Application of Cyclic Shells in Architecture, Machine Design, and Bionics

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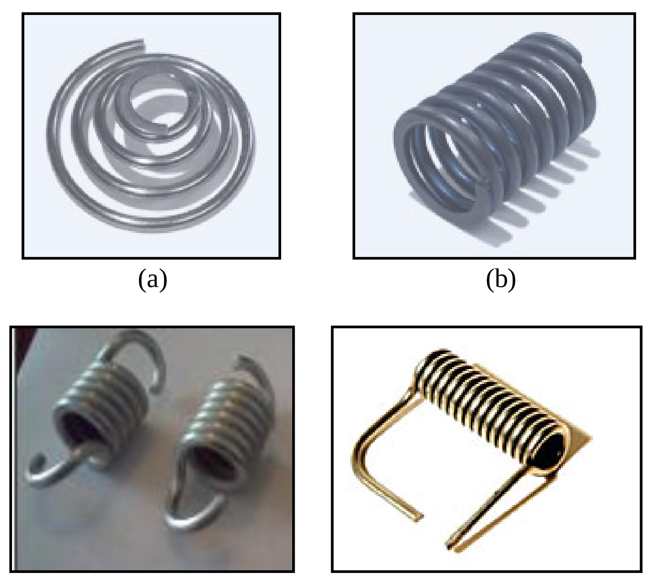
ABSTRACT
The principal achievements of science and engineering in the sphere of static and vibrational analysis of thin-walled objects, structures, and buildings in the shape of cyclic surfaces with circular generators are used for practical needs of people. These shells are useful as fragments of pipelines, spiral chambers of refrigerating units, as well as in spiral chambers of turbines in hydroelectric power stations, in high pressure units, in public and commercial buildings such as coverings of stadiums, and more. This review paper contains 18 references, and these are practically all original sources dealing with application of thin-walled cyclic shells. The additional photos can be found on the Internet.

***Keywords*: architecture, bionics, canal surfaces of Joachimsthal, cyclic surfaces with circular generators, Dupin's cyclides, epitrochoidal shell, machine design, spiral chamber, tubular shell**

1. Introduction
To date great progress has been achieved in the strength analysis of thin elastic shells [1]. Thin-walled shell structures combining lightness with considerable strength find widespread use in modern engineering and building. Many have noted the rapid progress of the practice and theory of the application of thin shells and thin-walled shell structures in the last 10 years. But shells used in constructions belong to a limited class of surfaces. An accurate tendency in world practice is the application of spatial structures of arbitrary form giving expressive architectural images and solving functional problems. Today we have a new generation of young architects and engineers who have shown interest in designing wide-span spatial coverings. This process amplifies occurrence of new materials, such as fibrous reinforced polymeric composites, which can be used in covers due to its ability to curve. One family of forming curves in cyclic surfaces is represented by circles of constant or variable radius that considerably reduces the cost, and simplifies the process of manufacturing thin shells in the form of these surfaces without a corresponding decrease in the operational possibilities.

2. A circular helical surface with the forming circle lying in the adjoining plane of a helical line of the centers of circles
The defining vector of a forming circle of this surface is directed along a binomial of a helical line of the centers of generatrix circles. This surface is used in the theory of screw surfaces in designing cutting tools [2].

3. Tubular surfaces
Tubular surfaces are used widely in building and machine design. One may see tubular helical surfaces in contours of cylindrical helical springs which can be classified due to given loading (Fig. 1) or design. These are conic helical springs of compression (Fig. 1a), cylindrical helical springs of compression (Fig. 1b), extension (Fig. 1c), torsion (Fig. 1d), and flat spiral springs take into account the perceived loading of springs. Due to the cross-section, cylindrical helical springs are solid [3] and hollow [4].



(a) (b) (c) (d)

Figure 1. Classification of springs depending on a kind of perceived loading

The tubular spiral surface with a flat line center is used as a spiral heating element in electrical tiles (Fig. 2), for installation in a fan heater (Fig. 3), in drying apparatus in the food industry, or for heating of preparations in the industry.

Figure 2. A spiral heating element in electrotile Saturn ST-EC 1165.1

Figure 3. A tubular heating element [www.scat-technology.ru]

The tubular helical surface is undertaken as a basis of designing of coils [5] of diverse function (Figs. 4-6). In particular, the cartridge of a heating element (Fig. 5) is used in the combined catching heating appliance of water for indirect heating. The heating appliance of series MORA E NTR is intended for preparation of hot water for technical and economic purposes using warmth of water heated with the help of an outside source of energy, for example, with the help of gas reactor. In addition they have an electric heating element. It allows the possibility to use a water heater as an electric boiler, for example, in the summer, when there is no necessity for use of a heating element. It is especially effective in a system with a diesel copper.

Figure 4. A heating element of the hair dryer for Lukey 852D+fan, 701,702 [www.gsm-komplekt.com.ua]

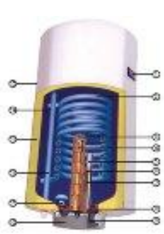


Figure 5. A cartridge of a heating element heating element [www.moramoravia.ru]

Figure 6. A coil for cooling of a liquid [clear moon.livejournal.com]

For modeling spiral tan such as Microcoil (MC) (Fig. 7), it is possible to use a tubular helical surface with a centerline of a variable step (Fig. 8). Tubular surfaces with a flat or spatial centerline are the basis for any design of pipelines [6] of constant radii. The connections of pipelines at right angles are usually taken in the form of a circular tube segment (Fig. 9a, 9b), or torus is approximated by fragments of several cylindrical surfaces (Fig. 9c, 9d). In their book, St. Polanski and L. Pianowski [7] have offered various approximations of tubular surfaces by fragments of cylindrical surfaces.

Figure 7. Spiral tan Microcoil (MC) [www.mirnagreva.ru]

Figure 8. A tubular helical surface with a centerline of variable step

The most widespread constructive elements in power installations are pipelines with a complex outline of an axial centerline. They are applied in units of a high pressure, circular pumps, and turbine units [8].

Tubular circular helicoids and spiral surfaces are widely used in designing descents in water attractions (Fig. 10). A tubular surface on the sphere or tubular loxodrome (Fig. 11) may be recommended as a form of a trigger trench in water attractions. Sometimes the tubular surface with any centerline is used as an exotic interior of water pool (Fig. 12) or even the whole building (Fig. 13).

(a) A tube in plastic [www.pcvexpro.ru]

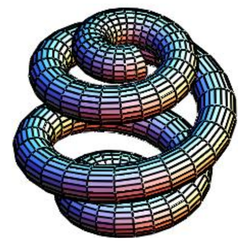
(b) Gas tube [www.caspenergy.com]

(c) Connecting details of pipelines

(d) Details for connection of pipelines of identical and different diameter

Figure 9. Pipelines of diverse function

Figure 10. Water attractions



Figire 11. A tubular loxodrome

Figure 12. The Complex “Yalta,” Russia [«Всеслав-Донбасс»]

Figure 13. Imaginations of the architect

4. Dupin’s cyclides

Dupin’s cyclides are two-canal surfaces [11]. Both families of flat lines of the principle curvatures lying in two pencils of planes are the circles. It is necessary to notice that geometrical researches on Dupin’s cyclides enjoy wide popularity of geometers. However, engineers-designers and architects, until today, have not paid enough attention to these surfaces. Only two offers on the application of Dupin’s cyclides in real constructions are published in the technical literature [12, 13]. For example, in a work [13], the problem of covering a building with a double shell was stated, but the lines of contours should be by lines of curvatures, and both families of lines of curvatures of the shell surface should be by families of circles. All requirements of the problem can be executed if a compartment of Dupin's cyclide is used as a geometrical image of the shell (Fig. 14).

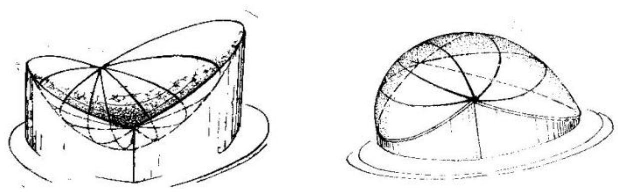


Figure 14. Offers on use of Duplin's cyclides

5. Canal surfaces of Joachimsthal

Joachimsthal's surface is a surface with a family of plane lines of curvature lying in the planes of the pencil [14]. Canal surface is called a cyclic surface with a family of circles *R* (*u*) being lines of curvature of a surface. If circles of canal surfaces lay in the planes of pencil, the surface is canal surface of Joachimsthal. A centerline of canal surfaces of Joachimsthal is a flat curve *r*(*u*), therefore canal surfaces of Joachimsthal are included into a group of cyclic surfaces with circles in planes of a pencil with a flat centerline. Post-graduate student of a chair of strength of materials of the Peoples Friendship University of Russia, N.J. Abbushi [15] has constructed gyps models of canal surfaces of Joachimsthal explaining three methods of their formation (Fig. 15).

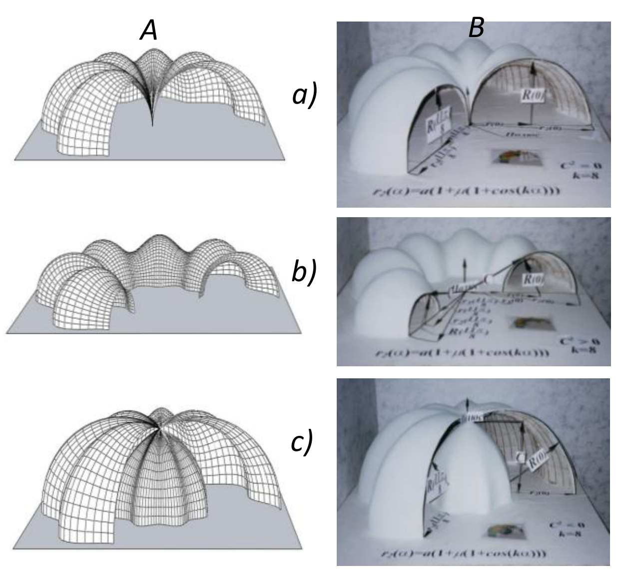


Figure 15. Mathematical and gyps models of canal surfaces of Joachimsthal [15]

6. Epitrochoidal surface

The *M* point located on a plane of a circle with the *a* radius, which rolls without sliding on other motionless circles with *b* radius, forms an epitrochoidal line. The planes of these two circles constitute a constant corner γ. The distance from a point of *M* to the center of a mobile circle is equal to μa ($\mu = 1$, or $\mu < 1$, or $\mu > 1$). Changing parameter γ from 0 to 2π, it is possible to receive a family of epitrochoidal curves which will form epitrochoidal surfaces (Fig. 16) [16].

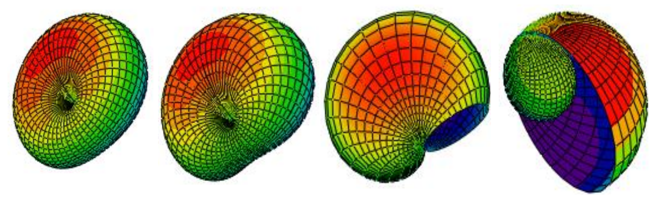


Figure 16

Now epitrochoidal surfaces draw attention to themselves only from the point of view of geometrical researches. Recommendations in the scientific and technical literature on application of these surfaces in real designs were not discovered.

7. Cyclic surfaces with circles in planes of pencil

The cyclic surface with circles in planes of the pencil is formed by circles of the constant or variable radius in which the center moves along the given centerlines of a surface, and the generatrix circles lie in planes of the pencil [11].

The cyclic surface with generatrix circles of constant or variable radius in planes of pencil and with a flat centerline in the form of a logarithmic spiral well approximates the form of spirally-curtailed ammonite bowls (Fig. 17) and bowls of the mollusks existing now [17].

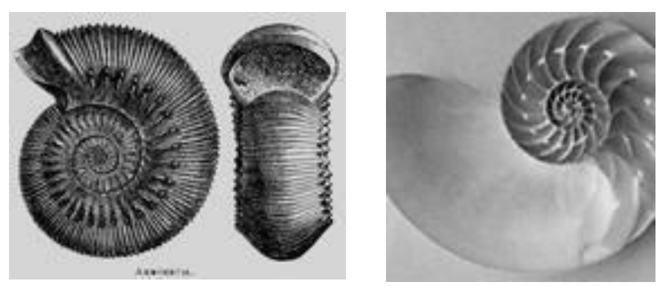


Figure 17. Ammonite bowls

The spiral chamber of the waterwheel represents the cyclic surface, the cross-sections of which are circles of variable radii. This chamber winds around a cylindrical part of a rotor of the waterwheel and serves as the water admission. The diameter of the spiral chamber in each section is defined from the conditions of the best entrance of the expense of water. The function of change of the radius of the chamber has a difficult structure; however, with sufficient accuracy it may be defined by the following formula:

$$\overline{R} = \overline{R}_0 (1 + k\beta) ,$$

where $\overline{R}_0$ is the radius of the entrance aperture of the chamber, and *k* is the factor dependent on the expense of water. Some variants of spiral chambers of waterwheels are presented in Figs. 18 and 19. The spiral chamber of the waterwheel provides uniform water inflow on all perimeters of the directing apparatus, i.e. an axisymmetric operating mode of all directing blades. The section of the spiral chamber of the waterwheel is in regular intervals narrowed on a stream course. On HYDROELECTRIC POWER STATIONS with pressures exceeding 50–60 m, steel spiral chambers of round sections (Fig. 18), winding the stator almost completely, are applied.

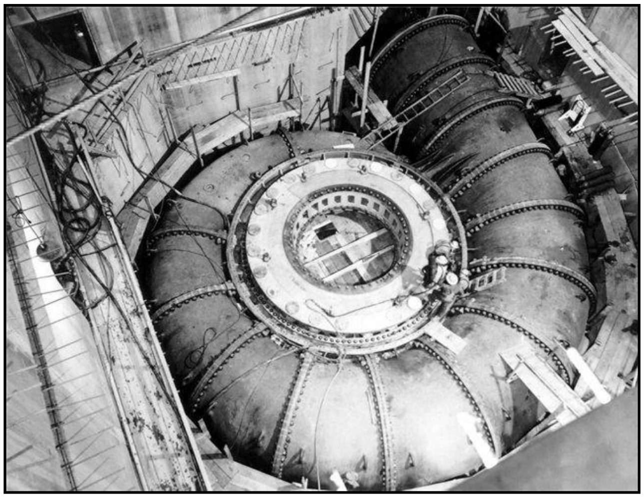


Figure 18. Process of assemblage of the spiral chamber [www.tvscience.ru]

Figure 19. Process of assemblage of the spiral chamber

On HYDROELECTRIC POWER STATIONS with the lesser pressure, the spiral chambers are made of reinforced concrete. Steel chambers of high-powered HYDROELECTRIC POWER STATIONS are difficult to make because of great thickness of walls. Therefore reinforced concrete chambers find more and more wide application.

Let's consider one more surface from a subclass «Cyclic surfaces with circles in planes of pencil», namely a circular helical surface with generatrix circles in planes of pencil. This surface is used mainly in a theory of helical surfaces for design of cutting tools [2], but it can find application in architecture as well.

Due to the names of following three cyclic surfaces shown in Figs. 20–22, one may understand that they describe the form of sea or river cockle-shells.

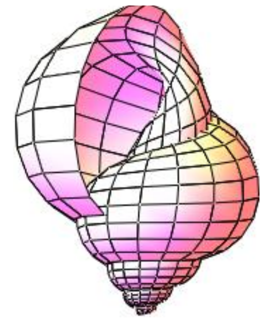


Figure 20. A spiral surface "cockle-shell without the peak"

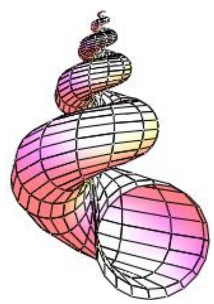


Figure 21. A spiral-type surface "cockle-shell with the peak"

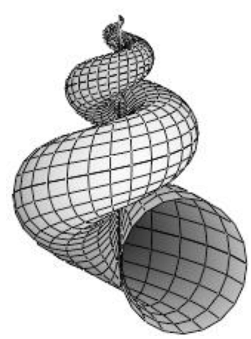


Figure 22. Seashell

8. Normal cyclic surface

A normal cyclic surface is formed by the movement of a circle of variable or constant radius along an arbitrary director curve, but the circle generatrix must lie in the normal plane of the given director curve [11].

Two types of normal cyclic surfaces with an elliptic centerline and with a generatrix circle of variable radius (Fig. 23) and the connecting channel for two cylindrical surfaces with parallel axes (Fig. 9c; Fig. 24 [18]) are used widely as connecting parts of pipelines.

The widely-known one-sided surface «Klein's Bottle» can also be constructed in the form of a normal cyclic surface with circle generatrixes of variable radius (Figure 25). Cyclic normal surfaces one may see in the forms of wind instruments (Fig. 26).

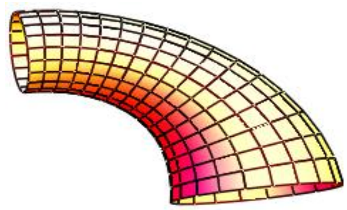


Figure 23. Norrmal cyclic surface with an elliptic line of the centers and with a generating circle of variable radius (type 1)

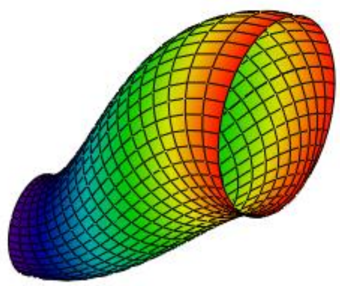


Figure 24. The connecting channel for two cylindrical surfaces with parallel axes

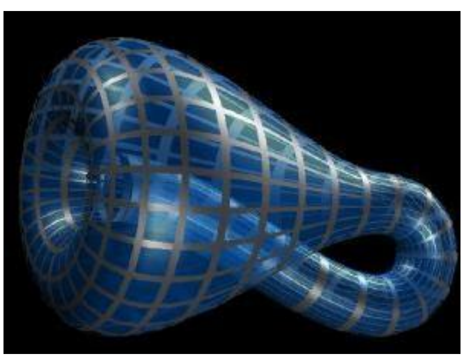


Figure 25. Klein bottle

Figure 26. Wind instrument [ru.wikipedia.org/wiki/wind]

8. Circular translational surfaces

Directing and generating curves of circular translational surface lie in mutually perpendicular planes. Surfaces of direct translation can be defined by the equation:

$$z = z(x,y) = z_1(x) + z_2(y)$$

Circular translational surfaces were taken as a basis of many erected wide-span thin-walled shells; two of them are presented in Fig. 27. The covers represented in Fig. 27 are also called a «Bohemian dome».

(a) Shopping center in Chelyabinsk (102 x 102 m), Russia

(b) The market in Cheremushki, Moscow, Russia

Figure 27. Circular transition shells

9. A surface with a plane of parallelism

The shed cover erected in Bulgaria (Fig. 28) consisting of shells of conoidal type with a thickness of 3 cm, a span of 18 m, and the distance between trusses of 7 m may be considered to be a subclass of cyclic surfaces with a plane of parallelism. The form of the cover differs from the form of a conoid because their generatrixes are not straight lines but flat circular curves. This form provides more favorable static conditions, and allows a smaller thickness for the cover. The cover is gathered from precast reinforced concrete fragments made in advance and its complex shape does not result in a price increase. Each shell was gathered from four elements with weight of 4.5 tons considering possibilities of elevating means.

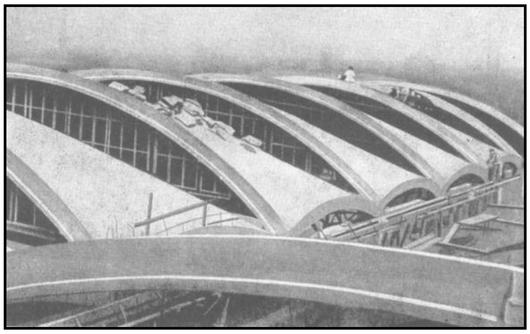


Figure 28. The shed cover

10. Surface of helical pole

Sometimes the surface of a column was taken in the form of a pole with a helical line of centers of circular generatrices of constant radius. In Fig. 29, the surface of the helical column has two generating circles, while in Fig. 30, the surface of the helical column has four generating circles.

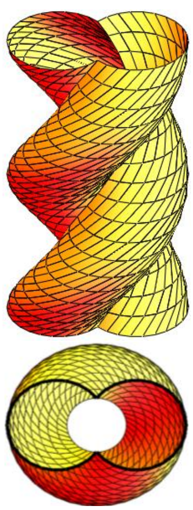


Figure 29. The surface of the helical pole with two generating circles

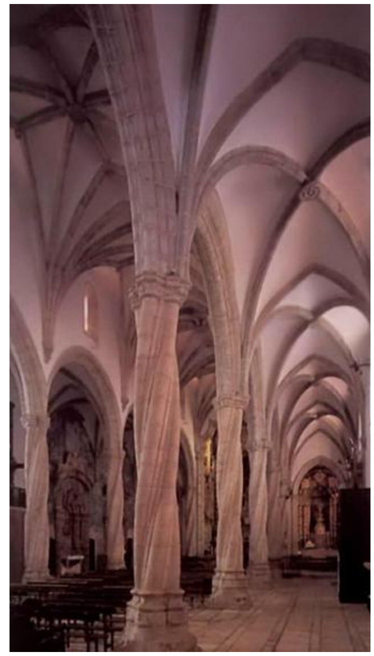


Figure 30. Olivenza, Spain, Church, the 16th century

11. Conclusion

The materials contained in monographs, dissertational works, scientific papers, in proceedings of conferences, and listed in the review include almost all spheres of the application of cyclic shells known in the world today.

As to the engineering possibilities of the application of cyclic shells, they have found wide application in curvilinear pipes of constant radius, basically in the form of torus, in spiral chambers of waterwheels, and in the circular translation shells used as spatial coverings of buildings. The helical cyclic surfaces help to describe the forms of some cockle-shells and to work out mathematic models of their shapes.

Cyclic surfaces with a straight center line, or shells of revolution, are widely used in architecture. Sometimes, architects especially underline the presence of generatrix circles in building design. As to cyclic structures of the general non-degenerate type, only single examples of their application are known. The author hopes that the literature resulting from the review and the analysis of tendencies of development of the application of cyclic shells will help architects, civil engineers, and mechanical engineers expand and diversify projected spatial structures using cyclic surfaces, and will help post-graduate students choose themes of scientific research.

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