Peristaltic Transport of Incompressible Non-Newtonian Second Order Fluid through A Flexible Cylindrical Channel

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ABSTRACT:-
In this Chapter we discuss, peristaltic transport of Incompressible Non-Newtonian second order fluid through a flexible cylindrical channel, making use of long wavelength approximation. The perturbation analysis is carried out to obtain the velocity field, the streamlines, and shear stress. The computational analysis has been carried out for drawing streamlines, velocity profiles, and the stress which are plotted for different sets of governing parameters.

KEY WORDS: - Non –Newtonian Second order fluid.

1. INTRODUCTION:-

Peristalsis is well known to physiologists to be one of the major mechanisms for fluid transport in many biological system. In particular peristaltic mechanism may be involved in urine transport form kidney to bladder through the ureter, movement of chime in the gastrointestinal tract, transport of spermatozoa in the ducts, Efferentes of the mole reproductive tracts and in the cervical canal, movement of ovum in the fall opian tubes, transport of lymph in the lymphatic vessel is and in the vasomotion of small blood vessels. In addition, peristaltic pumping occurs in many practical applications involving biomechanical systems.

The study of the mechanism of peristalsis, in both mechanical and physiological situations, has become the object of scientific research. Since the first investigation of Latham (2), Several theoretical and experimental attempts have been made to understand peristaltic action in different situations. All such investigations seem to differ in various details. Taking muscle action in the tube wall, into in a book by Liron (3). The roller pump is an engineering example working on the principle of peristalsis with the tube being compressed by rotating rollers or by a series of mechanical fingers. All the important literature upto 1978 on peristaltic transport has been documented by Rath (4). Later, Srivastava and Srivastava (5) have presented an exhaustive list of theoretical contributions to this field, classifying them according to the geometry under consideration and the parameters describing the flow. Tackabatako et.al. (6) have studied numerically the influence of finite wave length and Reynolds number on the efficiency of peristaltic pumping.

In a more recent paper, Srivastava investigated the problem of peristaltic transport of blood by assuming a single layered casson fluid, which ignores the presence of peripheral layer.

2 .FORMULATION AND SOLUTION OF THE PROBLEM:-

Consider the peristaltic transport of an incompressible second order fluid through a co-axial cylinders with inner wall being rigid and outer wall flexible. The unsteady axisymmetric flow is generated by imposing peristaltic wave on the flexible outer cylinder.

Following 'Coleman & Noll (1) the constitutive equation for an impressible second order fluid is

$$\tau_{ij} = - P_{ij} + 2\,\mu_1\, d_{ij} + 2\,\mu_2\, e_{ij} + 4\,\mu_3\, d_i^{\alpha}\, d_{\alpha j} \tag{1.1}$$

where

$$d_{ij} = ½\,(V_{i,j} + V_{j,i})$$

$$e_{ij} = ½\,(a_{i,j} + a_{j,i} + 2\,V_{,j}^{m}\,V_{m,j}\,) \tag{1.2}$$

τ_{ij} is the stress – tensor of the second order fluid.
P_{ij} is the stress tensor in its hydrostatic state.
V_i and a_i indicate components of fluid velocity and acceleration and J indicates their derivatives in J^{th} direction.
μ_1, μ_2 and μ_3 are the co-efficients of viscosity, elastico – viscosity and cross-viscosity respectively and ρ is the density of the fluid.
The governing equation of linear momentum in the tensor form is

$$\rho\left(\frac{\partial v_i}{\partial t} + V_j V_{i,J}\right) = \tau_{ij,J} \tag{1.3}$$

The equation of continuity is $V_{i,i} = 0$ (1.4)

Choosing cylindrical frame reference (r,θ, z) the equation of motion of the axisymmetry flow of an incompressible second order fluid are

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}\right)=\frac{\partial}{\partial r}\tau_{rr}+\frac{1}{r}(\tau_{rr}-\tau_{\theta\theta}) \quad (1.5)$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)=\frac{\partial}{\partial r}\tau_{rz}+\frac{1}{r}\tau_{rz} \quad (1.6)$$

The equation of continuity is

$$\frac{\partial}{\partial r}(ru)+\frac{\partial}{\partial z}(rw)=0 \quad (1.7)$$

where
$$\tau_{rr}=-P+2\mu_1\frac{\partial u}{\partial r}+2\mu_2\left[u\frac{\partial^2 u}{\partial r^2}+\frac{\partial u}{\partial z}\frac{\partial w}{\partial r}+w\frac{\partial^2 u}{\partial r\partial z}+2\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial w}{\partial r}\right)^2\right]$$

$$+\mu_3\left[4\left(\frac{\partial u}{\partial r}\right)^2+\left(\frac{\partial w}{\partial r}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2\right] \quad (1.8)$$

$$\tau_{zz}=-P+2\mu_1\frac{\partial w}{\partial z}+2\mu_2\left[u\frac{\partial^2 w}{\partial z\partial r}+\frac{\partial u}{\partial z}\frac{\partial w}{\partial r}+\left(\frac{\partial w}{\partial z}\right)^2+w\frac{\partial^2 w}{\partial z^2}+\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]$$
$$+\mu_3\left[4\left(\frac{\partial w}{\partial z}\right)^2+\left(\frac{\partial w}{\partial r}+\frac{\partial u}{\partial z}\right)^2\right] \quad (1.9)$$

$$\tau_{rz}=\mu_1\left[\frac{\partial w}{\partial r}+\frac{\partial u}{\partial z}\right]+\mu_2\left[\frac{\partial u}{\partial r}\frac{\partial w}{\partial r}+u\frac{\partial^2 w}{\partial r^2}+\frac{\partial w}{\partial r}\frac{\partial w}{\partial z}+w\frac{\partial^2 w}{\partial r\partial z}+\frac{\partial u}{\partial z}\frac{\partial u}{\partial r}\right.$$

$$\left.+u\frac{\partial^2 u}{\partial z\partial r}+\frac{\partial w}{\partial z}\frac{\partial u}{\partial z}+w\frac{\partial^2 u}{\partial z^2}\right]+\mu_3\left[2\left(\frac{\partial w}{\partial r}+\frac{\partial u}{\partial z}\right)\left(\frac{\partial w}{\partial z}+\frac{\partial u}{\partial r}\right)\right] \quad (1.10)$$

$$\tau_{\theta\theta}=-P+2\mu_1\frac{u}{r}+2\mu_2\left[\frac{u}{r}\frac{\partial u}{\partial r}+\frac{u^2}{r^2}\right]+4\mu_3\frac{u^2}{r^2} \quad (1.11)$$

$$\tau_{r\theta}=\tau_{\theta z}=0 \quad (1.12)$$

The inner rigid pipe is of radius 'a' and a wave of contraction and expansion is imposed on the outer flexible pipe r = b. The fluid motion is due to the peristaltic action of this wave.

$$r = b + \delta \sin 2\pi\left(\frac{z+ct}{\lambda}\right)$$

imposed on the outer cylinder.

where
- c is the wave speed
- λ is the wave length
- δ is amplitude of the wave
- b is the mean radius of the outer pipe

The flow becomes steady with reference to the wave frame moving along with the wave and some speed 'c'.
The relevant boundary conditions are

$$\left.\begin{array}{lll} w=0 & \text{on} & r=r_1 \\ w=0 & \text{on} & r=r_2+b\ (=f) \\ u=0 & \text{on} & r=r_1 \end{array}\right\} \quad (1.13)$$

$$2\pi\int_{r_1}^{f} wrdr = q$$

Introduce the following non-dimensional variables as

$$z^* = \frac{z+ct}{\lambda}; r^* = \frac{r}{\lambda}, \in = \frac{b}{\lambda} (\in << 1);$$

$$(w^*, u^*) = \left(\frac{w}{c}, \frac{U}{c\in}\right); p^* = \frac{P}{\rho c^2}; t^* = \frac{ct}{\lambda}; \chi = \frac{\delta}{b}$$

$$\phi = \frac{a}{b}; q^* = \frac{\bar{q}}{\pi a^2 c}; Q^* = \frac{\bar{q}}{\pi a^2 c}; \theta^* = \frac{\bar{\theta}}{\pi a^2 c}$$

and $$q^* = \theta^*\left(2\phi - \frac{\phi^2}{2}\right)$$

and making use of the long wavelength approximation $(\in << 1)$,

The expression for the stream function is

$$\psi = \psi_0 + \in \psi_1 + \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\Rightarrow \psi = C_1 + C_2 r + C_3 r \log r + C_4 r^3 + \in [C_5 + C_6 r^2 + C_7 r^2 \log r + C_8 r^4 + R [- \frac{1}{3} l_1 r - \frac{1}{8} l_2 r^2 (\log r)^2 - \frac{1}{3} l_3 r^3 + \frac{1}{16} l_4 r^4 \log r + \frac{1}{140} l_5 r^5 - \frac{1}{32} l_6 r^6 + \frac{1}{3} l_7 (3 r \log r + 4r) - \frac{1}{16} l_8 r^2 \log r + \frac{1}{9} l_9 r^3 (3 \log r - 4 + \frac{1}{32} l_{10} r^2 (4 (\log r)^2 + 21)] - \frac{R}{S_1} [- \frac{1}{16} \log r - 3 l_{12} r - \frac{1}{8} l_{13} r^2 (\log r)^2 + \frac{9}{8} l_{14} r^4 \log r - \frac{1}{64} l_{15} (2 (\log r)^2 + 5 \log r) + \frac{1}{45} l_{16} r^{-1} + \frac{3}{8} l_{17} r^2 \log r + l_{18} (\frac{(\log r)^3}{3} + \frac{5}{4} (\log r)^2 + \frac{17}{8} \log r) - \frac{1}{45} l_{19} r^{-1} (45 \log r + 84) - 2\frac{R}{S_2} [- \frac{1}{16} l_{20} \log r - 3 l_{21} r - \frac{1}{8} l_{22} r^2 (\log r)^2 + \frac{27}{8} l_{23} r^4 \log r - \frac{1}{45} l_{24} r^{-1} - \frac{5}{32} l_{25} (2(\log r)^2 + 5 \log r) + \frac{1}{16} l_{26} r^2 \log r - \frac{1}{135} l_{27} r^{-1} (45 \log r + 84)] \quad (1.14)$$

$$u = -\frac{1}{r}\frac{\partial \psi}{\partial z} = -\frac{1}{r} [C_{1z} + C_{2z} r + C_2 (2 \pi \lambda \cos 2 \pi z) + C_{3z} r \log r + C_3 (2 \pi \chi \cos 2 \pi z) \log r + C_3 (2 \pi \chi \cos 2 \pi z) + C_{4z} r^3 + 3 C_4 r^2 (2 \pi \chi \cos 2 \pi z)] \quad (1.15)$$

$$w = \frac{1}{r}\frac{\partial \psi}{\partial r} = \frac{C_2}{r} + C_3\left(\frac{\log r}{r} + \frac{1}{r}\right) + 3C_4 r + \in [2 C_6 + 2 C_7 \log r + C_7 + 4 C_8 r^2 + R\{ - \frac{1}{3} l_1 \frac{1}{r} - \frac{1}{4} l_2 ((\log r)^2 + \log r) - l_3 r + \frac{1}{4} l_4 r^2 \log r + \frac{1}{16} l_4 r^2 + \frac{1}{28} l_5 r^3 - \frac{3}{16} l_6 r^4 + \frac{1}{3} l_7 (3\frac{1}{r} \log r + \frac{7}{r}) - \frac{1}{16} l_8 (2 \log r + 1) + l_9 (r \log r - r) + \frac{1}{32} l_{10} (8 (\log r)^2 + 8 \log r + 42)\} - \frac{R}{S_1}\{ - \frac{1}{16} l_{11} \frac{1}{r^2} - 3 l_{12} \frac{1}{r} - \frac{1}{4} l_{13} ((\log r)^2 + \log r) + \frac{9}{8} l_{14} (4r^2 \log r + r^2) - \frac{1}{64} l_{15} (\frac{4}{r^2} \log r + \frac{5}{r^2}) - \frac{1}{45} l_{16} \frac{1}{r^3} + \frac{3}{8} l_{17} (2 \log r + 1) + l_{18} (\frac{(\log r)^2}{r^2} + \frac{5}{2r^2} \log r + \frac{17}{8r^2}) + l_{19} (\frac{1}{r^3} \log r + \frac{39}{45 r^3})\} - 2\frac{R}{S_2}\{[- \frac{1}{16} l_{20} \frac{1}{r^2} - 3 l_{21} \frac{1}{r} -$$

$$\frac{1}{8} l_{22} ((\log r)^2 + \log r) + \frac{27}{8} l_{23} (4 r^2 \log r + r^2) - \frac{1}{45} l_{24} \frac{1}{r^3} - \frac{5}{32} l_{25} (4 \frac{\log r}{r^2} + \frac{5}{r^2}) + \frac{1}{16} l_{26} (2 \log r + 1) + \frac{1}{3} l_{27} (\frac{\log r}{r^3} + \frac{39}{45 r^3}))\}] \qquad (1.16)$$

3. STRESS ON THE WALL:-

The stress on the flexible wall of the pipe in the non-dimensional form is

$$(\tau)_{r=s} = \frac{\left(\frac{1}{2}\left(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z}\right)(1 - s_z^2) + \left(\frac{\partial u}{\partial r} - \frac{\partial w}{\partial z}\right)s_z\right)}{(1 + s_z^2)}$$

Substituting w and u; and then τ has been evaluated for different values of the governing parameters for the first order approximation. The stresses on the boundaries are evaluated and tabulated in tables 1-3. We observe that the stress τ increases (& decreases) with an increase in S_1 (& S_2) for fixed values of other parameters while an increase in R increases the stress for variation in all other parameters.

Table - 1
Stress (□) on the flexible boundary with S_1 Variation

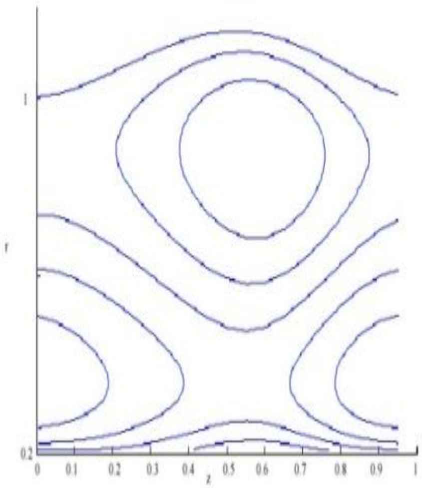
	I	II	III	IV	V	VI	VII	VIII	IX	X
S_1=0.1	21.6156	11.8361	8.57624	6.94633	6.94633	11.0482	5.8447	4.11019	3.24293	2.72258
S_1=0.2	20.1432	10.3637	7.10384	5.47392	5.47392	10.2745	5.07092	3.3364	2.46915	1.94879
S_1=0.3	19.6524	9.87286	6.61303	4.98312	4.98312	10.0165	4.81299	3.07848	2.21122	1.69086
S_1=0.4	19.407	8.62746	6.36763	4.73772	4.73772	9.88757	4.68403	2.94951	2.08225	1.5619
S_1=0.5	19.2597	9.48022	6.22039	4.59048	4.59048	9.81019	4.60665	2.87213	2.00488	1.48452

	I	II	III	IV	V	VI	VII	VIII	IX	X
□	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01
R	20	20	20	20	20	20	20	20	20	20
Z	1/.8	1/.8	1/.8	1/.8	1/.8	1/.8	1/.8	1/.8	1/.8	1/.8
S	1.01414	1.01414	1.01414	1.01414	1.01414	1.00701	1.00701	1.00701	1.00701	1.00701
S_2	0.1	0.2	0.3	0.4	0.5	0.1	0.2	0.3	0.4	0.5
□	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

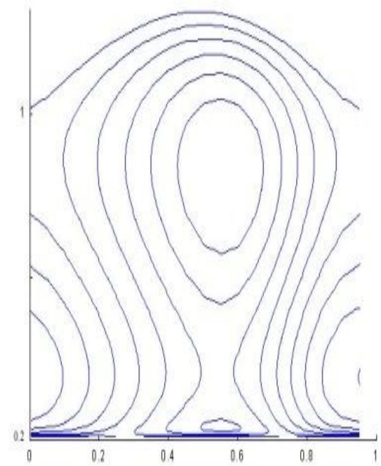
4 . DISCUSSION:-

The streamlines are plotted for different variations in the governing parameters in a unit distance along axial direction. We observe from streamline pattern for all variations in R, S_1, S_2 & χ clearly exhibit the flow separation in the vicinity of the inner cylinder. An increase in R the trapping of the fluid with formation of bolus (closed stream lines) may be observed from (Fig 1). A further increase in R, (Fig 2) give rise to an interesting pattern with fluid near by bolus circulating around the bolus and the flow on either sides being separated by this trapped fluid. When S_1 is increased this formation of bolus is weakened with only one bolus appearing near the flexible cylinder(Fig 3). However the trapping once again flowing when the amplitude of the boundary wave is increased (Fig 4).When R is sufficiently large irrespective of the values of S_1 & S_2 or similar interesting pattern of bell shaped streamlines are observed (Fig 5).

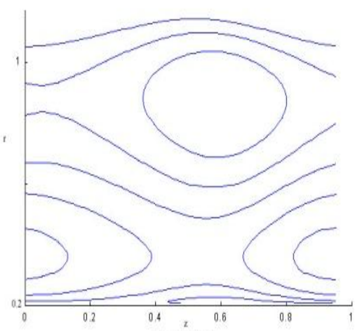
Fixing R and S_1, S_2 greater than 0.3 whether $S_1 \gtrless S_2$,the streamline pattern shows two bolus formation. But $S_1 < S_2$ these two bolus are not separated from the lower region near the rigid cylinder while for $S_1 > S_2$ this trapped fluid is separated from the lower region (Figs 6 & 7). This phenomenon of appearance of single bolus may also be observed even for higher and almost equal values of S_1 & S_2 (Figs 8 & 9). An increase in S_1, retards w in the lower region and enhances near the outer cylinder (Figs 10&11). For a fixed S_1 the behaviour of w with increase in S_2 is similar to that of variation in S_1 (Fig 12&13), although the retardation near the flexible cylinder is comparably faster. The magnitude of w at lower values of S_2 (≤ 0.5) is higher compared to its values at S_2 (> 0.5). Fixing S_1 and S_2 and other parameters an increase in R enhances w in the lower region and retards the same in the upper region adjacent to the flexible boundary (Figs 14&15).



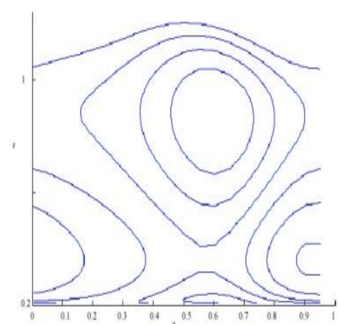
Streamlines Fig. 8
$R = 10\ S_1 = 0.5\ S_2 = 0.3\ \phi = 0.2\ \chi = 0.01$



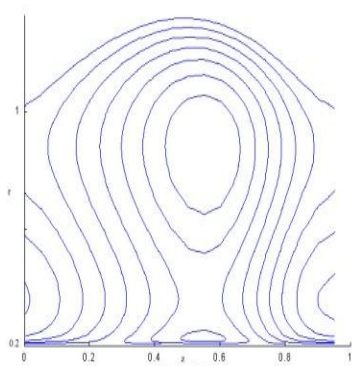
Streamlines Fig. 3
$R = 50\ S_1 = 0.1\ S_2 = 0.5\ \phi = 0.2\ \chi = 0.01$



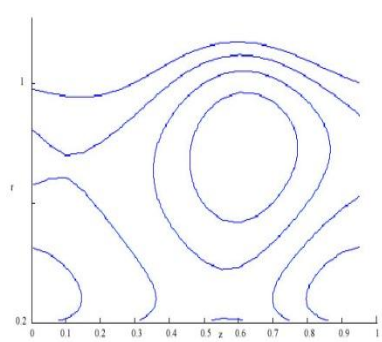
Streamlines Fig. 4
$R = 10\ S_1 = 0.2\ S_2 = 0.4\ \phi = 0.2\ \chi = 0.01$



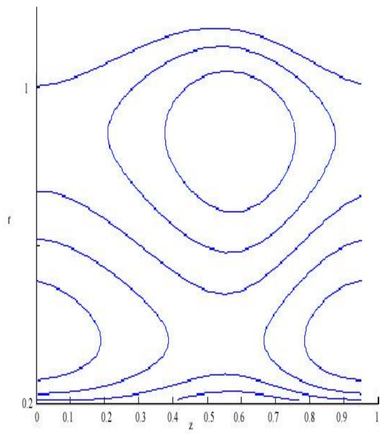
Streamlines Fig. 5
$R = 10\ S_1 = 0.2\ S_2 = 0.4\ \phi = 0.2\ \chi = 0.02$



Streamlines Fig. 6
$R = 50\ S_1 = 0.2\ S_2 = 0.4\ \phi = 0.2\ \chi = 0.01$

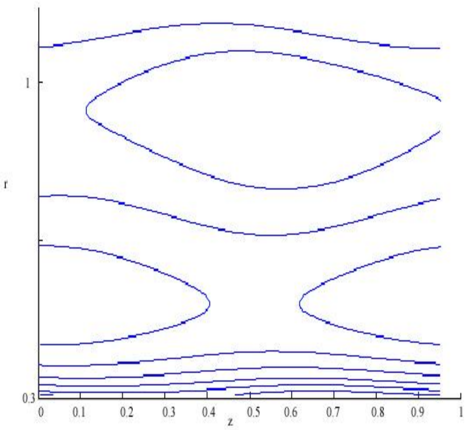


Streamlines Fig. 7
$R = 50\ S_1 = 0.2\ S_2 = 0.4\ \phi = 0.2\ \chi = 0.01$



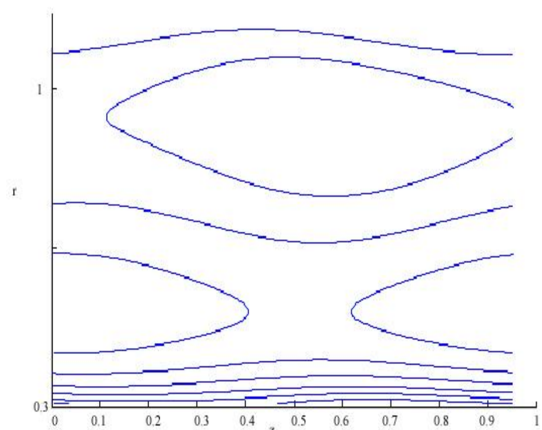
Streamlines Fig. 8

$R = 10$ $S_1 = 0.5$ $S_2 = 0.3$ $\phi = 0.2$ $\chi = 0.01$



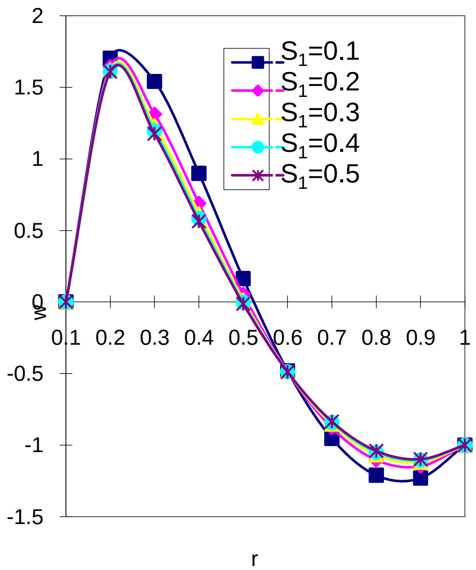
Streamlines Fig. 12

$R = 20$ $S_1 = 0.5$ $S_2 = 0.5$ $\phi = 0.3$ $\chi = 0.01$



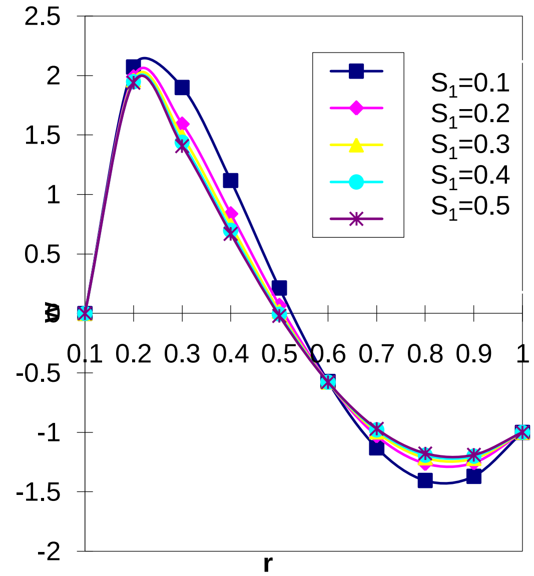
Streamlines Fig. 12

$R = 20$ $S_1 = 0.5$ $S_2 = 0.5$ $\phi = 0.3$ $\chi = 0.01$



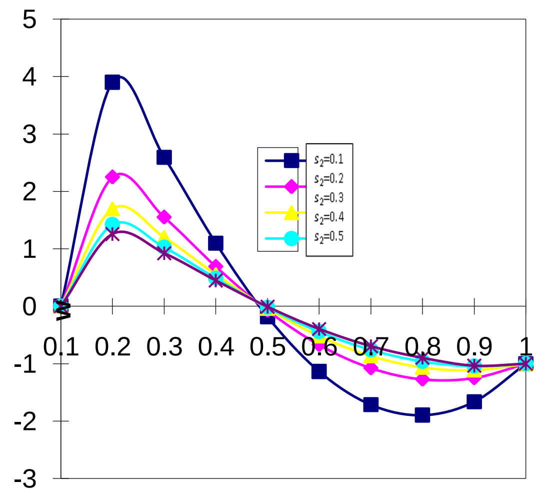
Axial Velocity profile Fig. 1

χ=0.01, S_2=0.5,Q=1 R=10,Z=0,ϕ=0.1



Axial Velocity profile Fig. 2

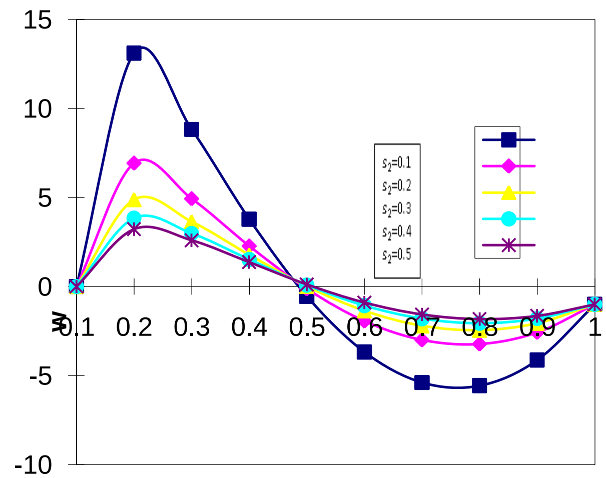
χ=0.01, S_2=0.5,Q=1 R=20,Z=1/8,ϕ=0.1



Axial Velocity profile Fig. 3

r

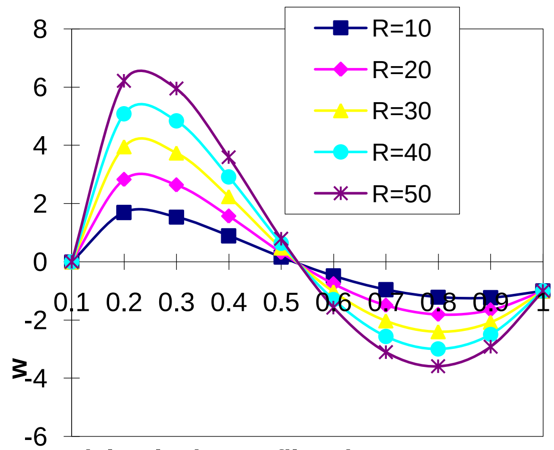
χ=0.01, S_1=0.5 R=10,Z=1/8,ϕ=0.1



Axial Velocity profile Fig.4

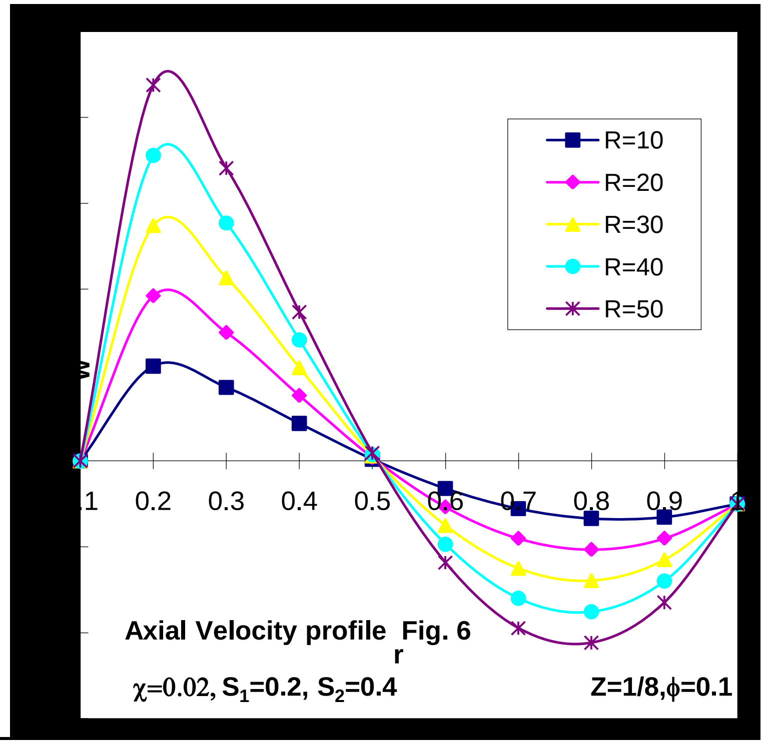
r

χ=0.02, Q=1S_1=0. R=20,Z=1/8,ϕ=0.1, E=0.01



Axial Velocity profile Fig. 5

χ=0.01, S_1=0.1, S_2=0.5 z =0,ϕ=0.1



Axial Velocity profile Fig. 6

χ=0.02, S_1=0.2, S_2=0.4 Z=1/8,ϕ=0.1

5. REFERENCES:-

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