ABSTRACT
By introducing a new fuzzy evaluation sheet, we believe it portrays additional information on students’ performances in answering each question in a test or examination compared to conventional marking method. Moreover, this approach can be used to compare students’ performances which have the same final linguistic terms by looking into each question and each criterion. This paper presents a new method for students’ learning achievement evaluation by automatically generating the weights of the attributes “accuracy rate”, “time rate”, “difficulty”, “complexity”, “answer-cost” and “importance”, respectively, with the fuzzy reasoning capability. The proposed method normalizes the adjustment quantity to insure the fairness of the adjustment in each inference result. It can provide us much fairer and more reasonable inference results for students’ learning achievement evaluation. It can evaluate students’ answer scripts in a more flexible and more intelligent manner.

Keywords: Degrees Of Confidence, Index of Optimism (İO), Interval-valued fuzzy sets, Fuzzy Evaluation Marksheet, Fuzzy reasoning, Fuzzy rules, Grade Membership Functions, Result Transformer, Satisfaction Levels, Student Answer-script.

I. INTRODUCTION
Evaluation of students’ answer scripts normally done by the two popular existing systems: grading system and traditional marking system. Types of questions are assumed to be such that answers are of subjective types only. In traditional system of evaluation, “marking” i.e. awarding of marks is done, whereas in the Grading system of evaluation, “grading” i.e. awarding of grades is done. Evaluation of students’ learning achievement is the process of determining the performance levels of individual students in relation to educational objectives.

I.1 Theoretical Issues
At present, students’ answer scripts have subjective evaluation by evaluator in many universities/ institutions. In current scenario, the evaluation done by evaluators has some limitations as follows:

1. Single vs. Multiple evaluator
Single evaluator evaluates all the answer scripts as per his own judgement so there is no unfairness to those students. But if some papers are evaluated by one evaluator while others are by different evaluator, then depending on the nature of assessment (i.e. strict, normal, lenient) of evaluator there is a chance that some students get good marks as compared to others because of diversity in subjective assessment level of different evaluators.

2. Level of satisfaction
In subjective evaluation there is no flexibility to consider the evaluator’s different levels of satisfaction so as to accurately assess the answer scripts.

3. Degree of confidence
In current situation, evaluator’s degree of confidence in awarding a particular grade/mark is not considered which reflects in less accurate evaluation.

4. Lack of details in result
Student is provided only with the final marks of subject and not the detailed additional information in assessment like how marks are awarded based on accuracy, coverage, difficulty, complexity, etc. in answering each question in a test/examination.

5. Assurance for similar scoring criteria
Assessing a particular student’s answer script by an evaluator can be problematic because this makes it hard to ensure that the scoring criteria are applied to one student is also applied in the same way to other students.

6. Increasing number of evaluators and answer scripts
As the number of evaluators and the number of papers to be evaluated increases, there is less and less likelihood of applying the scoring criteria the same way every time.

7. Personal factors
Personal factors like fatigue and myriad may affect consistency in the evaluation process.

II. RELATED WORK
Until now, some methods have been presented for dealing with students’ evaluation:

In [1], Bai and Chen presented a method for automatically constructing grade membership functions of lenient-type grades, strict-type grades, and normal-type grades given by teachers. Based on the constructed grade membership functions, system can perform fuzzy reasoning to infer the scores of students. It provides a useful way to evaluate students’ answer scripts in a smarter and fairer manner.

In [2], Biswas presented Fuzzy Evaluation Method (fem) and a Generalized Fuzzy Evaluation Method (gfem) for applying fuzzy sets in students’ answer scripts evaluation is
developed. These methods have the drawbacks. (1) Because a matching function is used to measure the degrees of similarity between the standard fuzzy sets and the fuzzy marks of the questions, it will take a large amount of time to perform the matching operations. (2) Two different fuzzy marks may be translated into the same awarded grade and it is unfair for students’ evaluation.

Chen and Lee [3] presented two methods for applying fuzzy sets in students’ answerscripts evaluation to overcome these drawbacks. The methods presented in [3] are much faster in execution and fairer in the task of student evaluation. The method has the drawback. It cannot deal with the situation where the evaluating values are represented by fuzzy numbers associated with degrees of confidence between zero and one and they do not consider the degree of optimism of the evaluator in evaluating students’ answerscripts. If these factors are considered, there is room for flexibility.

In [4], Chen and Wang presented new methods for evaluating students’ answerscripts based on interval-valued fuzzy grade sheets. Marks awarded to the answers in the students’ answerscripts are represented by interval-valued fuzzy sets. The degree of similarity between an interval-valued fuzzy mark and standard interval-valued fuzzy set is calculated by a similarity function. An index of optimism determined by the evaluator is used to indicate the degree of optimism of the evaluator.

In [5] Hui-Yu Wang and Shyi-Ming Chen presents a new approach for evaluating students’ answerscripts using fuzzy numbers associated with degrees of confidence of the evaluator. The satisfaction levels awarded to the questions of students’ answerscripts are represented by fuzzy numbers associated with degrees of confidence between zero and one.

In [6], Saleh and Kim proposed a method for evaluation of students’ answerscripts using fuzzy system. This method applies a fuzzification, fuzzy inference, and defuzzification considering the difficulty, the importance and the complexity of questions. This method has an advantage. The transparency, objectivity, and easy implementation provide a useful way to automatically evaluate students’ achievement in a more reasonable and fairer manner. It persuades students who are skeptical and not satisfied with the evaluation results. This system has a drawback. It requires the domain experts to decide the values of complexity and importance. Experts’ decision is the challenging task.

In [7], Li and Chen proposed the method for answerscripts’ evaluation in which the weights of the attributes accuracy rate, time rate, difficulty, and importance are generated automatically with the fuzzy reasoning capability. This method normalizes the quantity to insure the fairness of adjustment in each inference results for student’s learning achievement evaluation.

Nolan has discussed the design and development of an expert system fuzzy classification scoring system [8]. The main function of the expert fuzzy classification scoring system is to support teachers in the evaluation by providing them with uniform framework for generating ratings based on the consistent application of scoring rubrics. An experiment demonstrated that teachers using expert fuzzy classification scoring system can make assessments in less time and with a level of accuracy.

In [9], Wang and Chen presented a new method for evaluation using vague values. The vague mark awarded to each question can be regarded as a vague set, where each element in the universe of discourse belonging to the vague set is represented by the vague values. Methods presented in [2] have used the fuzzy sets. In a fuzzy set, the grade of membership of an element is represented by a real value between zero and one. Single value between zero and one tells nothing about the accuracy of a number. So if the number is presented as a vague set, then there is a room for more flexibility.

In [10], Bardul proposed a method to evaluate the students’ performances as individual and as a group. The main objective of this study is to improve the existing fuzzy approach in assessing students’ performance. This study focuses on two types of assessments namely students’ answer scripts assessment and students’ group assessment. In the students’ answer scripts assessment the trapezoidal fuzzy number is used to represent the standard satisfaction level for the grading scales and the students’ fuzzy scores. The center points of both standard satisfaction levels and the fuzzy score is calculated using the center of gravity method. In the students’ group assessment instructors as well as students are involved in selecting and determining the assessment criteria. The pair-wise comparison technique based on fuzzy scales is used to find the relative strength between each criterion. The weights of selected criteria are represented by the normalized fuzzy eigenvectors. The fuzzy relation composition method is employed in order to combine the instructor and students’ evaluation, which finally gives the overall students’ group performance. Both the answer scripts assessment and group assessment processes can be easily performed with the aid of fuzzy assessment sheet. This integrated fuzzy approach provides additional information on students’ performance and can be used as an option for instructors to assess students’ performance.

Wang and Chen presented new methods for evaluating the answerscripts of students [11], where the evaluating values are represented by fuzzy numbers, and an optimism index λ determined by the evaluator is used to indicate the degree of optimism of the evaluator for evaluating the answerscripts of students, where the value of λ is between zero and one. The universe of discourse is formed by a set of satisfaction levels. The fuzzy mark awarded to the answer of each question of the answerscript of a student is represented by a type-2 fuzzy set. The proposed methods can overcome the drawbacks of the methods presented in [2] and [3]. It can evaluate the answerscripts of students in a more flexible and more intelligent manner.
In [12], the analysis of students’ evaluation, especially in the case of the students’ answerscripts under the evaluation grade of linguistic data is discussed. Chih-Hsun Hsieh transfer mostly linguistic data, subjective message into triangular fuzzy numbers, and use the function principle instead of the extension principle to calculate the students’ score. The principle does not change the type of membership function and will reduce the trouble and tediousness of operations. In addition, the degree of similarity between two fuzzy numbers is defined with the utility value of fuzzy number to transfer the students’ score into letter-grade score.

In [13], Bardul and Mohamad presented a method that uses normalized values to represent some of extreme cases of satisfaction levels and utilize fuzzy numbers to generate more consistent fuzzy marks. After the instructors mark the scripts by using the traditional method, the satisfaction levels of each question will be identified by using fuzzy numbers. Then, the degree of satisfactions of each question will be calculated. The fuzzy marks will be generated to produce the total score. Finally, the fuzzy grade will be obtained. The result that based on the fuzzy sets approach could provide more and better information which portrays the student performance of each question. Thus, this paper attempts to overcome the drawbacks identified in technique carried out by Chen and Lee [3].

The drawbacks and the alternative ways suggested to overcome them are explained as follows:

1. Chen and Lee [3] have a fixed value for each satisfaction level. However, in [12], researchers used normalized values to represent the degree of satisfaction for lower extreme cases (i.e. grade E) and for upper extreme cases (i.e. grade A) while keeping the degree of satisfaction of A-, B+, B, B-, C+, C, C-, D+, and D.

2. Chen and Lee [3] did not utilize the advantage of using fuzzy numbers in their evaluation method. In [12], fuzzy numbers are used as this technique can provide equations for each grade and finally produce a consistent result.

3. Chen and Lee [3] also mentioned that the higher the degree of satisfaction the more the fuzzy mark satisfies the instructor’s opinion. The drawback is that the fuzzy marks given are merely based on the instructor’s opinion. In [12], the authors used fuzzy numbers to generate a more consistent fuzzy mark.

III. Workflow of Proposed Fuzzy Evaluation System

In the proposed project, a new method for evaluating students’ answerscripts is presented where evaluating marks awarded to the questions in answerscripts are represented by vague values. An index of optimism $\lambda$ is determined using common answerscript assessed by each evaluator. This indicates the degree of optimism of the evaluator, where $\lambda \in [0, 1]$. The evaluation satisfaction levels awarded to each questions are obtained by using the expected truth values of each vague satisfaction values. Degree of confidence associated with satisfaction level is used to calculate $\alpha$-cut which in turn is used to calculate the total mark of each student. A method is presented to automatically construct the grade membership functions of lenient-type grades, strict-type grades and normal-type grades, given by teachers, respectively. Based on the constructed grade membership functions, the system performs fuzzy reasoning to infer the scores of students. Automatic generation of weights of attributes “accuracy rate”, “coverage rate”, “difficulty”, “complexity”, “answer-cost” and “importance” is done with the fuzzy reasoning capability. It is used to normalize the adjustment quantity with which the result is adjusted to insure fairness. It provides a useful way to evaluate students' answerscripts in a smarter and intelligent manner.

\[ \lambda \in [0, 1] \]

A common answerscript is awarded by all evaluators

<table>
<thead>
<tr>
<th>Module 1</th>
<th>Index of Optimism Calculator</th>
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<tr>
<td>Module 2</td>
<td>Fuzzy Evaluation Marksheet</td>
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<td>Module 3</td>
<td>Grade Membership Function Constructor (GMF)</td>
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<tr>
<td>Module 4</td>
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**Fig. 1 Workflow of proposed Fuzzy Evaluation System**

**Modules:**

1. Index of Optimism Calculator of a Teacher
2. Fuzzy Evaluation Marksheet
3. Grade Membership Function Constructor
4. Marks Transformer
5. Result Adjuster

The formulation for this proposed system done as follows:

**Module 1:** The Evaluators’ index of optimism $\lambda$ is calculated, where $\lambda \in [0, 1]$. A common answerscript is shared with all the evaluators to assess by traditional marking method. All these marks are collected to identify the diversity of range in awarding marks to students. Minimum & Maximum numbers are identified from these numbers to set the evaluators index of optimism $\lambda$.

**Module 2:** Evaluator uses vague grade sheet as shown in Table 2 to award his/her satisfaction level of answer to each question rather than marks. The system converts vague marksheets into fuzzy marksheet as shown in Table 3 from
fuzzy marksheet, total mark of each student is calculated which is a crisp number.

**Module 3:** This optional module is considered when results obtained by module 2 are adjusted three node fuzzy evaluation system. The three node fuzzy evaluation system is shown in figure 6. The adjustment is done by considering the factors like complexity, difficulty of question paper which is decided by domain expert. This step gives adjusted mark that is scaling up or down the total marks.

**Module 4:** Using the marks obtained by module 2 / module 3, the grade membership functions are constructed for mapping strict / lenient mark to normal mark. The grade membership functions are formed using interpolation technique.

**Module 5:** Marks obtained by module 2 / module 3 are transformed to normal type using grade membership functions and detailed evaluation report is generated for each student.

**IV. DESIGN CONSIDERATION**

**Module 1:** Index of Optimism Calculator of a Teacher
Marks of single shared common answerscript assessed by all evaluators are input to this module. The aim of this module is to calculate the index of optimism of a teacher. This module is shown in figure 2.

![Fig 2 Calculation of index of optimism](image)

**Input:**
1. Common subjective answerscript,
2. Grade vector [G] denoting assigned maximum mark of each question

**Output:**
1. Index of optimism of each evaluator

**Algorithm:**

**Step 1:** Common subjective answerscript is given to all evaluators to assess it by traditional method.

**Step 2:** The evaluated marks are stored in database.

**Step 3:** Maximum (Max) & Minimum (Min) marks are extracted.

**Step 4:** Index of optimism of Evaluator is then calculated as,

\[
\lambda = \frac{\text{Evaluator_s score} - \text{Min}}{\text{Max} - \text{Min}}
\]  

**Step 5:** Index of optimism is stored in database.

**Module 2:** Fuzzy Evaluation Marksheet
Evaluator’s degree of confidence is taken into account to generate the fuzzy interval of marks \([x, y]\). Question level degree of satisfaction is judged by evaluator with the help of satisfaction level rubric. Then defuzzify the question level interval marks to calculate total marks of each student. This module is shown in figure 3.

![Fig 3 Fuzzy evaluation of student’s answerscripts](image)

**Input:**
1. Subjective answerscript of each student \(S_j\),
2. Grade vector \([G]\) denoting assigned maximum mark of each question,
3. Satisfaction level Table 1,
4. Evaluators’ index of optimism \(\lambda\), where \(\lambda \in [0,1]\)

**Output:**
1. Total marks stored in database

**Algorithm:**

**Step 1:** Calculate the expected truth value \(E(X_i)\) (Table 3) of each vague truth value \(X_i\) in the vague grade sheet shown in Table 2, where \(E(X_i) \in [0,1]\) and \(1 \leq i \leq 11\).

\[
E(X) = (1 - \lambda) \times t_x + \lambda \times (1 - f_x)
\]

**Step 2:** Calculate the corresponding expected truth value \(E(Y)\) of each satisfaction level \(Y\) in the vague grade sheet shown in Table 2, where \(Y \in \{\text{EG, VVG, VG, G, MG, F, MB, B, VB, VVB, EB}\}\) and \(E(Y) \in [0,1]\).

The degree of satisfaction \(D(Q_i)\) of the question \(Q_i\) of the student’s answerscript can be evaluated by the function \(D\),
\[ D(Q,i) = \left[ \frac{\sum E(X_{i1}) \times E(VG) + \sum E(X_{i2}) \times E(VVG) + \cdots + E(X_{i11}) \times E(EB)}{\sum E(X_{i1}) + \sum E(X_{i2}) + \cdots + \sum E(X_{i11})} \right] \]  

\[ E(X_i) \text{ is expected satisfaction value of vague satisfaction value } X_i, 1 \leq i \leq 11, \text{ and } 0 \leq D(Q,i) \leq 1. \text{ The larger the value of } D(Q,i), \text{ the higher the degree of satisfaction that the answer of question } Q_i \text{ satisfies the evaluator’s opinion.} \]

**Step 3:** Based on Step 2 output, find the matching satisfaction level. Input that in fuzzy grade sheet as shown Table 4.

**Step 4:** Calculate \( \alpha \)-cut of each fuzzy mark \( F_i \), based on degree of confidence of evaluator

\[ (F_i)_\alpha = [a_{i1}, a_{i2}], \text{ where } \alpha \in [0,1] \]  

**Step 5:** Calculate interval-valued mark \([m_{i1}, m_{i2}]\) of each question \( Q_i \), where

\[ [m_{i1}, m_{i2}] = \left[ \frac{s_j}{s_1 + s_2 + \cdots + s_n} \right] \times [a_{i1}, a_{i2}] \]  

**Step 6:** Calculate defuzzified crisp mark of each question \( Q_i \) using optimism index \( \lambda \).

\[ Q_{i\text{-mark}} = (1 - \lambda) \times m_{i1} + \lambda \times m_{i2} \]  

**Step 7:** Store these \( \text{Total Mark of Student} \) in Mark database along with the corresponding evaluators’ index of optimism \( \lambda \).

Marks are of three categories depending upon \( \lambda \) value,

- If \( \lambda < 0.5 \), Evaluator and the marks are strict \( (g_{L_i}) \),
- If \( \lambda = 0.5 \), Evaluator and the marks are normal \( (g_{N_i}) \),
- If \( \lambda > 0.5 \), Evaluator and the marks are lenient \( (g_{H_i}) \)

**Module 3: Grade Membership Function Constructor**

Index of optimism is used to extract strict / normal / lenient type marks using which the grade membership functions are constructed. This module is shown in figure 4.

- **Input:**
  - Evaluators’ index of optimism \( \lambda \), where \( \lambda \in [0,1] \)
  - Total marks

- **Output:**
  - Grade membership functions (Strict / Lenient to Normal)

**Algorithm:**

**Step 1:** Calculate the total average strict-type grade \( \text{Avg}_{L_i} \) of \( g_{L_i} \)

\[ \text{Avg}_{L_i} = \frac{\sum_{i=1}^{m} g_{L_i}}{m} \]  

**Step 2:** Use parabolic curve interpolation techniques to get the most appropriate relational function between \( (g_{L_i} \) and \( g_{N_i} \) ) and between \( (g_{H_i} \) and \( g_{N_i} \) ) respectively.

**Module 4: Marks Transformer**

Final evaluation mark sheet is generated after marks are transformed to normal type. This marksheet shows the detailed information. This module is shown in figure 5.
Algorithm:

1. Convert (Strict type / Lenient type) marks obtained in Step 6 of module 2 to normal marks using grade membership function obtained in Step 2 of Module 3 and store final result in database.

2. Produce final student evaluation mark sheet showing detail information.

**Module 5: Result Adjuster**

Difficulty and Complexity level of each question in the question paper are taken from domain expert. Accuracy & Coverage rate of each answer is generated using marks and coverage level obtained from Module 2 respectively. Result obtained by Module 2 is adjusted with respect to complexity and difficulty of question paper. This module is shown in figure 6(a) [6]. Fuzzy logic controller is shown in 6(b) [6].

Input:
1. Each Question level marks & coverage level obtained from module 2,
2. Grade Vector [G] denoting the assigned maximum score of each question.
3. Complexity & Importance matrices given by Domain Expert

Output:
1. Adjusted total marks stored in database

Algorithm:

1. **Step 1:** Generate the Accuracy matrix, \( A[a_{ij}]_{m \times n} \),

   \[
   a_{ij} = \frac{Q_{work}}{Wageage \_ Mark \_ of \_ Question \_ Q} 
   \]  

   Where, \( m \) is the total number of questions in question paper and \( n \) is the total number of students. Generate the coverage matrix \( T[c_{ij}]_{m \times n} \), where \( c_{ij} \) is (coverage level / total number of coverage points) of each question \( Q_i \). Coverage level is obtained from Step 1 of Module 1.

2. **Step 2:** Based on the accuracy rate matrix A and the coverage matrix T, calculate the average accuracy rate.
$AvgA_i$ and the average coverage rate $AvgT_i$ for each question $Q_i$ as:

$$AvgA = \frac{\sum_{i=1}^{n} w_i}{n} \quad (12)$$

$$AvgT = \frac{\sum_{i=1}^{n} t_i}{n} \quad (13)$$

Where $n$ is total number of students.

Based on fuzzy sets “low”, “more or less low”, “medium”, “more or less high” and “high” shown in figure 7, fuzzy the average accuracy rate $AvgA_i$ and average coverage rate $AvgT_i$ for each $Q_i$ and calculate their membership grades belonging to each fuzzy set, respectively. Then, we can get the fuzzy score matrix $F_A$ for the average accuracy and can get the fuzzy score matrix $F_T$ for the average answer-coverage rate, shown as follows:

$$F_A = [f_{a_i}]_{m \times 5} \quad (14)$$

$$F_T = [f_{t_j}]_{l \times 5} \quad (15)$$

where $f_{a_i} \in (0,1)$ and $f_{t_j} \in (0,1)$ denotes the membership value of the score and coverage of ith question $Q_i$ belonging to the jth fuzzy set shown in Figure 7.

**Step 3:**

3.1 Obtain the weights of accuracy rate $W_A$ and coverage rate $W_T$ to perform fuzzy reasoning. These weights are decided by domain experts ($W_A + W_T = 1$).

Based on fuzzy accuracy rate matrix $F_A$, fuzzy coverage rate matrix $F_T$ and fuzzy rules $R_D$ given in the form of IF-THEN rules, the fuzzy difficulty matrix is derived as $D = [d_{ik}]_{m \times l}$ where $m$ is total number of questions and $l$ is total number of levels as per the fuzzy sets in figure 7. $d_{ik} \in (0,1)$ denotes the membership value of the difficulty of question $Q_i$ belonging to level $k$.

The value of $d_{ik}$ is obtained as:

$$d_{ik} = \max_{[0,1]} \{|WA \ast f_{a_i} + WT \ast f_{t_j}|} \quad (16)$$

Weight of the difficulty $D$ & complexity $C$ matrix $WD$ & $WC$ can be determined by domain expert such that ($WD + WC = 1$).

3.2 Based on the fuzzy difficulty matrix $D$, and the fuzzy complexity matrix $C$, given the fuzzy rules $RE$, we obtain the effort (i.e., answer cost) matrix of dimension $m \times 1$, in the same manner as we obtained the difficulty matrix above,

$$E = [e_{ik}]_{m \times l} \quad (17)$$

Where $e_{ik} \in [0, 1]$ denotes the membership value of the effort to answer question $Q_i$ belonging to level $k$, which is a measure of effort required by students to answer question $Q_i$.

3.3 Weight of the effort $E$ & importance $P$ matrices, $W_E$ & $W_P$ can be determined by a domain expert such that ($W_E + W_P = 1$).

3.4 Based on the fuzzy effort matrix $E$, and fuzzy importance matrix $P$ with their weight $W_E$, $W_P$, we obtain the adjustment matrix of dimension $m \times 1$:

$$W = [W_{ik}]_{m \times l} \quad (18)$$

Where $w_{ik} \in [0, 1]$ denotes the membership value of the adjustment to answer question $Q_i$ belonging to level $k$.

We use the following formula to obtain the adjustment vector,

$$\bar{W} = [w_{ik}]_{m \times l} \quad (19)$$

Where $w_{ik} \in [0, 1]$ denotes the final adjustment value required by question $Q_i$ obtained by

$$w_{ik} = \frac{0.1 \ast w_{i1} + 0.3 \ast w_{i2} + 0.5 \ast w_{i3} + 0.7 \ast w_{i4} + 0.9 \ast w_{i5}}{0.1 + 0.3 + 0.5 + 0.7 + 0.9} \quad (20)$$

Where $0.1, 0.3, 0.5, 0.7$ and $0.9$ are the centres of fuzzy membership functions shown in Figure 7.

**Step 4:**

We calculate the bias $B$ as follows, here $AV$ is nothing but $W$,

$$(g_{1} \ast AV_{1}) \ast B + (g_{1} \ast AV_{1}) \ast B + \ldots (g_{m} \ast AV_{m}) \ast B = 100 \quad (21)$$

$$B = \frac{\sum_{i=1}^{m} R_i}{\sum_{i=1}^{m} g_{i} \ast AV_{i}} \quad (22)$$

**Step 5:** Calculate the assigned score $AS_{i}$ after the adjustment of the ith question $Q_{i}$, shown as:

$$AS_{i} = g_{i} \ast W \ast B \quad (23)$$

Finally, we can obtain the adjusted total score $ATS_{i}$ of the student $S_{p}$ where

$$ATS_{p} = a_{p} \ast AS_{i} \quad (24)$$

**V. RESULTS**

We are still in progress for implementation of module 3, 4, and 5. Following are results of MATLAB implemented module 1, and 2 of proposed fuzzy evaluation system.
**Module 1: Index of Optimism Calculator**

Total Evaluators = 10.
Students’ marks assessed by different teachers,
T = [69, 70, 64, 65, 61, 71, 75, 63, 62, 66];

Index of Optimism $\lambda =$

**Module 2: Fuzzy Evaluation Marksheet**

Assume Index of optimism ($\lambda$) = 0.60

### Table 1: Satisfaction Levels and Their Corresponding Vague Satisfaction Values

<table>
<thead>
<tr>
<th>Satisfaction levels</th>
<th>Vague satisfaction values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extremely Good (EG)</td>
<td>[1, 1]</td>
</tr>
<tr>
<td>Very Very Good (VVG)</td>
<td>[0.90, 0.99]</td>
</tr>
<tr>
<td>Very Good (VG)</td>
<td>[0.80, 0.89]</td>
</tr>
<tr>
<td>Good (G)</td>
<td>[0.70, 0.79]</td>
</tr>
<tr>
<td>More or less Good (MG)</td>
<td>[0.60, 0.69]</td>
</tr>
<tr>
<td>Fair (F)</td>
<td>[0.50, 0.59]</td>
</tr>
<tr>
<td>More or less Bad (MB)</td>
<td>[0.40, 0.49]</td>
</tr>
<tr>
<td>Bad (B)</td>
<td>[0.25, 0.39]</td>
</tr>
<tr>
<td>Very Bad (VB)</td>
<td>[0.10, 0.24]</td>
</tr>
<tr>
<td>Very Very Bad (VVB)</td>
<td>[0.01, 0.09]</td>
</tr>
<tr>
<td>Extremely Bad (EB)</td>
<td>[0.00, 0.00]</td>
</tr>
</tbody>
</table>

### Table 2: Vague Mark Represented by Vague Values of the Question Q.1 in a Vague Grade Sheet

<table>
<thead>
<tr>
<th>Question No.</th>
<th>Points covered (pc)</th>
<th>EG</th>
<th>VVG</th>
<th>VG</th>
<th>G</th>
<th>MG</th>
<th>F</th>
<th>MB</th>
<th>B</th>
<th>VB</th>
<th>VVB</th>
<th>EB</th>
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<td></td>
<td></td>
<td>X1</td>
<td>X2</td>
<td>X3</td>
<td>X4</td>
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<td>X6</td>
<td>X7</td>
<td>X8</td>
<td>X9</td>
<td>X10</td>
<td>X11</td>
</tr>
</tbody>
</table>

### Table 3: Expected Truth Values of Vague Truth Values of the Question Q.1 of Table 3.2

<table>
<thead>
<tr>
<th>Question No.</th>
<th>Points covered (pc)</th>
<th>EG</th>
<th>VVG</th>
<th>VG</th>
<th>G</th>
<th>MG</th>
<th>F</th>
<th>MB</th>
<th>B</th>
<th>VB</th>
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</table>

### Table 4: Fuzzy Grade Sheet with Satisfaction Levels Associated with Degree of Confidence

<table>
<thead>
<tr>
<th>Question No.</th>
<th>Satisfaction levels</th>
<th>Degree of confidence of satisfaction levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q.1</td>
<td>F1</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>Q.2</td>
<td>F2</td>
<td>$\beta$</td>
</tr>
<tr>
<td>Q.3</td>
<td>F3</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>Q.n</td>
<td>Fn</td>
<td>$\delta$</td>
</tr>
</tbody>
</table>

Total Mark = Degree of Confidence of Total Mark
### Table 3.5: Fuzzy Rule Bases to Infer Difficulty and Effort

(a) Fuzzy Rule base RD for obtaining Difficulty

<table>
<thead>
<tr>
<th>Accuracy</th>
<th>Coverage rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5 5 4 4 3 3</td>
</tr>
<tr>
<td>2</td>
<td>5 4 3 4 3 2 2</td>
</tr>
<tr>
<td>3</td>
<td>4 4 3 2 2 2</td>
</tr>
<tr>
<td>4</td>
<td>4 3 2 2 1 1</td>
</tr>
<tr>
<td>5</td>
<td>3 2 2 1 1 1</td>
</tr>
</tbody>
</table>

(b) Fuzzy Rule base for RE obtaining Effort

<table>
<thead>
<tr>
<th>Difficulty</th>
<th>Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 1 2 2 3 4</td>
</tr>
<tr>
<td>2</td>
<td>1 1 2 2 3 4</td>
</tr>
<tr>
<td>3</td>
<td>2 2 3 4 4 5</td>
</tr>
<tr>
<td>4</td>
<td>2 3 4 4 5</td>
</tr>
<tr>
<td>5</td>
<td>3 4 4 5 5</td>
</tr>
</tbody>
</table>

1: “Low”, 2: “more or less low”, 3: “medium”, 4: “more or less high”, 5: “high”

### Table 5: Satisfaction Levels, Corresponding Vague Satisfaction Values, Expected Vague Truth Values

<table>
<thead>
<tr>
<th>Satisfaction levels</th>
<th>Vague satisfaction values (Y)</th>
<th>E(Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extremely Good (EG)</td>
<td>[1,1]</td>
<td>1.0</td>
</tr>
<tr>
<td>Very Very Good (VVG)</td>
<td>[0.9,0.99]</td>
<td>0.954</td>
</tr>
<tr>
<td>Very Good (VG)</td>
<td>[0.8,0.89]</td>
<td>0.854</td>
</tr>
<tr>
<td>Good (G)</td>
<td>[0.7,0.79]</td>
<td>0.754</td>
</tr>
<tr>
<td>More or less Good (MG)</td>
<td>[0.6,0.69]</td>
<td>0.654</td>
</tr>
<tr>
<td>Fair (F)</td>
<td>[0.5,0.59]</td>
<td>0.554</td>
</tr>
<tr>
<td>More or less Bad (MB)</td>
<td>[0.4,0.49]</td>
<td>0.454</td>
</tr>
<tr>
<td>Bad (B)</td>
<td>[0.2,0.39]</td>
<td>0.334</td>
</tr>
<tr>
<td>Very Bad (VB)</td>
<td>[0.1,0.24]</td>
<td>0.184</td>
</tr>
<tr>
<td>Very Very Bad (VVB)</td>
<td>[0.01,0.09]</td>
<td>0.058</td>
</tr>
<tr>
<td>Extremely Bad (EB)</td>
<td>[0,0]</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table 6: Vague Mark Represented by Vague Values of the Question Q.1 in a Vague Grade Sheet

<table>
<thead>
<tr>
<th>Question No.</th>
<th>Degree of Confidence</th>
<th>Satisfaction levels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EG</td>
<td>VVG</td>
</tr>
<tr>
<td>Q.1</td>
<td>0.75</td>
<td>[0.8,0.9]</td>
</tr>
<tr>
<td>Q.2</td>
<td>1.0</td>
<td>[0,0]</td>
</tr>
<tr>
<td>Q.3</td>
<td>0.75</td>
<td>[0,0]</td>
</tr>
<tr>
<td>Q.4</td>
<td>0.95</td>
<td>[0,0]</td>
</tr>
</tbody>
</table>

### Table 7: Expected Truth Values of Vague Truth Values of the Question Q.1 of Table 4.10

<table>
<thead>
<tr>
<th>Question No.</th>
<th>Degree of Confidence</th>
<th>Satisfaction levels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EG</td>
<td>VVG</td>
</tr>
<tr>
<td>Q.1</td>
<td>0.75</td>
<td>0.86</td>
</tr>
<tr>
<td>Q.2</td>
<td>1.0</td>
<td>0</td>
</tr>
<tr>
<td>Q.3</td>
<td>0.75</td>
<td>0</td>
</tr>
<tr>
<td>Q.4</td>
<td>0.95</td>
<td>0</td>
</tr>
</tbody>
</table>

Calculate $\alpha$ -cut of each fuzzy mark $F_\alpha$ based on degree of confidence of evaluator:

- (Very Very Good) $0.75 = [94, 96]$
- (More or Less good) $1.0 = [65, 65]$
- (Good) $0.75 = [74, 76]$
- (Very Bad) $0.95 = [17, 17]$
TABLE 8: FUZZY GRADE SHEET WITH SATISFACTION LEVELS ASSOCIATED WITH DEGREE OF CONFIDENCE

<table>
<thead>
<tr>
<th>Question No.</th>
<th>Satisfaction levels</th>
<th>Degree of confidence of satisfaction levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q. 1</td>
<td>Very Very Good</td>
<td>0.75</td>
</tr>
<tr>
<td>Q. 2</td>
<td>More or Less good</td>
<td>1.0</td>
</tr>
<tr>
<td>Q. 3</td>
<td>Good</td>
<td>0.75</td>
</tr>
<tr>
<td>Q. 4</td>
<td>Very Bad</td>
<td>0.95</td>
</tr>
</tbody>
</table>

**Total Mark obtained by Module 2 =** 59

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VI. CONCLUSION

By introducing a new fuzzy evaluation sheet, we believe it portrays additional information on students’ performances in answering each question in a test or examination compared to conventional marking method. Moreover, this approach can be used to compare students’ performances which have the same final linguistic terms by looking into each question and each criterion. This information can be very useful and beneficial for students, instructors, and other authorized or related bodies to have overall picture of students’ performances.

Time reduction is yet another goal. The task of grading students’ answer scripts is very repetitive and labour intensive. Faithful application of scoring rubric takes considerable amount of time. The proposed system leads to quicker and valid evaluation as it maintains the consistency while evaluating the answer scripts.

REFERENCES


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Fig. 7: Fuzzy membership functions of five levels