

# Power Rate Reaching Law Based Second Order Sliding Mode Control

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**ABSTRACT:** In this article the second order sliding control (2-SMC) is proposed with the power rate reaching law. For sliding mode control here a proportional, integral, derivative sliding surface is used. This law increases the reaching speed; it provides the results with faster convergence of sliding surface & low chattering mode. The performance of closed loop system is analyzed to get better tracking specifications though it consists of external disturbances. The electromechanical plant is used to show suitability and effectiveness of the proposed second-order sliding mode control and factors involved in the design. The sliding mode control law is derived using direct Lyapunov stability approach and asymptotic stability is proved theoretically. The 2-SMC using power rate reaching law is compared with 2-SMC for uncertain plant results.

**Keywords :** Sliding mode control, DC motor model, PID sliding surface.

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## I. INTRODUCTION

Now-a-days sliding mode control is the most interesting concept from the technical point of view. Sliding mode control (SMC) is known to be a robust control method appropriate for uncertain systems. High robustness is maintained against various kinds of uncertainties such as external disturbances and measurement error.

To obtain a second-order sliding mode control based on a PID sliding surface with independent gain coefficients and address issues related to sliding mode control. Equivalent control approach is used in solution based on the second-order plant model. Experimental validation of the present design proves that the control and tracking performance is improved in the presence of uncertainties and disturbances while maintaining the stability. The formal descriptions of traditional SMC, 2-SMC, model description is given with control strategy and results [1].

The study of second order sliding mode control is done for an uncertain plant which uses the equivalent approach to show the improved performance of the system. In recent years, control of such systems has attracted greater search interest. It is well known that most of the control systems have nonlinear and time-varying behavior with various uncertainties and disturbances. The SMC method has some advantages such as robustness to parameter uncertainty, insensitivity to bounded disturbances, fast dynamic response are mark able computational simplicity with respect to other robust control approaches and easy implementation of the controller[2,4].

In the early sixties this method has gained the significant research attention it the former USSR and its wide variety of applications are seen in the late seventies [3]. The aim of the controller is to bring the error to sliding surface and further it is bounded to remain on the sliding surface. The control process is based on these two phases: sliding phase and reaching phase. So that two types of control laws can be derived separately, the equivalent controls law and the switching law. The second order sliding mode approach proposed to reduce the chattering problem and it gives good robustness for the closed loop system [5].

For the robust finite time controller design higher order sliding mode control algorithm is given as: design finite time controller and discontinuous control laws, gives finite time stabilization of the nominal system and rejects the uncertainties of the system respectively [6].Systematic assessment of chattering problem and sliding mode control solution for real life engineering problems are well covered in [7].The second-order sliding

mode control, compared to first-order SMC has the advantage that it provides a smooth control and better performance in the control implementation yielding less chattering and better convergence accuracy while preserving the robustness properties [8]. The peculiar focus is to reduce the error to zero, not only the sliding surface, but also its second-order derivative. It means that the second-order sliding mode corresponds to the control acting on the second derivative of the sliding surface [9]. There are different types of second order sliding mode control algorithms methods. For these methods proposed solution is classified into model based and non-model based technique. Model-based control design is systematic and can be applied in general cases, and specifications in terms of robustness and tracking accuracy can be priori assigned, as well as various criteria can be fulfilled [4].

The description of electromechanical plant and its online identification is available in this paper. In short we get the examination of dc motor behavior which constitutes a useful effort for analysis and control of many practical applications [10].

In this paper second order sliding mode results are improved by using the power rate reaching law. The reaching laws are of different classical types out of which power rate reaching law is one. The problem of chattering is better overcome by this law and the response we get is also faster from the previous responses as in traditional SMC, 2-SMC [1]. The equivalent control approach is used in the plant model and the stability is proved using the direct Lyapunov approach. The dc motor drive is the component of the electromechanical systems, have been used extensively in several industrial applications as actuating elements to follow a predetermined as speed or position trajectory under load for their advantages of easy speed and position control and wide adjustable range. For the analysis and control of many practical applications examination of dc motor behavior constitutes a useful effort.

## II. SLIDING MODE CONTROL

### 2.1. Theory of sliding mode control

The sliding mode control or SMC, is a nonlinear control method that alters the dynamics of a nonlinear system by application of a discontinuous control signal that forces the system to "slide" along a cross-section of the system's normal behavior. The state-feedbacks control law is not a continuous function of time. Instead, it can switch from one continuous structure to another based on the current position in the state space. Hence, sliding mode control is a variable structure control method. The multiple control structures are designed so that trajectories always move toward an adjacent region with a different control structure, and so the ultimate trajectory will not exist entirely within one control structure. Instead, it will slide along the boundaries of the control structures. The motion of the system as it slides along these boundaries is called a sliding mode and the geometrical locus consisting of the boundaries is called the sliding surface [12].

### 2.2. Equations for linear uncertain plant

Let a single-input second-order linear uncertain plant [1]:

$$\ddot{y}_m(t) = -(A_n \pm \Delta A) \dot{y}_m(t) - (B_n \pm \Delta B) y_m(t) + (C_n \pm \Delta C)u(t) + d(t) \quad (1)$$

Where  $y_m(t) \in \mathbb{R}^+$ ,  $y_m(t)$  is the output,  $u(t)$  is control input,  $u(t) \in \mathbb{R}^+$ ,  $A_n$ ,  $B_n$  &  $C_n$  are the nominal plant parameters,  $\Delta A$ ,  $\Delta B$  and  $\Delta C$  are the unknown model uncertainties introduced by the plant parameters, nonlinear friction and unmodeled dynamics,  $d(t)$  denotes uncertain external disturbance,  $t$  is the independent time variable,  $D$  denotes set of real constants. The second-order plant model, Eq.(1), can be rearranged as:

$$\ddot{y}_m(t) = -A_n \dot{y}_m(t) - B_n y_m(t) + C_n u(t) + D(t, u(t)) \quad (2)$$

Where,

$D(t, u(t))$  denoting the lumped uncertainty that is bounded and unknown satisfying  $|D| \leq D_{max}$ , is given

$$D(t, u(t)) = \pm \Delta A \dot{y}_m(t) \pm \Delta B y_m(t) \pm \Delta C u(t) + d(t) \quad (3)$$

The upper bound for the uncertainty  $D_{max}$  is,

$$D_{max} = \Delta A |\dot{y}_m(t)| + \Delta B |y_m(t)| + \Delta C |u(t)| + |d(t)| \quad (4)$$

Where  $\bar{u}$  is the input hard constraint,  $|u(t)| \leq \bar{u}$ ,  $D_{max} \in \mathbb{R}^+$ ,  $\mathbb{R}^+$  set of positive real constants. The control problem is to find suitable control input such that the output tracks desired command asymptotically in the presence of model uncertainties & disturbances. The tracking error  $e(t)$ ,  $e(t) \in \mathbb{R}$ , in terms of command signal  $y_r(t)$  ( $y_r(t) \in \mathbb{R}$ ) and measured output signal  $y_m(t)$ , is defined as,

$$e(t) = y_r(t) - y_m(t) \quad (5)$$

The Sliding mode control is of different types one is first order sliding mode control and another is the second order sliding mode control.

In the first order sliding mode control the condition is given like this  $s(t) = 0$  and  $s(t)\dot{s}(t) < 0$ . The aim of the first order sliding mode control is to force the error to move on the switching surface. The sliding surface in traditional SMC depends on the tracking error.[1]

The sliding surface for the traditional SMC it depends on tracking error and derivatives of tracking error, given as,

$$s(t) = \lambda_c + \left(\frac{d}{dt}\right)^{(n-1)} e(t) \tag{6}$$

Where  $\lambda_c$  is positive constant it belongs to  $\mathbb{R}^+$ ,  $n$  denotes the order of uncontrolled system. The error starting from a certain initial value will converge to the boundary layer and move inside the boundary layer towards the horizontal axis until it reaches  $\dot{e}(t) = 0$ .

The first time derivative of the sliding surface Eq. (6),

$$\dot{s}(t) = \lambda_c \dot{e}(t) + \ddot{e}(t) \tag{7}$$

Now take the second time derivative of error in term of plant parameters. Using the Eq. (5),

$$\ddot{e}(t) = \ddot{y}_r(t) - \ddot{y}_m(t) \tag{8}$$

Substitute the value of  $\ddot{y}_m(t)$  by using the Eq. (2) in such a manner as follows,

$$\ddot{e}(t) = \ddot{y}_r(t) + A_n \dot{y}_m(t) + B_n y_m(t) - C_n u(t) - D(t, u(t)) \tag{9}$$

This equation is further used at the time of deriving second order sliding mode control derivation.

### 2.3. Second-order sliding mode control

In the 2-SMC the condition is given like this “for any  $r^{\text{th}}$  order sliding mode,  $s(t) = \dot{s}(t) = \dots = s^{(r-1)}(t) = 0$ . The purpose of higher order sliding mode control is to enable the error to move on the switching surface  $s(t) = 0$  and the first successive derivative  $(r-1)$  null. The PID sliding surface introduced for the second order sliding mode control is give as,

$$\dot{s}(t) + \beta s(t) = k_p e(t) + \int_0^t k_i e(\tau) d\tau + k_d \dot{e}(t) \tag{10}$$

Where  $k_p, k_i$  and  $k_d$  are the independent positive constants denoting proportional, integral and derivative gains, respectively, all these constants belong to  $\mathbb{R}^+$ ,  $\beta$  is also a positive constant and belongs to  $\mathbb{R}^+$ . After the sliding mode is enforced it determines the rate of decay for  $s(t)$  and contributes in damping too. For the flexibility of the sliding surface the gains are provided in the above equation.

The system is initially in the region  $s(t) > 0$  and that input is not sufficient to drive the error towards sliding surface. So that it results in increasing  $s(t)$  and error moves far from the sliding surface. To force the error to move towards the sliding surface the integral action is used so that it increase the control action accordingly and it satisfy the condition  $\dot{V}(t) = 0$ . Now as the  $s(t)$  reaches the sliding surface, the control action gets reduced because  $s(t)$  is decreasing. The system is said to be in sliding mode when  $s(t)$  is on the sliding surface and so the problem of tracking set point is equivalent to that of remaining on the zero sliding surface for  $t > 0$ . The control input is given as,

$$U(t) = u_{eq}(t) + u_{sw}(t)$$

Where  $u_{eq}(t)$  and  $u_{sw}(t)$  are the equivalent control and the switching control respectively. The equivalent control is given by Utkin is based on the nominal plant parameters with  $D(t, u(t)) = 0$  and it provides the main control action. And the switching control ensures the discontinuity of the control law across the sliding surface. The controller must be designed such that it can drive the error to sliding surface and when it reach to sliding surface it is said to be the reaching phase.

#### 2.3.1. Equivalent control equation

The equivalent control is obtained from the equation (6), take the second time derivative of the sliding surface as such below,

$$\ddot{s}(t) + \beta \dot{s}(t) = k_p \dot{e}(t) + k_i e(t) + k_d \ddot{e}(t) \tag{11}$$

The error converges to zero exponentially if the system trapped on the sliding surface and the coefficients,  $k_p, k_i$  and  $k_d$ , are selected properly. The plant Eq. (1) satisfies second order mode with respect to the sliding surface  $s(t)$  if its error lies on the intersection of  $s(t) = 0$  and  $\dot{s}(t) = 0$ . Now substitute Eq. (9) into Eq. (11), as such,

$$\ddot{s}(t) + \beta \dot{s}(t) = k_i e(t) + k_p \dot{e}(t) + k_d (\ddot{y}_r(t) + A_n \dot{y}_m(t) + B_n y_m(t) - C_n u(t) - D(t, u(t))) \tag{12}$$

By recognizing  $\dot{s}(t) = 0$  the equivalent control is found and it's the necessary condition for error to stay on sliding surface,  $D(t, u(t))$  is not taken into account. When  $\dot{s}(t) = 0$  we get the equivalent control as,

$$u_{eq}(t) = (k_d C_n)^{-1} k_i e(t) + k_p \dot{e}(t) + k_d (\ddot{y}_r(t) + A_n \dot{y}_m(t) + B_n y_m(t) - \beta \dot{s}(t)) \tag{13}$$

The equivalent signal with the uncertainty is given as such below,

$$u_{eq}^* = u_{eq} + C_n^{-1} D(t, u(t)) \tag{14}$$

**2.3.2. Switching control equation**

If the switching control is function is introduced directly as,

$$U_{sw}(t) = \lambda_1 s(t) + k_s \text{sign}(\dot{s}(t)) \tag{15}$$

Where  $\lambda_1, k_s \in \mathbb{R}^+$  with  $\lambda_1 > \frac{1}{C_n k_d}, k_s > \frac{U_{max}}{C_n}$  with  $D_{max} = \sup_{\forall t, s, \dot{s}=0} \{D(t, u(t))\}$ . If Eqs. (13) and (15) are substituted into Eq. (12) one has

$$\ddot{s}(t) = -k_d D(t, u(t)) - k_d C_n \lambda_1 s(t) - k_d C_n k_s \text{sign}(\dot{s}(t)) \tag{16}$$

The second order sliding mode switching controller using the hyperbolic tangent function instead of the sin function can be given as,

$$U_{sw}(t) = \lambda_1 s(t) + k_s \tanh(\dot{s}(t)/\Omega) \tag{17}$$

**2.3.3. Lyapunov Stability**

For the stability here we are using the Lyapunov stability function as such given below,

$$V(t) = \frac{1}{2} s^2(t) + \frac{1}{2} \dot{s}^2(t) \tag{18}$$

With  $V(0)=0$  and  $V(t)>0$  for  $s(t)=0, \dot{s}(t)=0$ . The stability is guaranteed if the derivative of the Lyapunov function is negative definite, also known as the reaching condition[1]:  $\dot{V}(t) < 0, s(t) = 0, \dot{s}(t) = 0$

Taking the first time derivative of Eq. (18) yields:

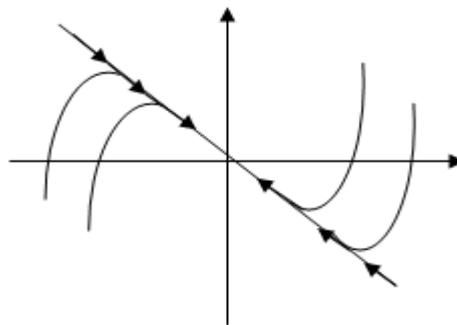
$$\dot{V}(t) = s(t) \dot{s}(t) + \dot{s}(t) \ddot{s}(t) \tag{19}$$

$$\begin{aligned} &= s(t) \dot{s}(t) + \dot{s}(t) (-k_d D(t, u(t)) - k_d C_n \lambda_1 s(t) - k_d C_n k_s \text{sign}(\dot{s}(t))) \\ &= s(t) \dot{s}(t) - k_d C_n \lambda_1 s(t) \dot{s}(t) - \dot{s}(t) k_d D(t, u(t)) - k_d C_n k_s |\dot{s}(t)| \\ &\leq |\dot{s}(t)| (s(t) - k_d C_n \lambda_1 s(t) - k_d D(t, u(t)) - k_d C_n k_s) \\ &\leq |\dot{s}(t)| (|s(t)| - k_d C_n \lambda_1 |s(t)| - k_d D(t, u(t)) - k_d C_n k_s) \\ &\leq |\dot{s}(t)| (|s(t)| - k_d C_n \lambda_1 |s(t)| + k_d D_{max} - k_d C_n k_s) \\ &= -|\dot{s}(t)| (|s(t)| (k_d C_n \lambda_1 - 1) + k_d C_n k_s - k_d D_{max}) < 0. \end{aligned} \tag{20}$$

**III. SLIDING MODE CONTROL BASED ON REACHING LAW**

Sliding mode based on reaching law includes reaching phase and sliding phase. The reaching phase drive system is to maintain a stable manifold and the sliding phase drive system ensures slide to equilibrium. One of the classical reaching law is given below. The equation for the power rate reaching law is given as [11]:

$$\dot{s}(t) = -k |s|^{\alpha} \text{sgn}(s) \quad k > 0, 1 > \alpha > 0 \tag{21}$$



**Fig.2. The idea of sliding mode**

This law increases the reaching speed when this state is far away from the switching manifold. Power rate reaching law reduces the rate when state is near the manifold and the result gives us fast & low chattering reaching mode. There are different classical reaching laws out of them here the power rate reaching law is used.  $x_1$

**3.1. Equivalent control equation**

As we know for the second order sliding mode the condition is  $s(t) = \dot{s}(t) = \dots = s^{(r-1)} = 0$  and in power rate reaching law the Eq.(20) is used while deriving the equation for equivalent control.

Now use Eq.(12),as given below:

$$\ddot{s}(t) + \beta \dot{s}(t) = k_i e(t) + k_p \dot{e}(t) + k_d (\ddot{y}_r(t) + A_n \dot{y}_m(t) + B_n y_m(t) - C_n u(t) - D(t, u(t))) \quad (22)$$

If we substitute the  $\ddot{s}(t) = 0$  and the Eq. (20) we get,

$$\beta (-k |s|^\alpha \text{sgn}(s)) = k_i e(t) + k_p \dot{e}(t) + k_d (\ddot{y}_r(t) + A_n \dot{y}_m(t) + B_n y_m(t) - C_n u(t) - D(t, u(t))) \quad (23)$$

By rearranging the terms we get,

$$k_d C_n u(t) = k_i e(t) + k_p \dot{e}(t) + k_d (\ddot{y}_r(t) + A_n \dot{y}_m(t)) + \beta (k |s|^\alpha \text{sgn}(s)) \quad (24)$$

Considering the necessary conditions we kept the error on the sliding surface,  $D(t, u(t))$  is not taken into account so the equivalent control is obtained as follows:

$$u_{eq}(t) = (k_d C_n)^{-1} (k_i e(t) + k_p \dot{e}(t) + k_d (\ddot{y}_r(t) + A_n \dot{y}_m(t)) + \beta (k |s|^\alpha \text{sgn}(s))) \quad (25)$$

### 3.2. Switching control equation

The equation for the switching control is given as follows:

$$u_{sw}(t) = \lambda_1 s(t) + k_s \tanh(\dot{s}(t)/\Omega) \quad (26)$$

## IV. DC MOTOR MODEL

### 4.1. Description

Almost every mechanical movement that we see around us is accomplished by an electric motor. Electric machines are a means of converting energy. Motors take electrical energy and produce mechanical energy. Some examples of large motor applications include elevators, electric trains, hoists, and heavy metal rolling mills. Examples of small motor applications include motors used in automobiles, robots, hand power tools and food blenders. Micro-machines are electric machines with parts the size of red blood cells, and find many applications in medicine.

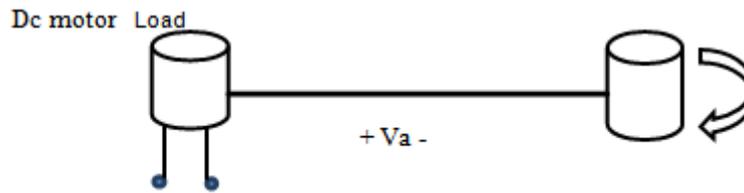


Fig.2. Diagram of Electromechanical plant

A dc motor connected to a load via a shaft, is shown in Fig.1, the dc motor, as components of electromechanical systems, have been widely used in several industrial applications as actuating elements to follow a predetermined speed or position trajectory under load for their advantages of easy speed and position control and wide adjust ability range. Consequently, examination of dc motor behavior constitutes a useful effort for analysis and control of many practical applications [10].The electrical and mechanical equations for the electromechanical plant, shown in Fig.2, consisting of a dc motor connected to a load via along shaft can be given as above:

### 4.2. Equations

$$V_a(t) = L_a \frac{d}{dt} i_a(t) + R_a i_a(t) + K_m \omega_t \quad (27)$$

$$J_m \left( \frac{d\omega_m(t)}{dt} \right) = T_m(t) - T_s(t) - R_m \omega_m(t) - T_f(\omega_m) \quad (28)$$

$$J_L \left( \frac{d\omega_L(t)}{dt} \right) = T_s(t) - T_d(t) - R_L \omega_L(t) - T_f(\omega_m) \quad (29)$$

$$s(t) = k_s(\theta_m(t) - \theta_L(t)) + B_s(\omega_m(t) - \omega_L(t)) \quad (30)$$

$$(d\theta_m(t))/dt = \omega_m(t) , (d\theta_L(t))/dt = \omega_L(t)$$

Where  $v_a$  is the motor armature voltage,  $R_a$  &  $L_a$  are the armature coil resistance and inductance respectively,  $i_a$  is the armature current,  $k_m$  is the torque coefficient,  $T_m$  is the generated motor torque,  $\omega_m$  are the rotational speeds of the motor,  $j_m$  and  $j_L$  are the moment of inertia  $R_m$  and  $R_L$  are the coefficient of the viscous friction,  $T_d$  is the external load disturbance,  $T_f$  is the nonlinear friction,  $T_s$  is the transmitted shaft torque and  $t$  is the time.

There are several plant parameters in the plant model so in this case it is very much difficult while doing the experiments. We are not able to define each and every parameter at that time the simplification is needed. So to simplify the modeling process the plant can be approximated using a

second order model including disturbances and uncertainties as:

$$\ddot{\omega}_L(t) = -(A_n \pm \Delta A) \dot{\omega}_L(t) - (B_n \pm \Delta B) \omega_L(t) + (C_n \pm \Delta C) v_a(t) + f(t) \quad (31)$$

$$\ddot{\omega}_L(t) = -A_n \dot{\omega}_L(t) - B_n \omega_L(t) + v_a C_n(t) + L(t, u(t)) \quad (32)$$

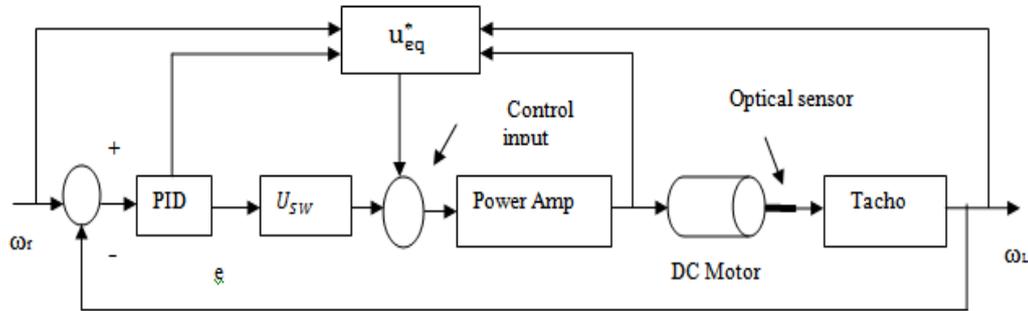


Fig.3. Diagram of Experimental Setup

Where the linear plant parameters are based on the nominal operating point should be determined experimentally ( $t, u(t)$ ) in Eq. (38) denotes the lumped uncertainty that is bounded by unknown,  $|L(t, u(t))| \leq L_m, L_m \in \mathbb{R}^+$ , and it is defined by,

$$L(t, u(t)) = -\Delta A \omega_L(t) - \Delta B \dot{\omega}_L(t) + \Delta C u(t) + f(t) \quad (33)$$

And the upper bound  $L_m$ .

$$L_m = \Delta A |\omega_L(t)| + \Delta B |\dot{\omega}_L(t)| + \Delta C \bar{u} + |f(t)| \quad (34)$$

Where  $\bar{u}$  is the input hard constraint,  $|u(t)| \leq \bar{u}$ .

The control problem is to find suitable control input such that the output tracks desired command asymptotically in the presence of model uncertainties & disturbances. The tracking error  $e(t), e(t) \in \mathbb{R}$ , in terms of command signal  $\omega_r(t) (\omega_r(t) \in \mathbb{R})$  and measured output signal  $\omega_i(t)$ , is defined as,

$$e(t) = \omega_r(t) - \omega_i(t) \quad (35)$$

The second time derivative of the error with DC motor model can be given as follows. With help of the Eq.(8) we can write it directly.

$$\ddot{e}(t) = \ddot{\omega}_r(t) - \ddot{\omega}_i(t) \quad (36)$$

The single input second order linear uncertain plant equation we get it from the Eq.(2) in terms of dc motor as follows:

$$\ddot{\omega}_i(t) = -A_n \dot{\omega}_i(t) - B_n(t) \omega_i(t) + C_n v_a(t) + D(t, v_a(t)) \quad (37)$$

By substituting the value of  $\ddot{\omega}_i(t)$  in the above equation we get the  $\ddot{e}(t)$  as follows:

$$\ddot{e}(t) = \ddot{\omega}_r(t) + A_n \dot{\omega}_i(t) + B_n(t) \omega_i(t) - C_n v_a(t) - D(t, v_a(t)) \quad (38)$$

Where  $\omega_i(t)$  is the output,  $\omega_i(t) \in \mathbb{R}$ ,  $v_a(t)$  is the control input,  $v_a(t) \in \mathbb{R}$ ,  $A_n, B_n$  &  $C_n$  are the nominal plant parameters.  $D(t, v_a(t))$  it denotes the lumped uncertainty. The control input can be given as follows:

$$V_a(t) = v_{eq}(t) + v_{sw}(t) \quad (39)$$

Considering the necessary condition we kept error on the sliding surface,  $D(t, v_a(t))$  is not taken in to account so the equivalent control is obtained as ,

$$v_{eq}(t) = (k_d C_n)^{-1} (k_i e(t) + k_p \dot{e}(t) + k_d (\ddot{\omega}_r(t) + A_n \dot{\omega}_i(t) + \beta (k |s|^\alpha \text{sgn}(s))) \quad (40)$$

The equation for the switching control is given as follows:

$$v_{sw}(t) = \lambda_1 s(t) + k_s \tanh(\dot{s}(t)/\Omega) \quad (41)$$

## V. SECOND ORDER SLIDING MODE CONTROL WITH POWER RATE

### 5.1. Description

The control diagram of the experimental setup is shown in the Fig.2. In this setup a dc motor is connected to load via a long shaft,  $\omega_r$  is the command signal,  $\omega_L$  is the output measured. The proposed second order sliding mode controller with power rate reaching law is implemented in Simulink Matlab software.

The PID sliding surface is used and surface parameters of the system  $k_p$ ,  $k_i$ ,  $k_d$ , are given as 40, 2, 0.5 respectively. It is known that mathematical model of a plant based on physical describes dynamic behaviors. However, parameters in the model of a plant cannot be obtained precisely. Therefore, as a preliminary work, the present plant should be tested to calculate nominal plant parameters,  $A_n$ ,  $B_n$  and  $C_n$ . This process is needed before the closed-loop operation is allowed. First-order plus dead-time model is one of the effective model types to approximate real plants, if the open-loop response to an applied signal does not possess any overshoot.

The approximate plant model in transfer function form, based on the process reaction curve method, can be given:

$$G(s) \cong \frac{ke^{-T_d s}}{T_s + 1} \cong \frac{k}{(1 + T_d s)(1 + T_s)} \quad (42)$$

Where  $k$ ,  $T_d$  and  $T$  are the steady-state gain, time delay and time constant respectively. It's obtained from the experimental results.

The step input signal is provided & the output is measured from the approximated model response. The parameter values are calculated from the measured plant Output to be  $k = 0.822$ ,  $T_d = 0.009s$  and  $T = 0.1418s$ . Using the approximate plant model, the nominal parameters are calculated as  $A_n = 118.1663$ ,  $B_n = 783.5762$  and  $C_n = 644.0997$ , where  $A_n = (T_d + T)/(T_d T)$ ,  $B_n = 1/(T_d T)$ ,  $C_n = k/(T_d T)$ . Thus the transfer function of the electromechanical plant can be obtained approximately as,

$$G(s) = \frac{644.0997}{s^2 + 118.1663s + 783.5762}$$

It is clear that the transfer function of the plant,  $G(s)$  is a second-order over-damped and stable nominal transfer function with two poles located on the left half part of the complex  $s$ -plane. The plant model based on the state variable form with zero initials is,

$$\dot{\omega}_L(t) = -118.1663 \omega_L(t) - 783.5763 \omega_L(t) + 644.0997 v_a(t) \quad (43)$$

### 5.2. Results

The result of this PRRL based on SOSMC is as given below. The switching control should be minimized to provide reasonable control activity in practical implementation not to hurt the actuators. For the plant, the overshoot at the output response is not desired. Conventional PID parameters are determined using the Ziegler-Nichols Quarter amplitude (1/4 decay ratio) method based on the real experimental test on plant.

The speed response and associated control efforts of the proposed SOSMC and the PRRL based SOSMC is shown in the results. The performance specification of the PRRL based SOSMC is much better than the SOSMC. Such that smaller rise time, settling time and smaller output deviation in magnitude is seen in the PRRL.

The performance specifications of the 2-SMC system are much better than that of the first-order SMC system such that the smaller rise time, settling time and the smaller output deviations in magnitude were obtained from the proposed 2-SMC system. The large overshoot (57.8%) is obtained from the PID control system that is unacceptable. The responses in smaller time range, in 0.20 s, are illustrated to check transient performance specifications. The 2-SMC input converges faster and the variations are smaller in steady-state conditions. Traditional sliding mode control input converges more slowly and the PID control input has larger variations in magnitude at the transient and steady-state conditions. Larger variations in the control effort are not desired.

The sliding surface variations  $s(t)$  during the control are illustrated in Fig. 11 for the 2-SMC system and Fig. 12 for the traditional SMC system. The steady-state variations are smaller in the 2-SMC

system with faster convergence. It can be noted that the sliding surface is not zero,  $s(t) \neq 0$  when the error signal is not zero. This means that the sliding mode is in the reaching phase up to 0.1 s and then arrives sliding phase. The surface  $s(t)$  is near to zero when the error signal is very small or motor speed is trying to reach the command speed. Theoretically the sliding function is expected to be zero at steady-state conditions, but there are always unmatched uncertainties and disturbances, frictions and nonlinearities. The system is in the sliding phase since the steady state (average) value of the sliding function is zero. The system trajectories are plotted in the phase plane,  $e(t)$  and  $\dot{e}(t)$  of the traditional SMC algorithm and  $e(t)$  and  $\dot{e}(t)$  of the 2-SMC algorithm in figs. 7 and 8, respectively.

The tracking of the SOSMC control system is shown in the fig.3 and fig.9 shows the tracking of PRRL based SOSMC. With the help of this result we can analyze the difference in the result of SOSMC and PRRL based SOSMC. To test the tracking of the system, the closed-loop system is tested at 1500 rpm of speed such that a square wave set-point change corresponding to 1500 -100 rpm is applied to the system. The figures confirm the fact that the system with the second-order sliding mode controller has a better tracking performance than the system with the traditional sliding mode controller and PID controller. Smaller speed variations,  $\pm 7$  rpm in magnitude were obtained in the 2-SMC system. The control effort of the SOSMC is shown in the fig.4 and fig.10 show the control efforts of the PRRL. The response to speed change of the dc motor is shown in the fig.5 for the SOSMC and the fig.11 shows speed change response of the PRRL. The control signal for the SOSMC is shown in the fig.6 and for the PRRL is shown in the fig.12. Now the phase plane trajectory of the SOSMC system is shown in the fig.7 and the phase plane trajectory of PRRL is shown in the fig.13. The sliding surface  $s(t)$  of the SOSMC is shown in the fig.8 and the fig.14 shows sliding surface  $s(t)$  for the PRRL.

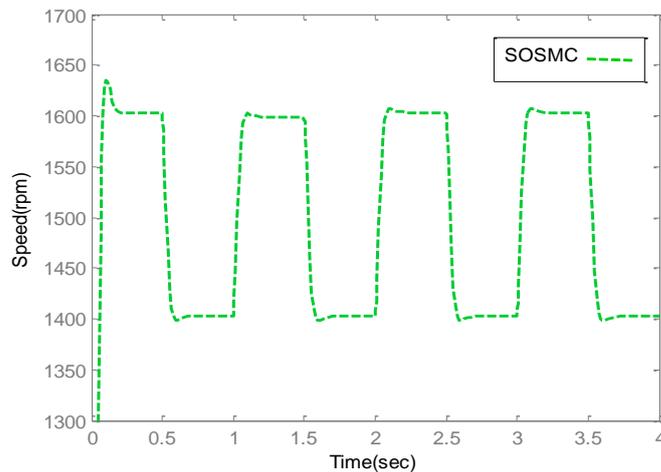


Fig.3.Tracking of 2-SMC control system

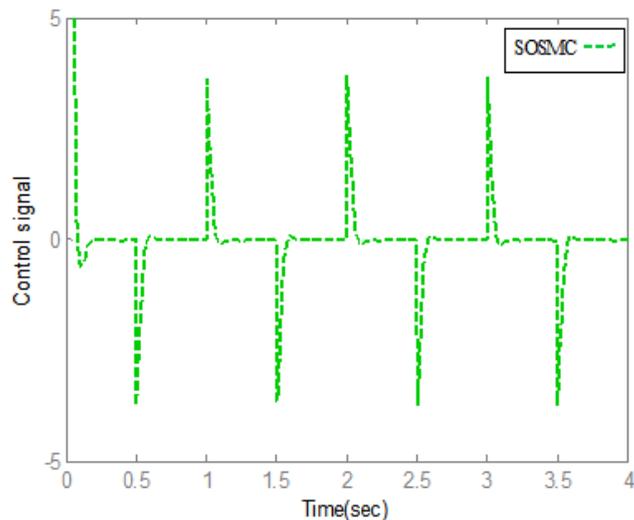


Fig.4. Control efforts of the 2-SMC system

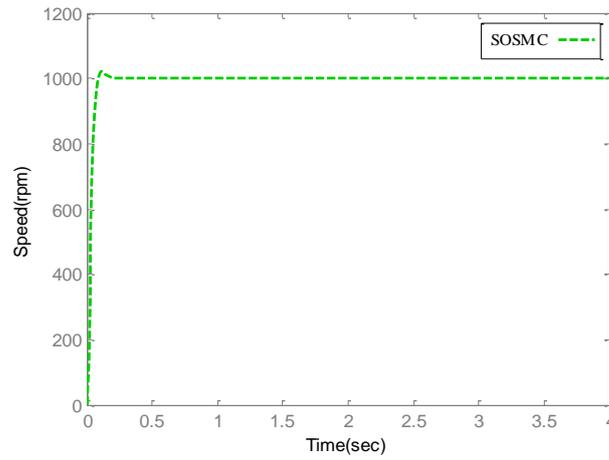


Fig.5. Response to speed change (0-1000 rpm)

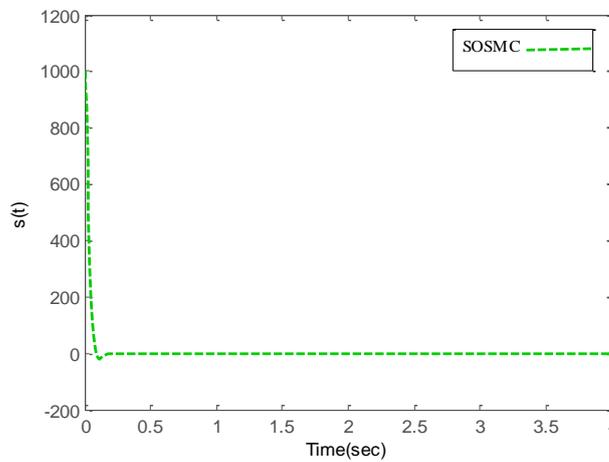


Fig.6. Control signal for the 2-SMC system

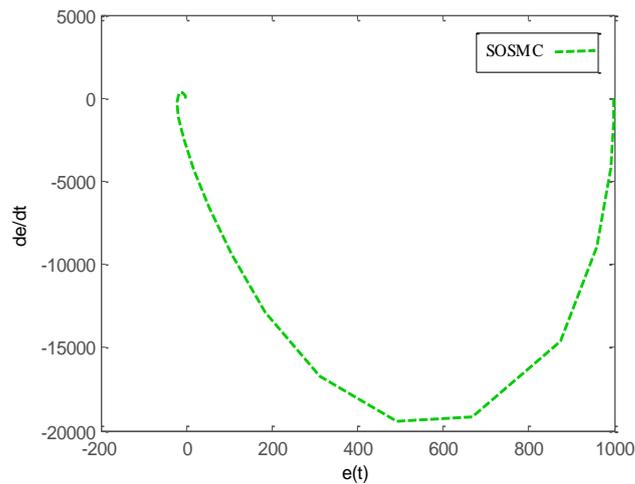


Fig.7. The phase plane trajectory of the 2-SMC system

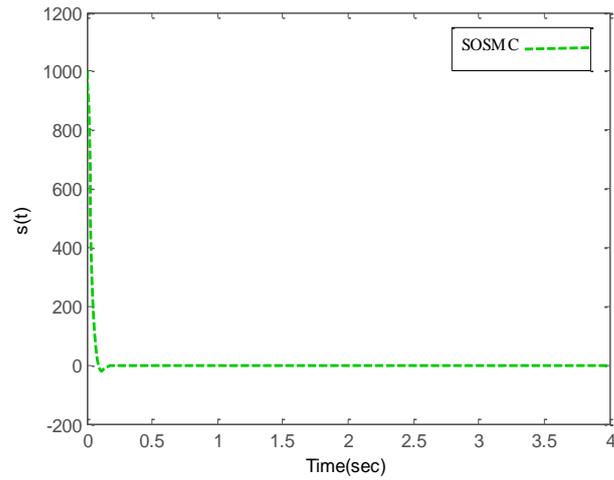


Fig.8. Sliding surface  $s(t)$  of the 2-SMC system.

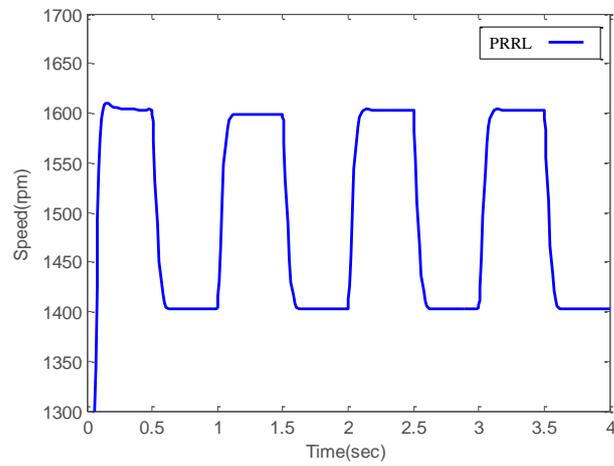


Fig.9. Tracking of PRRL control system

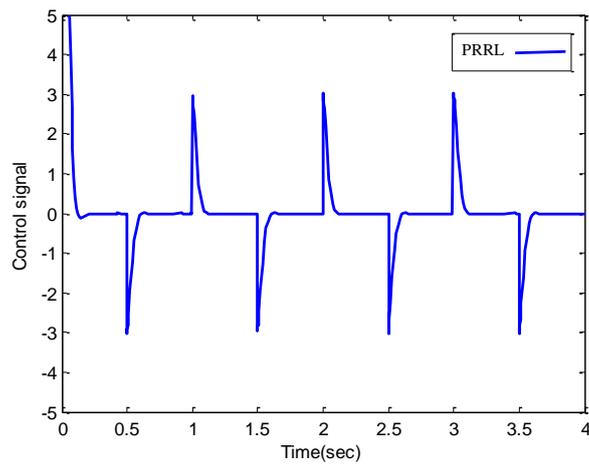


Fig.10. Control efforts of the PRRL system

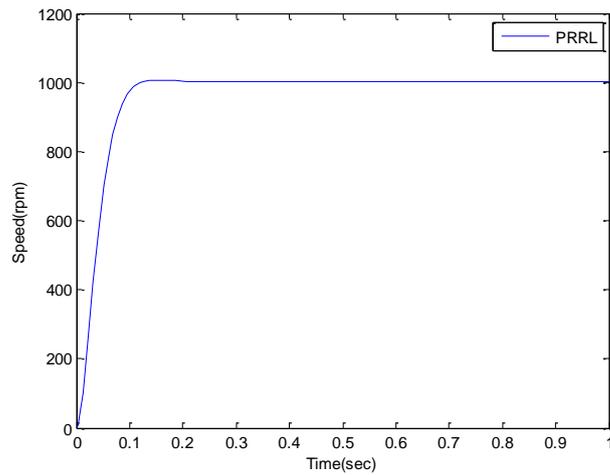


Fig.11. Response to speed change (0-1000 rpm)

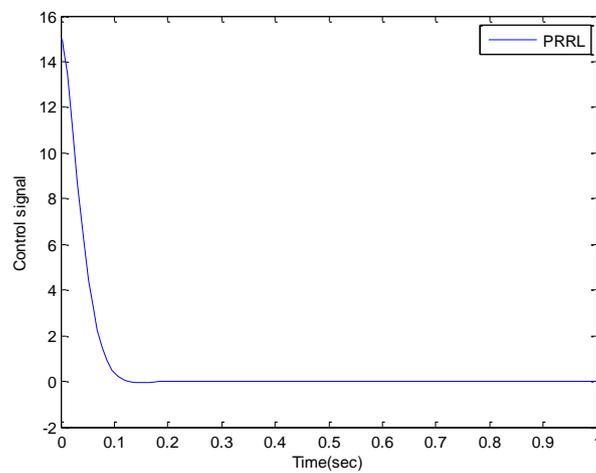


Fig.12 Control signal for the 2-SMC system

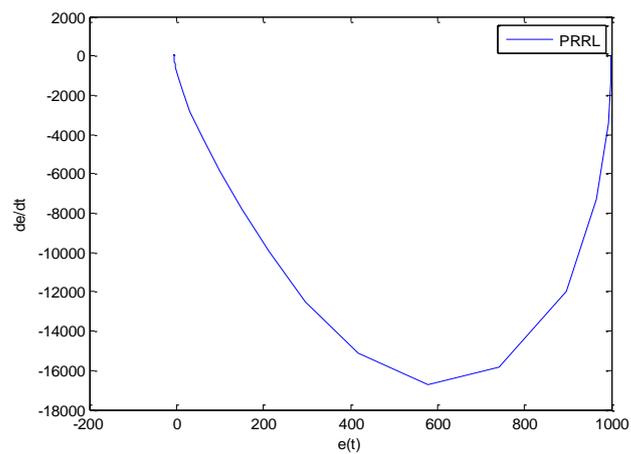


Fig.13 The phase plane trajectory of the PRRL system

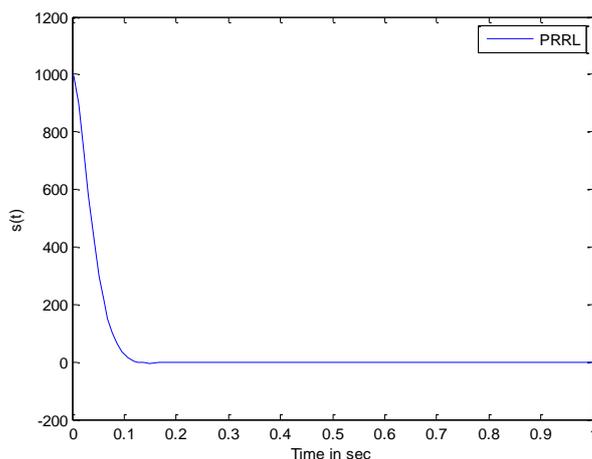


Fig.14 Sliding surface  $s(t)$  for the PRRL system

The speed responses and the associated control efforts of the proposed PRRL system, the 2-SMC system to a step command change (0\_1000 rpm speed change) are illustrated in Figs. 15. The performance specifications of the 2-SMC system are much better than that of the first-order SMC system such that the smaller rise time, settling time and the smaller output deviations in magnitude were obtained from the proposed 2-SMC system. The large overshoot (57.8%) is obtained from the PID control system & SMC that is unacceptable. Moreover, the switching control should be minimized to provide a reasonable control activity in practical implementation not to hurt actuators.

The figures below confirm the fact that the proposed PRRL control system provides better the transient and the steady-state performance specifications. These verify that the proposed PRRL system provides better performance specifications of the closed-loop system, a faster convergence of the sliding surface and better behavior of the output in case of external disturbances. At last the comparison of the speed, the control signal and the phase trajectory are given in the figures below. The Fig.15 shows the comparison for speed of SOSMC and PRRL. The Fig.16 shows the comparison for control signal of SOSMC and PRRL. The Fig.17 shows the phase trajectory comparison of SOSMC and PRRL.

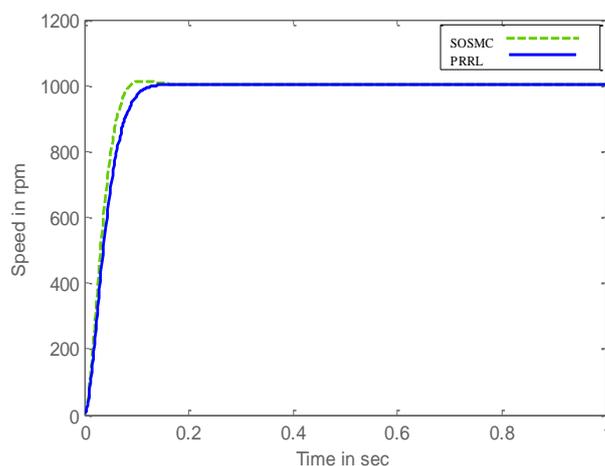


Fig.15 Combined result of SOSMC & PRRL for speed

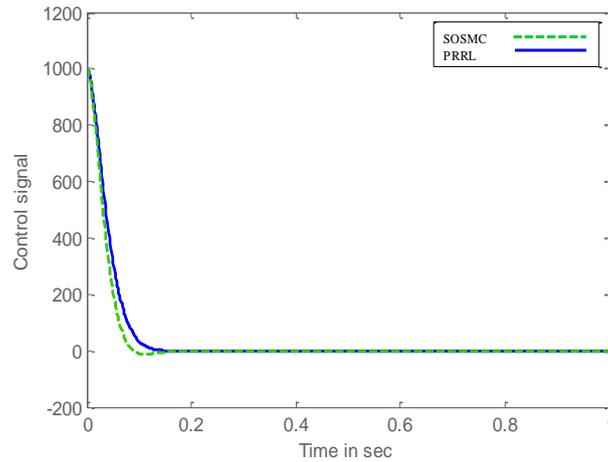


Fig.16 Combined result of SOSMC & PRRL for control signal

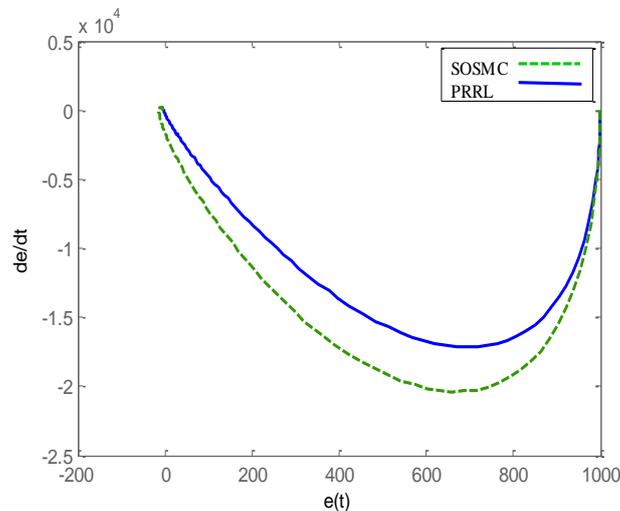


Fig.17 Combined result OF SOSMC & PRRL for phase trajectory.

### V. CONCLUSION

In the study of article, a second-order sliding mode control with power rate reaching law is proposed to improve performance of control systems. The stability of closed loop system is shown by using the specific approach. A second-order plant model is used in the present design, since many of the industrial plants can be modeled using a second-order model. Experimental results showing the effectiveness of the control method are presented for speed control of a dc motor while the nominal plant is assumed known.

To test the effectiveness of the present sliding mode controller experimental application was carried. From the above experimental results the proposed sliding mode controller is more suitable to be applied to the dc motor speed control problems due to the uncertainty handling capabilities and disturbance rejection of the 2-SMC. The hyperbolic function is used in order to smoothen the response of switching signal.

The tracking error converges to zero under the existence of parameter uncertainties and disturbances as the closed loop is in sliding mode. This system provides better performance specifications, a faster convergence of the sliding surface and better behavior of the output. So the chattering occurred is very low and response much faster and smoother than before.

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