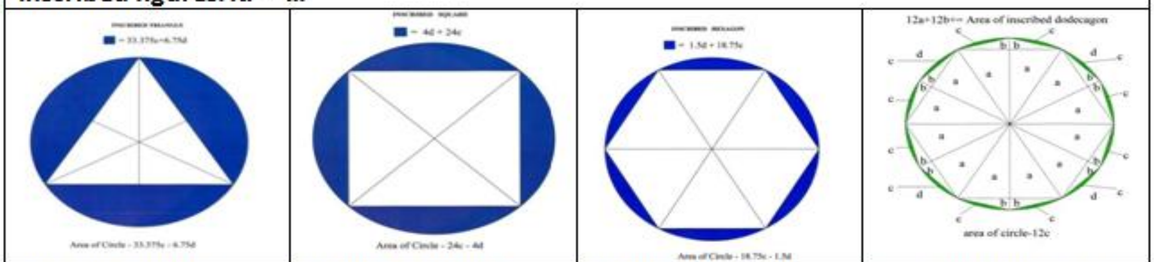
Area of inscribed dodecagon $= 3r^2 = 12(a + b)$	**Area of circle $= 12(a + b + c) = (17 - 8\sqrt{3})\, r^2$**
Side of square $= \sqrt{3}$	**$= \{3r^2 + [(2 - \sqrt{3})^2/2 \times 4]\} \quad = (17 - 8\sqrt{3})\, r^2$**



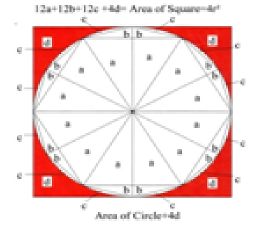
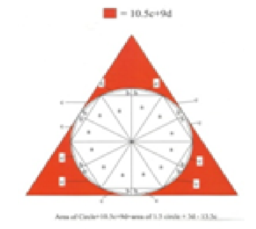
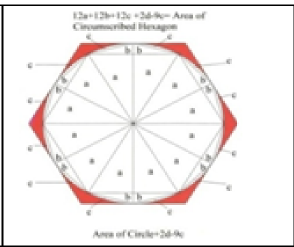
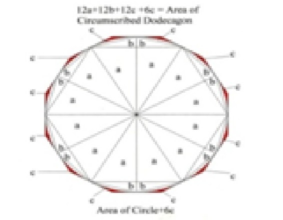
Area of circumscribed hexagon $= (2\sqrt{3})\, r^2 = 16a$	**Area of circumscribed dodecagon $= 12(2 - \sqrt{3})\, r^2 = 96b$**
$= \{3r^2 + [(2/\sqrt{3}) - 1]^2 \times 6\} \quad = (17 - 8\sqrt{3})\, r^2$	**Area of circle $= [8(2 - \sqrt{3}) + 1]\, r^2 \quad = (17 - 8\sqrt{3})\, r^2$**

Supported work: πr^2 method

Inscribed figures: πr^2 - ...



Area of ins. Triangle	**Area of ins. Square**	**Area of ins. Hexagon**	**Area of ins. Dodecagon**
$= (0.75\sqrt{3})\, r^2$	**$= 2r^2$**	**$= (1.5\sqrt{3})\, r^2$**	**$= 3r^2$**
$= \pi r^2 - (33.375c + 6.75d)$	**$= \pi r^2 - (24c + 4d)$**	**$= \pi r^2 - (18.75c + 1.5d)$**	**$= \pi r^2 - 12c$**

circumscribed figures: πr^2 + ...			
			
Area of cir. Square $= 4r^2$ $= \pi r^2 + 4d$	**Area of cir. Triangle $= (3\sqrt{3})\,r^2$ $= \pi r^2 + (10.5c + 9d)$**	**Area of cir. Hexagon $= (2\sqrt{3})\,r^2$ $= \pi r^2 + (2d - 9c)$**	**Area of cir. Dodecagon $= 12(2 - \sqrt{3})\,r^2$ $= \pi r^2 + 6c$**

Exact equations	Derived equations
Area of circle $= (\pi r^2)$	Area of inscribed triangle $= (0.75\sqrt{3})\,r^2 = \pi r^2 - (6.75d + 33.375c)$
Area of inscribed dodecagon $= 3r^2 = (\pi r^2 - 12c)$	Area of inscribed hexagon $= (1.5\sqrt{3})\,r^2 = \pi r^2 - (1.5d + 18.75c)$
Area of circumscribed square $= 4r^2 = (\pi r^2 + 4d)$	Area of inscribed square $= 2r^2 = \pi r^2 - (4d + 24c)$
	Area of circumscribed triangle $= (3\sqrt{3})\,r^2 = \pi r^2 + (9d + 10.5c)$
	Area of circumscribed hexagon $= (2\sqrt{3})\,r^2 = \pi r^2 + (2d - 9c)$
	Area of circumscribed dodecagon $= 12(2 - \sqrt{3})\,r^2 = \pi r^2 + 6c$

Proof of exact equation equal to derived equations
(Area of circumscribed 3 square + area of 6 inscribed dodecagon) $= [3(4r^2) + 6(3r^2)] = 30r^2$
= [area of cir. (2 triangle + 3 hex. + 1 Dodd.) + 3 ins. square]
$= \{2(3\sqrt{3})\,r^2 + 3(2\sqrt{3})\,r^2 + 1(24 - 12\sqrt{3})\}\,r^2 + 6r^2 = 30r^2$

Exact equations	= Derived equations
[Area of circumscribed 3 square $= 3(\pi r^2 + 4d)$ + area of inscribed 6 dodecagon $= 6(\pi r^2 - 12c)$] $= (9\pi r^2 + 12d - 72c)$	[area of circumscribed (2 triangle + 3 hexagon + 1 dodecagon) + 3 ins. Square] $= [2(\pi r^2 + 10.5c + 9d) + 3(\pi r^2 + 2d - 9c) + 1(\pi r^2 + 6c) + 3(\pi r^2 - 24c - 4d)]$ $= (9\pi r^2 + 12d - 72c)$

I got method by using it we can solve infinite examples similar to above. Let us see following table.

We know that: $(\pi r^2 - 12c = 3r^2)$ & $(\pi r^2 + 4d = 4r^2)$

Exact area	Derived area
Area of circle $= (\pi r^2)$	Area of inscribed triangle $= (0.75\sqrt{3})\,r^2 = \pi r^2 - (6.75d + 33.375c)$
Area of inscribed dodecagon $= 3r^2 = (\pi r^2 - 12c)$	Area of inscribed hexagon $= (1.5\sqrt{3})\,r^2 = \pi r^2 - (1.5d + 18.75c)$
Area of circumscribed square $= 4r^2 = (\pi r^2 + 4d)$	Area of inscribed square $= 2r^2 = \pi r^2 - (4d + 24c)$
	Area of circumscribed triangle $= (3\sqrt{3})\,r^2 = \pi r^2 + (9d + 10.5c)$
	Area of circumscribed hexagon $= (2\sqrt{3})\,r^2 = \pi r^2 + (2d - 9c)$
	Area of circumscribed dodecagon $= 12(2 - \sqrt{3})\,r^2 = \pi r^2 + 6c$

From above table using exact equations or derived equations or putting value of area we get appropriate answer in every example.

	Derived area		Sum of each row is $24r^2$			$=(\ldots\pi r^2+\ldots c+\ldots d)$	= exact area
s. r.	(Ins. Tri.	+ Ins. Hex.	+ Cir. Hex.	+ Cir. Tri.	+ Cri. Dodd.)		$=\ldots(\pi r^2-12c)+\ldots(\pi r^2+4d)$
1				(4	+ 1)	$=(5\pi r^2+48c+36d)$	$=-4(\pi r^2-12c)+9(\pi r^2+4d)$
2			(3	+ 2	+ 1)	$=(6\pi r^2+24d)$	$=6(\pi r^2+4d)$
3		(2		+ 3	+ 1)	$=(6\pi r^2+24d)$	$=6(\pi r^2+4d)$
4			(6		+ 1)	$=(7\pi r^2-48c+12d)$	$=4(\pi r^2-12c)+3(\pi r^2+4d)$
5		(4		+ 2	+ 1)	$=(7\pi r^2-48c+12d)$	$=4(\pi r^2-12c)+3(\pi r^2+4d)$
6	(2	+ 1		+ 3	+ 1)	$=(7\pi r^2-48c+12d)$	$=4(\pi r^2-12c)+3(\pi r^2+4d)$
7		(2	+ 3	+ 1	+ 1)	$=(7\pi r^2-48c+12d)$	$=4(\pi r^2-12c)+3(\pi r^2+4d)$
8		(6		+ 1	+ 1)	$=(8\pi r^2-96c)$	$=8(\pi r^2-12c)$
9	(4			+ 3	+ 1)	$=(8\pi r^2-96c)$	$=8(\pi r^2-12c)$
10		(4	+ 3		+ 1)	$=(8\pi r^2-96c)$	$=8(\pi r^2-12c)$
11	(2	+ 3		+ 2	+ 1)	$=(8\pi r^2-96c)$	$=8(\pi r^2-12c)$
12	(2	+ 1	+ 3	+ 1	+ 1)	$=(8\pi r^2-96c)$	$=8(\pi r^2-12c)$
13		(8			+ 1)	$=(9\pi r^2-144c-12d)$	$=12(\pi r^2-12c)-3(\pi r^2+4d)$
14	(2	+ 3	+ 3		+ 1)	$=(9\pi r^2-144c-12d)$	$=12(\pi r^2-12c)-3(\pi r^2+4d)$
15	(2	+ 5		+ 1	+ 1)	$=(9\pi r^2-144c-12d)$	$=12(\pi r^2-12c)-3(\pi r^2+4d)$
16	(4	+ 2		+ 2	+ 1)	$=(9\pi r^2-144c-12d)$	$=12(\pi r^2-12c)-3(\pi r^2+4d)$
17	(4		+ 3	+ 1	+ 1)	$=(9\pi r^2-144c-12d)$	$=12(\pi r^2-12c)-3(\pi r^2+4d)$
18	(2	+ 7			+ 1)	$=(10\pi r^2-192c-24d)$	$=16(\pi r^2-12c)-6(\pi r^2+4d)$
19	(4	+ 4		+ 1	+ 1)	$=(10\pi r^2-192c-24d)$	$=16(\pi r^2-12c)-6(\pi r^2+4d)$
20	(4	+ 2	+ 3		+ 1)	$=(10\pi r^2-192c-24d)$	$=16(\pi r^2-12c)-6(\pi r^2+4d)$
21	(6	+ 1		+ 2	+ 1)	$=(10\pi r^2-192c-24d)$	$=16(\pi r^2-12c)-6(\pi r^2+4d)$
22	(4	+ 6			+ 1)	$=(11\pi r^2-240c-36d)$	$=20(\pi r^2-12c)-9(\pi r^2+4d)$
23	(8			+ 2	+ 1)	$=(11\pi r^2-240c-36d)$	$=20(\pi r^2-12c)-9(\pi r^2+4d)$
24	(6	+ 1	+ 3		+ 1)	$=(11\pi r^2-240c-36d)$	$=20(\pi r^2-12c)-9(\pi r^2+4d)$
25	(6	+ 3		+ 1	+ 1)	$=(11\pi r^2-240c-36d)$	$=20(\pi r^2-12c)-9(\pi r^2+4d)$
26	(6	+ 5			+ 1)	$=(12\pi r^2-288c-48d)$	$=24(\pi r^2-12c)-12(\pi r^2+4d)$
27	(8		+ 3		+ 1)	$=(12\pi r^2-288c-48d)$	$=24(\pi r^2-12c)-12(\pi r^2+4d)$
28	(8	+ 2		+ 1	+ 1)	$=(12\pi r^2-288c-48d)$	$=24(\pi r^2-12c)-12(\pi r^2+4d)$
29	(8	+ 4			+ 1)	$=(13\pi r^2-336c-60d)$	$=28(\pi r^2-12c)-15(\pi r^2+4d)$
30	(10	+ 1		+ 1	+ 1)	$=(13\pi r^2-336c-60d)$	$=28(\pi r^2-12c)-15(\pi r^2+4d)$
31	(10	+ 3			+ 1)	$=(14\pi r^2-384c-72d)$	$=32(\pi r^2-12c)-18(\pi r^2+4d)$
32	(12			+ 1	+ 1)	$=(14\pi r^2-384c-72d)$	$=32(\pi r^2-12c)-18(\pi r^2+4d)$
33	(12	+ 2			+ 1)	$=(15\pi r^2-432c-84d)$	$=36(\pi r^2-12c)-21(\pi r^2+4d)$
34	(14	+ 1			+ 1)	$=(16\pi r^2-480c-96d)$	$=40(\pi r^2-12c)-24(\pi r^2+4d)$
35	(16				+ 1)	$=(17\pi r^2-528c-108d)$	$=44(\pi r^2-12c)-27(\pi r^2+4d)$

Area of Inscribed figures			Area of Circumscribed figures		
Triangle	$=(0.75\sqrt{3})\,r^2$	$=\pi r^2-(6.75d+33.375c)$	Triangle	$=3\sqrt{3})\,r^2$	$=\pi r^2+(9d+10.5c)$
Square	$=2r^2$	$=\pi r^2-(4d+24c)$	Square	$=4r^2$	$=\pi r^2+4d$
Hexagon	$=(1.5\sqrt{3})\,r^2$	$=\pi r^2-(1.5d+18.75c)$	Hexagon	$=(2\sqrt{3})\,r^2$	$=\pi r^2+(2d-9c)$
Dodecagon	$=3r^2$	$=\pi r^2-12c$	Dodecagon	$=12(2-\sqrt{3})\,r^2$	$=\pi r^2+6c$

From above table putting value or area we get appropriate answer in every example.

Sum of each row Area of circle $=\pi r^2$ $=(17-8\sqrt{3})\,r^2$								
s. r. no.	Inscribed figures				Circumscribed figures			
	(Triangle	Square	Hexagon	Dodd.	Triangle	Square	Hexagon	Dodd.)
1	-6	10	-1	-1			-1	
2	-4	7	-2	1			-1	
3	-2	5	-3	1		1	-1	
4	-2	3	-1	1	-1	2	-1	
5		1		1		3	-4	
6		1	-2	1	-1	3	-1	
7		2	-4	3		1	-1	
8	-2	2	-1	3	-1	1	-1	
9	-2	4	-3	3			-1	
10				3		2	-4	
11			-2	3	-1	2	-1	
12	-4	4		3	-1		-1	
13	-2	1	-1	5	-1		-1	
14		1	-4	5			-1	
15				-1	-2	5	-1	
16				-1		-1	2	1
17		1		-3			2	1
18		-1		1		-2	2	1

For example no. 1

Area of inscribed [(– 6 triangle + 10 square – 1 hexagon – 1 dodecagon) – 1 cir. Hexagon]

$= -6(0.75\sqrt{3})r^2 + 10(2r^2) - 1(1.5\sqrt{3})r^2 - 1(3r^2) - 1(2\sqrt{3})r^2$

$= (-4.5\sqrt{3})r^2 + (20r^2) – (1.5\sqrt{3})r^2 - (3r^2) – (2\sqrt{3})r^2$
$= (17 – 8\sqrt{3})\ r^2$
$= - 6(\pi r^2 - 6.75d – 33.375c) + 10(\pi r^2 - 4d – 24c) – 1(\pi r^2 - 1.5d – 18.75c) – 1(\pi r^2 - 12c) – 1(\pi r^2 + 2d – 9c)$
$= (- 6\pi r^2 + 40.5d + 200.25c) + (10\pi r^2 - 40d – 240c) + (- 1\pi r^2 + 1.5d + 18.75c) + (- 1\pi r^2 + 12c) + (- 1\pi r^2 – 2d + 9c)$
$= \pi r^2$

Algebra proof   Note x, y = any number
{Area of(x) inscribed dodecagon + area of circumscribed [2y hexagon + (2x + 3y) dodecagon]} - [area of (y) circumscribed square]   = area of (3x + 4y) circle]
$= [3x + 4y\ (17 - 8\sqrt{3})\ r^2]$

Area of circumscribed (103 dodecagon + 413 hexagons + 134 triangles]
$= 103(24 – 12\sqrt{3})\ r^2 + 413(2\sqrt{3})\ r^2 + 134(3\sqrt{3})\ r^2$ $= (2472 – 8\sqrt{3})\ r^2$
$= 103(\pi r^2 + 6c) + 413(\pi r^2 + 2d – 9c) + 134(\pi r^2 + 9d + 10.5c)$ $= (650\pi r^2 - 1692c + 2032d)$
= area of 141 inscribed dodecagon + 508 circumscribed square + 1 circle
$= 141(\pi r^2 - 12c) + 508(\pi r^2 + 4d) + 1(\pi r^2)$ $= 141(3r^2) + 508(4r^2) + 1(\pi r^2)$
$= [(423 + 2032 + (17 – 8\sqrt{3})]\ r^2$ $= (2472 – 8\sqrt{3})\ r^2$

Conclusions:-   Exact Area of circle = $(17 - 8\sqrt{3})\ r^2$

REFERENCES,

[1]. Archimedes method, Euclidean geometry, Basic Algebra & Geometry concept, History of pi (π) Complete thesis of my research titled as ‘‘Exact value of pi ’’ has being published in following journals: IOSR(international organization scientific research) journal of mathematics in May-June 2012. IJERA(international journal of Engineering research and applications) in July-August 2013 IJMSI (international journal of mathematics and Statistics Invention) in Feb. 2015 IOSR(international organization scientific research) journal of mathematics in Nov. – Dec 2016

[2]. Soft copy of my thesis is now also available on internet and one can get it by making search with following key words: “pi value gogavale”

3.143593539448981651780429267953021064457557969516954975553544164384535864729599703350830505942011394594990868676746375760240439240008380169506755903245010811775254144781555343456201439191322600613879877501253864773416906794621962648625296230183622946717057107295847600107700961124884258797219947391593947053214963151192419182502477399334833363808321336017681403933690284289336661222206688123297156818656335489453170671113254609752844973471183293555800786639811834310845690432072349073623302652015416804053628125735833708731168090090419699341461457348832056135937290376983568693043493503170341887846007145854150405692994870868623866057983181016045833351520812324192098685688817630817296332878169161176771391729177535484944221242711522482217017077886769418682412639830436126757772470549707723943718466799866485644676153231429782848001896585322963380493104839118982810890024187632140385691592196967888954680304704302216343115558056610214462418093899506948249263737170266536490803104663602089308836032494833938625173010934992712802756603001321702419158112017846461411438237557577276912676680231420563526346350379923838592294513017869120197730692551358794673170314821371690992621915342235916770557837619961610175364306735196663578544479847234132302592448850638005436618471725041500290239110517911325848717602237510383199491…

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