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Rough Anti Semigroups.

, Faraj.A.Abdunabi[1], Ahmed shletiet[2] Muatazz A Bashir[3]

[1] (*Department of Mathematics, Faculty of Science, AJdabyia University, Libya.*)

[2] (*Department of Mathematics, Faculty of Science, AJdabyia University, Libya.*)

[3](*Department of Mathematics, Faculty of Education, University of Benghazi, Benghazi, Libya)*

ABSTRACT *In this paper, we present the concepts of Anti semigroups, AntiRough subgroups, and AntiRough subsemigroups, and homomorphismes of AntiRough antisemigroups in approximation spaces. Furthermore, we give some properties of these rough structures.*

Keywords: *upper approximation, antisibgroups , , Rough set, , rough antisemigroup*

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I. INTRODUCTION

Rough set theory is an efficient and good tool for modeling and processing incomplete information in information system. It was an introduced by Pawlak [1] in 1982. Many researchers have developed this theory that based on the concept of upper approximation and lower approximation in many areas. Some of these researchers study algebraic structures of rough sets such as Bonikowaski [2], Iwinski [3], and Pomykala and J.Pomykala [4]. Miao et al. [5] improve definitions of rough group and rough subgroup, and considers some properties. B.Davvaz in[6], study the concept of rough subring with respect ideal. Y.Y. Yao in [7] consider the concepts of lower and upper approximations on lattice. In addition, some properties of the lower and the upper approximations with respect to the normal subgroups were studied in[8] .The concepts of rough set theory build of lower and upper approximations. The upper approximation of a given set is the union of all the equivalence classes that are subsets of the set, and the upper approximation is the union of all the equivalence classes that are intersection with set non-empty. The main purpose of this paper is to introduce rough antisemigroups of Finite antigroups of typs(4). Also, we introduce some properties of approximations and these algebraic structures. We introduced the notion of AntiRough semigroups. However, our definition of rough antisemigroup is similar to the definition of rough groups.

II. PRELIMINARIES

We start by given some definitions and results about rough sets.

Suppose that $\sim$ an equivalence relation on an universe set (nonempty finite set) U. Some authors say $\sim$ is indiscernibility relation. The pair $(U, \sim)$ is called an approximation space. We use $U/\sim$ to denote the family of all equivalent classes$[x]_\sim$. The empty set $\emptyset$ and the element of $U/\sim$ are called elementary sets. For any $X \subseteq U$, we write X^c to denote the complementation of X in U.

Definition 2.1: Suppose that $(U, \sim)$ is an approximation space. We define the upper approximation of X by $\overline{\sim X} = \{x \in U: [x]_\sim \cap X \neq \emptyset\}$ and the lower approximation of X by$\underline{\sim X} = \{x \in U: [x]_\sim \subseteq X\}$ the boundary is $BX_\sim = \overline{\sim X} - \underline{\sim X}$. If $BX_R = \emptyset$, we say X is exact (crisp) set otherwise, we say X is Rough set (inexact).

Preposition 2-1: Suppose that $(U, \sim)$ is an approximation space. Suppose that $X,Y \subseteq U$, *we have:*

1) $\underline{\sim X} \subseteq X \subseteq \overline{\sim X}$
2) $\underline{\sim \emptyset} = \overline{\sim \emptyset}, \underline{\sim U} = \overline{\sim U}$,
3) $\underline{\sim(X \cup Y)} \supseteq \underline{\sim(X)} \cup \underline{\sim(Y)}$,
4) $\underline{\sim(X \cap Y)} = \underline{\sim(X)} \cap \underline{\sim(Y)}$,
5) $\overline{\sim(X \cup Y)} = \overline{\sim(X)} \cup \overline{\sim(Y)}$.

6) $\overline{\sim(X \cap Y)} \subseteq \overline{\sim(X)} \cap \overline{\sim(Y)}$.
7) $\overline{\sim X^C} = \left(\underline{\sim X}\right)^c$.
8) $\underline{\sim X^C} = (\overline{\sim X})^c$.
9) $\sim(\underline{\sim X}) = \overline{\sim(\underline{\sim X})} = \underline{\sim X}$. -
10) $\overline{\overline{(\sim(\overline{\sim X})}} = \underline{\sim(\overline{\sim X})} = \overline{\sim X}$.

Proposition 2-2 [8] Let *(U, R)* be an approximation space. Let *X* and *Y* be nonempty subsets of *U*. Then

1) $\overline{\sim X}\ \overline{\sim Y} = \overline{\sim XY}$.
2) $\underline{\sim X}\ \underline{\sim Y} \subseteq \underline{\sim XY}$.

Now, we introduce the some concepts of antigroups .for more details see [9].

Definition 2.2. Suppose that G≠ϕ set. Let *: $\mathcal{R}\times\mathcal{R}\rightarrow\mathcal{R}$ be binary operations defined on G. The (G,*) is called a groups if satisfy the following conditions:
C_1: $\forall\, x, y \in \mathrm{G}, x * y \in \mathrm{G}$;
C_2: $\forall\, x, y, z \in \mathrm{G}, x * (y * z) = (x * y) * z$;
C_3: $\forall\, x \in \mathrm{G}$, there exists $\mathrm{e} \in \mathrm{G}$ such that $x * e = e * x = x$;
C_4: $\forall\, x \in \mathrm{G}$, there exists $-x \in \mathrm{G}$ such that $x * (-x) = (-x) * x = e$;
And
If we have,
C_5: $\forall\, x, y \in \mathrm{G},\ x * y = y * x$, then (G,∗)is called a abeilan group.
Remark 2.1. $S \neq \phi$ is called a semigroup if satisfies C_1,C_2.
Definition 2.3. Suppose that $S \neq \phi$ and $H \subseteq S$, we say *H* is a subsemigroup of *S*, if $a * b \in H$ *for all* $a, b \in S$.
Definition 2.4. [9] Suppose that $G \neq \phi$, We say that *G* is An AntiGroup we denoted by . if *G* has at least one AntiLaw or at least one flowing conditions:
C_6: $\forall\, (x, y) \in \mathfrak{C}, x * y \notin \mathfrak{C}$;
C_7: For all the triplets $(x, y, z) \in \mathfrak{C}, x * (y * z) \neq (x * y) * z$;
C_8:There does not exist an element $e \in \mathfrak{C}$ such that $x * e = e * x = x\ \forall x \in \mathfrak{C}$.
C_9: There does not exist $u \in \mathfrak{C}$ such that $x * u = u * x = e\ \forall x \in \mathfrak{C}$.

Definition 2.5. [9] We called anon empty group *G* is An AntiAbelianGroup if *G* is abelian group and has at least one of {C_6, C_7,C_8, C_9} and
*C10:$\forall\, (x, y) \in \mathfrak{C}, x * y \neq y * x$.

Remark 2.2. If G is a group with a binary an operation, then there are 65 types of AntiGroups . If (G, ∗) abelian group, then there are 211 types of AntiAbelianGroups(see [9])
Definition 2.6. Suppose that G is a nonempty. If an AntiGroups ($\mathfrak{C}$, ∗) has the conditions C_1, C_2, C_3 and C_5 are either partially true, partially indeterminate or partially false for some elements of $\mathfrak{C}$ C_4 is totally false for all the elements of $\mathfrak{C}$, we say G is an AntiGroup of type-AG(4).

Example 2.1. Suppose that *X* = {1, 2, 3, 4, 0, 5} ia a universe of discourse. Suppose that *G* = {1, 2, 3, 0} is a subset of *X*. Define the a binary operation on *G* as shown in the Cayley table below

*	1	2	3	0
1	4	1	3	0
2	1	4	0	3
3	2	1	5	0
0	1	2	3	5

We can see the conditions C_1, C_2, C_3, and C_5 are either partially true or partially false with respect to ∗ but C_4 is totally false for all the elements of G. Hence (G, ∗) is $\mathfrak{C}$ a finite AntiGroup of type-(4).
Proposition 2.3. Suppose that G is ($\mathfrak{C}$, ∗) an AntiGroup of type-(4). Suppoae that let *g, x, y* ∈ $\mathfrak{C}$. Then

(i) $g * x = g * y \not\Rightarrow x = y$.
(ii) $x * g = y * g \not\Rightarrow x = y$.

III. MAIN RESULT

Definition 3.1. [11] Suppose that G is non empty with an binary operation $*$ and $(\underline{G}, \overline{G})$ is a rough set in the approximation space $(U, \sim)$. Then G is called a rough group if the following conditions are satisfied:
G_1: $\forall x, y \in G, x * y \in \bar{G}$ $(closed)$;
G_2 :$(x * y) * z = x * (y * z), \forall x, y, z \in G$ $(associative\ law)$;
G_3: $\forall x \in G, \exists e \in \bar{G}$ $such\ that\ x * e = e * x = x$ $(e\ is\ the\ rough\ identity\ element)$;
G_4: $\forall x \in G, \exists y \in \bar{G}$ $such\ that\ x * y = y * x = e$.
$(y\ is\ the\ rough\ inverse\ element\ of\ x. It\ is\ denoted\ as\ x - 1)$
Remark 3.1. If $H \subseteq G$. We called H is a rough subgroup if it is a rough group itself.

Definition 3-2. suppose that $(U, \sim)$ is an approximation space and $(*)$ be a binary operation defined on U. A subset S of U is called rough Anti semigroup on approximation space, provided the following properties are satisfied:
G_5 :$\forall x, y \in S, x * y \notin \overline{\sim S}$,
G_6: $\forall x, y, z \in S, (x * y) * z \neq x * (y * z)$ in $\underline{\sim S}$.
Example 3.1. let consider Example 2.1. Suppose that $A = \{1, 2, 0\}$ as shown in the Cayley tables below:

*	1	2	0
1	4	1	0
2	1	4	3
0	1	2	5

We can that A is an AntiSubgroup of G.

Let suppose that $\sim = \{(1,1),(2, 2),(2, 0),(0, 1),(0, 2),(0, 0)\}$. Where $[x]\sim = \{y \in A : x \sim y\}$.
Then $[1]\sim = \{1\}$; $[2]\sim = \{2, 0\}$; $[0]\sim = \{1, 2, 0\}$ We can get from the definition of the upper approximation of A by $,\overline{\sim\{1,2,0\}} = \{1,2,0\}$. Witch anti AntiSubgroup. By definition of 3.2, A is rough AntiSubgroup.

Example 3.2. Suppose that $U = \{1, 2, 3, 4, 5, 6\}$ and let $\mathfrak{C} = \{1, 2,3,5\}$ be a subset of U. Let $*$ be a binary operation defined on $\mathfrak{C}$ as shown in the Cayley table below

*	1	2	3	5
1	4	1	3	5
2	1	4	5	3
3	2	1	6	5
5	1	2	3	6

We have $(\mathfrak{C}, *)$ is a finite AntiGroup of type-AG[4]. Suppose a classification of U is $U/\sim = \{E_1, E_2, E_3\}$, where $E_1 = \{1, 2, 3\}$, $E_2 = \{4,\}$, $E_3 = \{5\}$. let $\boldsymbol{A} = \{1, 2, 5\}$, Let $*$ be defined on $\boldsymbol{A}$ as shown in the Cayley tables below:

*	1	2	5
1	4	1	5
2	1	4	3
5	1	2	6

(e-6). It can easily be seen from the tables that $\boldsymbol{A}$ is an AntiSubgroup of .
Now, for , then $\overline{\sim A} = \{1, 2, 3, 5\} = \mathfrak{C}$ is AntiGroup. So, A is a rough Antisemigroup.

If we take $B = \{2,3, 5\}$ a subset of $\mathfrak{C}$. Let $*$ be defined on B as shown in the Cayley tables below:

*	2	3	5
2	4	5	3
3	1	6	5
5	2	3	6

. It can easily be seen from the tables that is a AntiSubgroup of $\mathfrak{C}$. Then $\overline{\sim B} = \{1, 2, 3, 5\}$ is a rough Antisemigroup.
Definition 3.3. Suppose $\mathfrak{C}$ is rough Anti smiGroup of type-(4}. Suppose that H and K are twow AntiSubgroups of $\mathfrak{C}$. The set $A \oplus B$ is defined by $A \oplus B = \{x \in\ : x = h \oplus k$ for some $h \in$ H, k $\in$ K}.

Proposition 3.1. Suppose that H ,K and L are Rough AntiSubgroups of an AntiGroup $\mathfrak{C}$ of type-(4). Then

1- $\overline{\sim H} \oplus \overline{\sim K} \neq \overline{\sim H}$
2- $\overline{\sim H} \oplus \overline{\sim K} \neq \overline{\sim H} \oplus \overline{\sim K}$
3- $\overline{\sim H} \oplus (\overline{\sim K} \oplus \overline{\sim L}) \neq (\overline{\sim H} \oplus \overline{\sim K}) \oplus \overline{\sim L}$.

Proof easy and intuitive.

Proposition 3.2. Suppose that H ,K are Rough AntiSubgroups of an AntiGroup $\mathfrak{C}$ of type-(4). Then $\overline{\sim H} \cup \overline{\sim K}$ *is* Rough AntiSubgroups.

Proposition 3-3. Suppose that $(U, \sim)$ be an approximation space and $(*)$ be a binary operation defined on U. Suppose that A and B be two rough anti subsemigroups of the rough antisemigroup $\boldsymbol{A}$. Then $\overline{\sim(A)} \cap \overline{\sim(B)} \subseteq \overline{\sim(A \cap B)}$.
A sufficient condition for intersection of two rough antisubsemigroups of a rough antisemigroup be a rough anti subsemigroup is $\overline{\sim(A)} \cap \overline{\sim(B)} = \overline{\sim(A \cap B)}$.
Example 3.3. Let consider example 3.1 and 3.-2. We have $\boldsymbol{A} = \{1, 2, 5\}$ and $= \{2,3, 5\}$, then $\boldsymbol{A} \cap \boldsymbol{B} = \{2,5\}$ then $\overline{\sim A} = \{1, 2, 3, 4\} \cap \overline{\sim B} = \{1, 2, 3, 5\} = \{\{1, 2, 3\}$, $\overline{\sim(A \cap B)} = \{1, 2, 3, 5\}$.

IV. HOMOMORPHISMS OF ROUGH ANTIGROUP

Suppose that $(\mathfrak{C}, *)$ and $(\mathfrak{B}, \circ)$ be any two AntiGroups of type-AG[4]. The mapping $\varphi : \mathfrak{C} \rightarrow \mathfrak{B}$ is called an AntiGroupHomomorphism if φ does not preserve the binary operations $*$ and $\circ$ that is for all the duplet $(x, y) \in \mathfrak{C}$, we have

$$\varphi(x * y) \neq \varphi(x) \circ \varphi(y).$$

The kernel of φ denoted by Kerφ is defined by Kerφ = $\{x : \varphi(x) = e_{\mathfrak{B}}$ for at least one $e_{\mathfrak{B}} \in \mathfrak{B}\}$ where $e_{\mathfrak{B}}$ is a NeutroNeutralElement in $\mathfrak{B}$. The image of φ denoted by Imφ is defined by Imφ = $\{y \in : y = \varphi(x)$ *for some* $x \in \mathfrak{C}\}$.
If in addition φ is an AntiBijection, then φ is called an AntiGroupIsomorphism.
Suppose that Let $(U_1, \sim)$, (U_2, ρ) be two approximation spaces, and $(\cdot)$be binary operation over universes U_1 and ,$(\circ)$ over universes U_2
Definition 4.1 Let $\boldsymbol{A} \subset U_1$, $\boldsymbol{B} \subset U_2$ be rough antisemigroups. If there exists a surjection $\phi : \overline{\sim(A)} \rightarrow \overline{\sim(B)}$ such that $\phi(x \cdot y) = \phi(x) \circ \phi(y)$ for all $x, y \in \overline{\sim A}$ then ϕ is called a rough homomorphism and $\boldsymbol{A}$, $\boldsymbol{B}$ are called rough homomorphic semigroups.
Definition 4.2 Let $\mathfrak{C} \subset U_1$, $\mathfrak{B} \subset U_2$ be rough anti groups. If there exists a surjection $\phi : \overline{\sim(\mathfrak{C})} \rightarrow \overline{\sim(\mathfrak{B})}$ such that $\phi(x \cdot y) = \phi(y) \circ \phi(x)$ for all $x, y \in \overline{\sim\mathfrak{C}}$ then ϕ is called a rough anti homomorphism.
Proposition4.1. Let $\mathfrak{C}$ be a rough antigroup and φ_1 be a rough anti-homomorphism and φ_2 be a rough homomorphism on $\mathfrak{C}$. Then the composition $\varphi_1 o \varphi_2$ is a rough antihomomorphism on $\mathfrak{C}$.
Proof. Let $\mathfrak{C}$ be a rough antigroup and let φ_1 be a rough anti-homomorphism on $\mathfrak{C}$and φ_2 be a rough homomorphism on $\mathfrak{C}$. Then φ_1, $\varphi2 : : \overline{\sim(\mathfrak{C})} \rightarrow \overline{\sim(\mathfrak{B})}$ such that $\forall x, y \in : \overline{\sim(\mathfrak{C})}$, $\varphi_1(x * y) = \varphi_1(y) * \varphi_1(x)$ and $\varphi(x * y) = \varphi_2(x) * \varphi_2(y)$ Now $\forall\ x, y \in : \overline{\sim(\mathfrak{C})}$ $(\varphi_1 o \varphi_2)(x * y) = \varphi_1(\varphi_2(x * y)) = \varphi_1(\varphi_2(x) * \varphi_2(y)) = (\varphi_1 o \varphi_2)(y) * (\varphi_1 o \varphi_2)(x)$ Therefore, $\varphi_1 o \varphi_2$ is a rough anti-homomorphism on $\mathfrak{C}$.
Proposition4.2.. Let $\mathfrak{C}$ be a rough antigroup and φ_1 and φ_2 be two rough anti-homomorphisms on $\mathfrak{C}$. Then the composition $\varphi_1 o \varphi_2$ is a rough homomorphism on $\mathfrak{C}$.
Proof. Let$\mathfrak{C}$ be a rough antigroup and let φ_1, φ_2 be two rough anti-homomorphisms on $\mathfrak{C}$. Then $\varphi_1, \varphi_2 : \overline{\sim(\mathfrak{C})} \rightarrow \overline{\sim(\mathfrak{C})}$ such that $\forall x, y \in \overline{\sim(\mathfrak{C})}$ $\varphi_1(x * y) = \varphi_1(y) * \varphi_1(x)$ and $\varphi_2(x * y) = \varphi_2(y) * \varphi_2(x)$.
Now $\forall$x, $y \in \overline{\sim(\mathfrak{C})}$ $(\varphi_1 o \varphi_2)(x * y) = \varphi_1(\varphi_2(x * y)) = \varphi_1(\varphi_2(y) * \varphi_2(x)) = (\varphi_1 o \varphi_2)(x) * (\varphi_1 o \varphi_2)(y)$ Therefore, $\varphi_1 o \varphi_2$ is a rough homomorphism on $\mathfrak{C}$.

V. CONCLUSION

In this paper we introduce the study the concepts of rough in Finite antigroups of typs[4]. Moreover, we introduce some properties of approximations and these algebraic structures. And we give the definition of homomorphism antiGroupd

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