

Forecasting HKD/CNY Exchange Rate Dynamics: An Optimized ARIMA(1,1,2) Time Series Approach

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Abstract: In this project, time series analysis is used to test the stationarity of 1305 data, and the stationarity is found. The first order difference equation is used to optimize the process, the appropriate ARIMA model (p,d,q) is selected, the ACF and PACF images are observed, BP test is used and the QQ graph and histogram are checked. Finally, the best ARIMA(1,1,2) model is obtained to analyze and forecast the trend of exchange rate between HKD and CNY. The research findings provide a reference for predicting future exchange rate fluctuations between the Hong Kong dollar and the Chinese yuan, while also offering practical benefits for mainland students studying in Hong Kong.

Keywords Time Series Analysis, Rate fluctuations, ARIMA model (p,d,q)

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I. Introduction

As a premier Special Administrative Region (SAR) of China, Hong Kong occupies a unique and indispensable position within the global economic landscape. It serves not merely as a bridge for mainland China's foreign trade but as a sophisticated international financial hub that facilitates the seamless flow of capital, information, and talent.

The HKD is uniquely defined by its Linked Exchange Rate System (LERS), pegging it to the US Dollar, while the CNY follows a managed floating exchange rate regime. This structural divergence necessitates rigorous quantitative analysis. By closely monitoring these fluctuations, enterprises can move beyond reactive financial management toward a predictive model of trade cost assessment. At the same time, for mainland students studying in Hong Kong, it is also of great significance to closely monitor and study the exchange rate dynamics of the Hong Kong dollar and the Chinese yuan; making exchanges between the two currencies at the appropriate time can effectively reduce daily expenses in Hong Kong and alleviate the economic burden of life. Consequently, the study of exchange rate trends emerges as a vital skill set for both institutional actors and private individuals, serving as a cornerstone for financial optimization in an increasingly integrated regional economy.

II. Stationary Test

2.1 multiplicative decomposition

Before the stability test of the data, the trend of the data was first observed. Through the observation of the data, the data showed an upward trend from 2018 to June 2020, while the rest of the data showed a downward trend. Therefore, the multiplicative decomposition is used to process the data graph, and the four graphs produced are the original time series, the trend component, the seasonal component and the residual component. The Figure 1 is shown below.

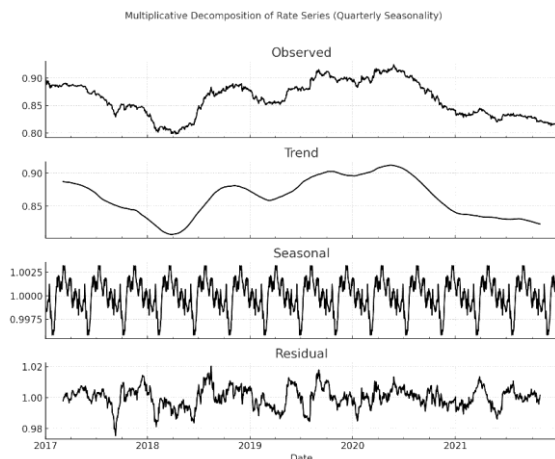


Figure 1 Multiplicative Time Series

In Figure 2, The value of ACF is decreased slowly, and the decline was not very large.

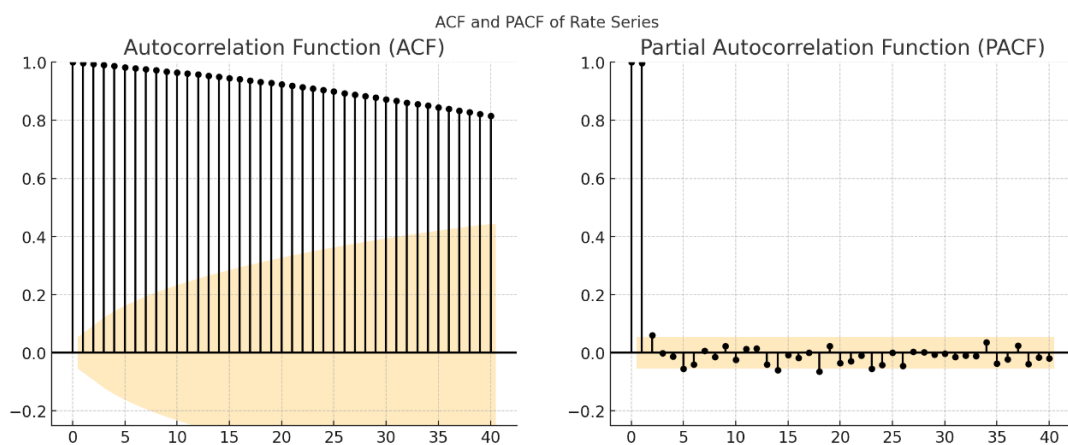


Figure 2 Plot of ACF and PACF

On the PACF of Figure 2, the value of PACF is cut off after lag two, lag 1 only drop a little, while lag 2 was a sharp drop, slightly above the threshold line.

The test results of ADF and KPSS further validate the previous image judgment results. In the test of ADF, its purpose is to detect whether the unit root exists, and the null hypothesis assumes the existence of the unit root, which also means that the time series is not stationary. The significance level of a P-value over 5% is about 70%, so we reject the null hypothesis. In the KPSS test, the null hypothesis is that the data is stable, and the P-value is only 0.01, which is far less than the significance level of 5%. So, this also shows that the data is non-stationary. The test results from ADF and KPSS show that the time series is not stable, so we need to use the difference method to make the data become stable, the result of ADF and KPSS test presents in Table 1.

Table 1 Stationary test of ADF and KPSS

Test Type	Null-hypothesis	Alternative-hypothesis	p-value
ADF	non-stationary	stationary	0.7645
KPSS	stationary	non-stationary	0.01

2.2 Differencing

In time series analysis, difference is a method used to make non-stationary time series data stationary. Through proper difference, trend and seasonality can be eliminated, so that the mean and variance of time series do not change with time, which provides a good data basis for the application of models such as ARIMA.

In time series analysis, the formula of first-order difference is used to eliminate linear trends in the data and make the series more stable. The first-order difference formula is as follows:

$$Y_t' = Y_t - Y_{t-1}$$

In the formula, Y_t is the value of the time series at the time t ; Y_{t-1} is the value of the time series at the time $t - 1$; Y_t' is

the value of the time series after the first difference of t imet.

2.3 Test after Differencing

After the first-order difference, the time series should be tested again to see if it is stationary. As shown in Table 2 below, the P-value of ADF is 1.4153e-29, which is far less than 5% of the significance level. the P-value of KPSS is 0.1, and both the ADF and KPSS tests of the data pass the stationarity test. the results are shown in the following table.

Table 2 Stationary test of ADF and KPSS

Test Type	Null-hypothesis	Alternative-hypothesis	p-value
ADF	non-stationary	stationary	1.4153e-29
KPSS	stationary	non-stationary	0.1

III. AIC and BIC model

AIC (Akaike Information Criterion) and BIC (Bayesian Information Criterion) are two criteria used in statistics for model selection. Their main role is to help select the optimal model by balancing the fit and complexity of the model.

AIC is used to measure the goodness of fit and complexity of the model. AIC is calculated as follows:

$$AIC = -2 \ln(L) + 2k$$

L is the maximum likelihood estimate of the mode, k is the number of parameters in the model.

AICc (Corrected Akaike Information Criterion) is a corrected version of AIC (Corrected Akaike Information Criterion), which is mainly used when the sample size is small. AICc adjusted the AIC under small sample conditions to reduce possible bias.

$$AICc = AIC + \frac{2k(k + 1)}{n - k - 1}$$

$$= -2 \ln(L) + 2k + \frac{2k(k + 1)}{n - k - 1}$$

BIC is an information criterion based on Bayesian theory, and the formula is as follows:

$$BIC = -2 \ln(L) + k \ln(n)$$

L is the maximum likelihood estimate of the mode, k is the number of parameters in the model, n is sample size.

Combine two equations AIC and BIC, we can explain BIC as follows:

$$BIC = AIC + [k \ln(n) - 2k]$$

$$BIC = AIC + k[\ln(n) - 2]$$

By using ARIMA(p, d, q) and two calculation models, we can get the results as shown in Table 3 below:

Table 3 Results of AIC and BIC model

Model	AIC	AICc	BIC
ARMIA(1,1,0)	-12361.803870	-12361.794653	-12351.457486
ARMIA(1,1,2)	-12365.857948	-12365.827179	-12345.165181
ARMIA(2,1,0)	-12360.185727	-12360.167280	-12344.666152
ARMIA(2,1,2)	-12362.456761	-12362.410572	-12336.590803

In practical applications, the AIC and BIC values of multiple models are usually calculated, and the model with smaller AIC or BIC values is selected as the optimal model. From my point of view, we suppose the ARIMA(1,1,2) is the best AIC model in this four models, and the ARIMA(1,1,0) is the best BIC model in this four models. In order to get the best and more accurate ARIMA, we need to find models with smaller errors and values of AIC or BIC.

Table 4 Results of six formulas

Model	ME	RMSE	MAE	MPE	MAPE	MASE
ARMIA(1,1,0)	0.000066	0.002111	0.001467	0.008070	0.169902	0.994065
ARMIA(1,1,2)	0.000052	0.002105	0.001465	0.006383	0.169724	0.993124
ARMIA(2,1,0)	0.000065	0.002111	0.001468	0.007927	0.170118	0.995315
ARMIA(2,1,2)	0.000057	0.002106	0.001468	0.006908	0.170050	0.995072

By observing the data in the table, the six measures are smaller, the model fits better. So ARIMA(1,1,2) is the best choice.

IV. Diagnostic Checking

4.1 The Box-Pierce Test

$$Q = n \sum_{k=1}^m \gamma_k^2$$

In the formula, n is the number of observations in the time series, $df = 1$, m is the number of lags to be tested, ρ_k is the sample autocorrelation of the residuals at lag k . Assume that H_0 is that the residuals are independently distributed, H_A is that the residuals are not independently distributed. If the p-value is less than $\alpha = 0.05$ with 95% confidence level, we will reject the H_0 . However, the p-value is greater, we fail to reject the null hypothesis, suggesting the residuals are approximately white noise.

Table 5 Results of The Box-Pierce Test

Model(p,d,q)	BP Test Statistic	BP Test p-value
ARIMA(1,1,0)	0.1661	0.9999
ARIMA(1,1,2)	0.1715	0.9999
ARIMA(2,1,0)	0.1685	0.9999
ARIMA(2,1,2)	0.1717	0.9999

4.2 QQ plot and Histogram

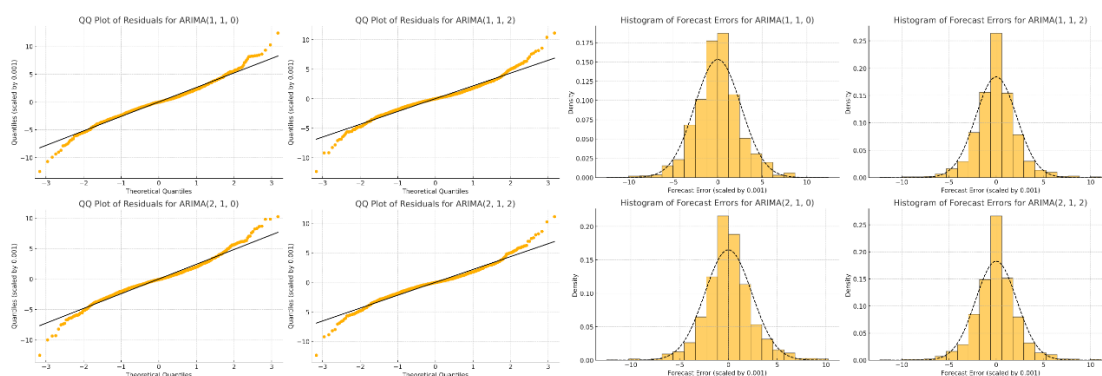


Figure 3 QQ plot and Histogram of four residuals ARIMA models

By observing the QQ plot and histogram, the residual points of ARIMA(1,1,2) are distributed along a 45 degree diagonal, which indicates that the residual is close to normal distribution, and the hypothesis of the model is reasonable. The histogram of the residuals shows a symmetrical bell-shaped distribution, which indicates that the residuals are close to normal distribution, and also indicates that the model fits the data well.

4.3 The Visualizing of Residual Analysis

The ACF and PACF plots are useful in residual analysis to identify the fitting effect of the model. It is assumed that the ACF and PACF plots have no significant lag and the residual is approximately white noise, indicating that the model fits well. Residuals ACF and PACF for Different ARIMA Models are shown below:

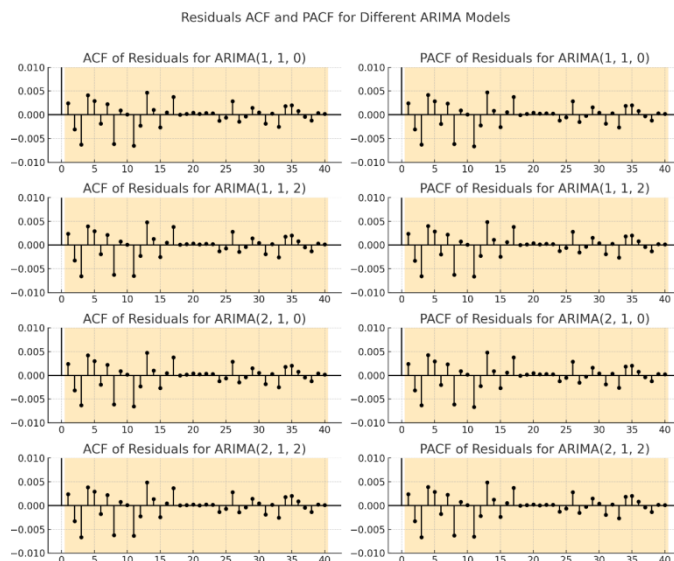


Figure 4 Residuals ACF and PACF for Different ARIMA Models

Through the figure, four ARIMA models are fit well. But ARIMA(1,1,2) is the best model in it. We can build such formula as followed:

$$(1 - 0.8833B)(1 - B)Z_t = (1 - 0.9601B + 0.0976B^2)a_t$$

V. Visualizing the Results

In the Figure 5, The fitting plot displays the original data and the fitting results of the ARIMA(1,1,2) model in the same graph, allowing you to visually see the model's performance on the known data. By comparing the difference between the two lines, we can tell which trend the model captures better in the data.

With these two different confidence intervals in the forecast plot, we can intuitively see the degree of uncertainty in the forecast. The 95% confidence interval is wider than the 80% confidence interval, indicating that we are more confident in the prediction of this range.

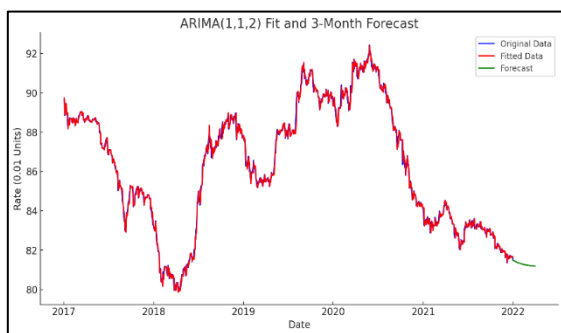


Figure 5 Fitting plot

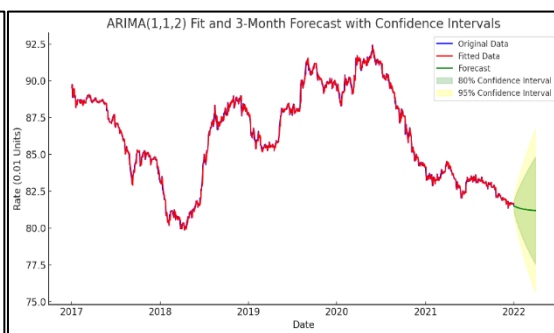


Figure 6 Forecast plot

VI. Conclusion

Under the combined influence of multiple factors, especially the cyclical adjustment of the Federal Reserve's monetary policy, the sustained strong recovery of the Chinese economy, and the significant differences in the epidemic prevention and control strategies of both sides, the exchange rate between the Hong Kong dollar and the Chinese yuan is expected to continue its downward trend in the next quarter.

To deeply explore and predict this exchange rate trend, this study systematically collected historical data from January 2, 2017 to December 31, 2021, a period of five years, obtaining 1,305 daily exchange rate observations of the Hong Kong dollar against the Chinese yuan as the basis for analysis. The research adopted a rigorous time series analysis method. Firstly, it conducted a stationarity test on the original data to confirm that it met the stationarity requirements. Then, by introducing a first-order difference equation to optimize the model, and based on this, combined with the characteristics of the autocorrelation function (ACF) and partial autocorrelation function (PACF) sequence graphs, it carefully selected and determined the most suitable ARIMA model parameter combination (p, d, q). Further, the study passed the BP test to ensure the independence of the

model residuals and supplemented with QQ plots and histograms to assess the normality of the residual distribution.

To select the optimal ARIMA model, this paper calculated the AIC, AICc, and BIC information criteria as well as the ME, RMSE, MAE, MPE, MAPE, and MASE six prediction error indicators for the four candidate models of ARIMA(1,1,0), ARIMA(1,1,2), ARIMA(2,1,0), and ARIMA(2,1,2). Additionally, the AIC and AICc values of ARIMA(1,1,2) were the smallest among the four models, indicating that it achieved the best balance between goodness of fit and parameter parsimony. Although the BIC criterion, due to its stricter penalty on the number of parameters, favored the simpler ARIMA(1,1,0), this divergence was a reflection of the inherent differences in the focus of the information criteria and did not affect the overall judgment. In terms of prediction errors, ARIMA(1,1,2) achieved the lowest values in all six indicators, with the smallest systematic prediction bias and the highest fitting accuracy. Based on the evaluation results of both types of criteria, the ARIMA(1,1,2) model performed the most stably. Therefore, this study ultimately established the ARIMA(1,1,2) model as the optimal prediction model and used it to conduct in-depth analysis and trend prediction of the future dynamics of the Hong Kong dollar and the Chinese yuan exchange rates.