Somewhat almost αg-continuous functions and Somewhat almost αg-open functions

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Abstract: *In this paper we tried to introduce a new variety of continuous and open functions called Somewhat almost αg-continuous functions and Somewhat almost αg-open functions. Its basic properties are discussed.*
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I. Introduction:

b-open[1] sets are introduced by Andrijevic in 1996. K.R.Gentry[9] introduced somewhat continuous functions in the year 1971. V.K.Sharma and the present authors of this paper defined and studied basic properties of *v*-open sets and *v*-continuous functions in the year 2006 and 2010 respectively. T.Noiri and N.Rajesh[11] introduced somewhat b-continuous functions in the year 2011. Inspired with these developments we introduce in this paper somewhat almost αg-continuous functions, somewhat almost αg-open functions and study its basic properties and interrelation with other type of such functions available in the literature. Throughout the paper (X, τ) and (Y, σ) (or simply X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For $A\subset(X;\tau)$, $cl\{A\}$ and A^o denote the closure of A and the interior of A in X, respectively.

II. Preliminaries

Definition 2.1: A subset *A* of X is said to be
(i) b-open[1] if $A\subset(cl\{A\})^o\cap cl\{A^o\}$.
(ii) αg-dense in X if there is no proper αg-closed set C in X such that $M\subset C\subset X$.

Definition 2.2: A function *f* is said to be
(i) somewhat continuous[9][resp: somewhat b-continuous[11]; somewhat gs-continuous[6]] if for $U\in\sigma$ and $f^{-1}(U)\neq\varphi$, there exists an open[resp: b-open; gs-open] set V in X such that $V\neq\varphi$ and $V\subset f^{-1}(U)$.
(ii) somewhat open[11][resp: somewhat b-open[9]; somewhat gs-open] provided that if $U\in\tau$ and $U\neq\varphi$, then there exists an open[resp: b-open; gs-open] set V in Y such that $V\neq\varphi$ and $V\subset f(U)$.

Definition 2.3: (X, τ) is said to be resolvable[8][b-resolvable[11]] if there exists a set A in (X, τ) such that both A and X - A are dense[b-dense] in (X, τ). Otherwise, (X, τ) is called irresolvable.

Definition 2.4: If X is a set and τ and σ are topologies on X, then τ is said to be equivalent[resp: αg-equivalent] to σ provided if $U\in\tau$ and $U\neq\varphi$, then there is an open[resp: αg-open] set V in *X* such that $V\neq\varphi$ and $V\subset U$ and if $U\in\sigma$ and $U\neq\varphi$, then there is an open[resp: αg-open] set V in (X, τ) such that $V\neq\varphi$ and $U\supset V$.

III. Somewhat almost αg-continuous function

Definition 3.1: A function *f* is said to be somewhat almost αg-continuous if for $U\in RO(\sigma)$ and $f^{-1}(U)\neq\varphi$, there exists a non-empty αg-open set V in X such that $V\subset f^{-1}(U)$.

It is clear that every almost continuous function is somewhat almost continuous and every somewhat almost continuous is somewhat almost αg-continuous. But the converses are not true.

Example 1: Let X = {a, b, c}, τ = {φ, {a}, {b, c}, X} and σ = {φ, {a}, X}. The function $f:(X,\tau)\to(X,\sigma)$ defined by *f*(a) = c, *f*(b) = a and *f*(c) = b is somewhat almost αg-continuous, somewhat αg-continuous but not somewhat continuous.

Example 2: Let X = {a, b, c}, τ = {φ, {b, c}, X}, σ = {φ, {b}, {a, c}, X} and η = {φ, {a}, X}. Then the identity functions $f:(X,\tau)\to(X,\sigma)$ and $g:(X,\sigma)\to(X;\eta)$ and $g\bullet f$ are somewhat almost αg-continuous.

However, we have the following

Theorem 3.1: If f is somewhat almost αg-continuous and g is continuous[r-continuous; r-irresolute], then $g\bullet f$ is somewhat almost αg-continuous.

Theorem 3.2: For a surjective function f, the following statements are equivalent:
(i) f is somewhat almost αg-continuous.
(ii) If C is regular closed in Y such that $f^{-1}(C) \neq X$, then there is a proper αg-closed subset D of X such that $f^{-1}(C) \subset D$.
(iii)If M is a αg-dense subset of X, then $f(M)$ is a dense subset of Y.
Proof: (i) ⇒(ii): Let $C\in RC(Y)$ such that $f^{-1}(C) \neq X$. Then $Y-C\in RO(Y)$ such that $f^{-1}(Y-C) = X - f^{-1}(C) \neq \varphi$ By (i), there exists $V\neq \varphi\in \alpha GO(X)$ and $V\subset f^{-1}(Y-C) = X - f^{-1}(C)$. Thus $X-V\supset f^{-1}(C)$ and $X - V = D$ is a proper αg-closed set in X.
(ii) ⇒(i): Let $U\in RO(\sigma)$ and $f^{-1}(U) \neq \varphi$ Then $Y-U\in RC(\sigma)$ and $f^{-1}(Y-U) = X-f^{-1}(U) \neq X$. By (ii), there exists a proper $D\in\alpha GC(X)$ such that $D\supset f^{-1}(Y-U)$. This implies that $X-D \subset f^{-1}(U)$ and X-D is αg-open and $X-D \neq \varphi$.
(ii) ⇒(iii): Let M be a αg-dense set in X. If $f(M)$ is not dense in Y. Then there exists a proper $C\in RC(Y)$ such that $f(M) \subset C\subset Y$. Clearly $f^{-1}(C) \neq X$. By (ii), there exists a proper $D\in\alpha GC(X)$ such that $M\subset f^{-1}(C) \subset D\subset X$. This is a contradiction to the fact that M is αg-dense in X.
(iii) ⇒(ii): If (ii) is not true. there exists $C\in RC(Y)$ such that $f^{-1}(C) \neq X$ but there is no proper $D\in\alpha GC(X)$ such that $f^{-1}(C) \subset D$. Thus $f^{-1}(C)$ is αg-dense in X. But by (iii), $f(f^{-1}(C)) = C$ is dense in Y, which contradicts the choice of C.

Theorem 3.3: Let f be a function and $X = A\cup B$, where $A,B\in RO(X)$. If $f_{/A}: (A; \tau_{/A})\to(Y, \sigma)$ and $f_{/B}:(B; \tau_{/B})\to(Y, \sigma)$ are somewhat almost αg-continuous, then f is somewhat almost αg-continuous.
Proof: Let $U\in RO(\sigma)$ such that $f^{-1}(U) \neq \varphi$. Then $(f_{/A})^{-1}(U) \neq \varphi$ or $(f_{/B})^{-1}(U) \neq \varphi$ or both $(f_{/A})^{-1}(U) \neq \varphi$ and $(f_{/B})^{-1}(U) \neq \varphi$. Suppose $(f_{/A})^{-1}(U) \neq \varphi$, Since $f_{/A}$ is somewhat almost αg-continuous, there exists $V\neq \varphi\in\alpha GO(A)$ and $V\subset (f_{/A})^{-1}(U) \subset f^{-1}(U)$. Since $V\in\alpha GO(A)$ and $A\in RO(X)$, $V\in\alpha GO(X)$. Thus f is somewhat almost αg-continuous. The proof of other cases are similar.

Theorem 3.4: Let $f:(X, \tau)\to (Y, \sigma)$ be a somewhat almost αg-continuous surjection and τ^* be a topology for X, which is αg-equivalent to τ. Then $f:(X, \tau^*)\to (Y, \sigma)$ is somewhat almost αg-continuous.
Proof: Let $V\in RO(\sigma)$ such that $f^{-1}(V) \neq \varphi$. Since f is somewhat almost αg-continuous, there exists $U\neq \varphi\in\alpha GO(X, \tau)$ such that $U\subset f^{-1}(V)$. But by hypothesis τ^* is αg-equivalent to τ. Therefore, there exists $U^*\neq \varphi\in\alpha GO(X; \tau^*)$ such that $U^*\subset U$. But $U\subset f^{-1}(V)$. Then $U^*\subset f^{-1}(V)$; hence $f:(X, \tau^*)\to (Y, \sigma)$ is somewhat almost αg-continuous.

Theorem 3.5: Let $f:(X, \tau)\to (Y, \sigma)$ be a somewhat almost αg-continuous surjection and σ^* be a topology for Y, which is equivalent to σ. Then $f:(X, \tau)\to (Y, \sigma^*)$ is somewhat almost αg-continuous.
Proof: Let $V^*\in RO(\sigma^*)$ such that $f^{-1}(V^*) \neq \varphi$. Since σ^* is equivalent to σ, there exists $V\neq\phi\in RO(Y, \sigma)$ such that $V\subset V^*$. Now $\varphi \neq f^{-1}(V) \subset f^{-1}(V^*)$. Since f is somewhat almost αg-continuous, there exists $U\neq \varphi\in\alpha GO(X, \tau)$ such that $U\subset f^{-1}(V)$. Then $U\subset f^{-1}(V^*)$; hence $f:(X, \tau)\to (Y, \sigma^*)$ is somewhat almost αg-continuous.

IV. Somewhat αg-irresolute function

Definition 4.1: A function f is said to be somewhat αg-irresolute if for $U\in\alpha GO(\sigma)$ and $f^{-1}(U) \neq \varphi$, there exists a non-empty αg-open set V in X such that $V\subset f^{-1}(U)$.

Example3: Let X = {a, b, c}, τ = {φ, {a}, {b, c}, X} and σ = {φ, {a}, {a, b}, X}. The function $f:(X, \tau)\to (X, \sigma)$ defined by $f(a) = c$, $f(b) = a$ and $f(c) = b$ is somewhat αg-irresolute but not somewhat-irresolute.

Example 4: Let X = {a, b, c}, τ = {φ, {a}, {a, b}, X} and σ = {φ, {a}, {b, c}, X}. The function $f:(X, \tau)\to (X, \sigma)$ defined by $f(a) = c$, $f(b) = a$ and $f(c) = b$ is not somewhat αg-irresolute and somewhat-irresolute.

Note 1: Every somewhat αg-irresolute function is slightly αg-irresolute.

Example 5: The identity functions f; g and $g\bullet f$ in Example 2 are somewhat αg-irresolute.

However, we have the following

Theorem 4.1: If f is somewhat αg-irresolute and g is irresolute, then $g\bullet f$ is somewhat αg-irresolute.

Theorem 4.2: For a surjective function f, the following statements are equivalent:
(i) f is somewhat αg-irresolute.
(ii) If C is αg-closed in Y such that $f^{-1}(C) \neq X$, then there is a proper αg-closed subset D of X such that $f^{-1}(C) \subset D$.
(iii)If M is a αg-dense subset of X, then $f(M)$ is a αg-dense subset of Y.
Proof: (i) ⇒(ii): Let $C\in\alpha GC(Y)$ such that $f^{-1}(C) \neq X$. Then $Y-C\in\alpha GO(Y)$ such that $f^{-1}(Y-C) = X - f^{-1}(C) \neq \varphi$ By (i), there exists $V\neq \varphi\in\alpha GO(X)$ and $V\subset f^{-1}(Y-C) = X - f^{-1}(C)$. This means $X-V\supset f^{-1}(C)$ and $X - V = D$ is proper αg-closed in X.

(ii) ⇒(i): Let $U\in\alpha GO(\sigma)$ and $f^{-1}(U) \neq \varphi$ Then $Y\text{-}U\neq \varphi\in\alpha GC(Y)$ and $f^{-1}(Y\text{-}U) = X\text{-}f^{-1}(U) \neq X$. By (ii), there exists $D\neq \varphi\in\alpha GC(X)$ such that $D\supset f^{-1}(Y\text{-}U)$. This implies that $X\text{-}D \subset f^{-1}(U)$ and X-D is αg-open and $X\text{-}D \neq \varphi$.
(ii) ⇒(iii): Let M be a αg-dense set in X. If $f(M)$ is not αg-dense in Y. Then there exists a proper $C\in\alpha GC(Y)$ such that $f(M) \subset C\subset Y$. Clearly $f^{-1}(C) \neq X$. By (ii), there exists a proper $D\in\alpha GC(X)$ such that $M\subset f^{-1}(C) \subset D\subset X$. This is a contradiction to the fact that M is αg-dense in X.
(iii) ⇒(ii): Suppose (ii) is not true. there exists $C\in\alpha GC(Y)$ such that $f^{-1}(C) \neq X$ but there is no proper $D\neq \varphi\in\alpha GC(X)$ such that $f^{-1}(C) \subset D$. This means that $f^{-1}(C)$ is αg-dense in X. But by (iii), $f(f^{-1}(C)) = C$ must be αg-dense in Y, which is a contradiction to the choice of C.

Theorem 4.3: Let f be a function and $X = A\cup B$, where $A,B\in RO(X)$. If $f_{/A}$: $(A; \tau_{/A}) \to (Y, \sigma)$ and $f_{/B}$:$(B; \tau_{/B}) \to (Y, \sigma)$ are somewhat αg-irresolute, then f is somewhat αg-irresolute.
Proof: Let $U\in\alpha GO(\sigma)$ such that $f^{-1}(U) \neq \varphi$. Then $(f_{/A})^{-1}(U) \neq \varphi$ or $(f_{/B})^{-1}(U) \neq \varphi$ or both $(f_{/A})^{-1}(U) \neq \varphi$ and $(f_{/B})^{-1}(U) \neq \varphi$. If $(f_{/A})^{-1}(U) \neq \varphi$, Since $f_{/A}$ is somewhat αg-irresolute, there exists $V\neq \varphi\in\alpha GO(A)$ and $V\subset (f_{/A})^{-1}(U) \subset f^{-1}(U)$. Since $V\in\alpha GO(A)$ and $A\in RO(X)$, $V\in\alpha GO(X)$. Thus f is somewhat αg-irresolute.

The proof of other cases are similar.

If f is the identity function and τ and σ are αg-equivalent. Then f and f^{-1} are somewhat αg-irresolute. Conversely, if the identity function f is somewhat αg-irresolute in both directions, then τ and σ are αg-equivalent.

Theorem 4.4: Let f:$(X, \tau)\to (Y, \sigma)$ be a somewhat αg-irresolute surjection and τ* be a topology for X, which is αg-equivalent to τ. Then f:$(X, \tau^*)\to (Y, \sigma)$ is somewhat αg-irresolute.
Proof: Let $V\in\alpha GO(\sigma)$ such that $f^{-1}(V) \neq \varphi$. Since f is somewhat αg-irresolute, there exists $U\neq \varphi\in\alpha GO(X, \tau)$ with $U\subset f^{-1}(V)$. But for τ* is αg-equivalent to τ, there exists $U^*\neq \varphi\in\alpha GO(X; \tau^*)$ such that $U^*\subset U$. But $U\subset f^{-1}(V)$. Then $U^*\subset f^{-1}(V)$; hence f:$(X, \tau^*)\to (Y, \sigma)$ is somewhat αg-irresolute.

Theorem 4.5: Let f:$(X, \tau)\to (Y, \sigma)$ be a somewhat αg-irresolute surjection and σ* be a topology for Y, which is equivalent to σ. Then f:$(X, \tau)\to (Y, \sigma^*)$ is somewhat αg-irresolute.
Proof: Let $V^*\in\sigma^*$ such that $f^{-1}(V^*) \neq \varphi$. Since σ* is equivalent to σ, there exists $V\neq \varphi\in(Y, \sigma)$ such that $V\subset V^*$. Now $\varphi \neq f^{-1}(V) \subset f^{-1}(V^*)$. Since f is somewhat αg-irresolute, there exists $U\neq \varphi\in\alpha GO(X, \tau)$ such that $U\subset f^{-1}(V)$. Then $U\subset f^{-1}(V^*)$; hence f:$(X, \tau)\to (Y, \sigma^*)$ is somewhat αg-irresolute.

V. Somewhat almost αg-open function

Definition 5.1: A function f is said to be somewhat almost αg-open provided that if $U\in RO(\tau)$ and $U\neq \varphi$, then there exists a non-empty αg-open set V in Y such that $V\subset f(U)$.

Example 6: Let X = {a, b, c}, τ = {φ, {a}, X} and σ = {φ, {a}, {b, c}, X}. The function f: $(X, \tau)\to (X, \sigma)$ defined by $f(a) = a$, $f(b) = c$ and $f(c) = b$ is somewhat almost αg-open, somewhat αg-open and somewhat open.

Theorem 5.1: Let f be r-open and g be somewhat almost αg-open. Then $g\bullet f$ is somewhat almost αg-open.

Theorem 5.2: For a bijective function f, the following are equivalent:
(i) f is somewhat almost αg-open.
(ii) If C is regular closed in X, such that $f(C) \neq Y$, then there is a αg-closed subset D of Y such that $D\neq Y$ and $D\supset f(C)$.
Proof: (i) ⇒(ii): Let $C\in RC(X)$ such that $f(C) \neq Y$. Then $X\text{-}C\neq \varphi\in RO(X)$. Since f is somewhat almost αg-open, there exists $V\neq \varphi\in\alpha GO(Y)$ such that $V\subset f(X\text{-}C)$. Put $D = Y\text{-}V$. Clearly $D\neq \varphi\in\alpha GC(Y)$. If $D = Y$, then $V = \varphi$, which is a contradiction. Since $V\subset f(X\text{-}C)$, $D = Y\text{-}V \supset (Y\text{-} f(X\text{-}C)) = f(C)$.
(ii) ⇒(i): Let $U\neq \varphi\in RO(X)$. Then $C = X\text{-}U\in RC(X)$ and $f(X\text{-}U) = f(C) = Y\text{-} f(U)$ implies $f(C) \neq Y$. Then by (ii), there is $D \neq \varphi\in\alpha GC(Y)$ and $f(C) \subset D$. Clearly $V = Y\text{-}D\neq \varphi\in\alpha GO(Y)$. Also, $V = Y\text{-}D \subset Y\text{-} f(C) = Y\text{-} f(X\text{-}U) = f(U)$.

Theorem 5.3: The following statements are equivalent:
(i) f is somewhat almost αg-open.
(ii)If A is a αg-dense subset of Y, then $f^{-1}(A)$ is a dense subset of X.
Proof: (i) ⇒(ii): Let A be a αg-dense set in Y. If $f^{-1}(A)$ is not dense in X, then there exists $B\in RC(X)$ such that $f^{-1}(A) \subset B\subset X$. Since f is somewhat almost αg-open and $X\text{-}B\in RO(X)$, there exists $C \neq \varphi\in\alpha GO(Y)$ such that $C\subset f(X\text{-}B)$. Therefore, $C\subset f(X\text{-}B) \subset f(f^{-1}(Y\text{-}A)) \subset Y\text{-}A$. That is, $A\subset Y\text{-}C \subset Y$. Now, Y-C is a αg-closed set and $A\subset Y\text{-}C \subset Y$. This implies that A is not a αg-dense set in Y, which is a contradiction. Therefore, $f^{-1}(A)$ is a dense set in X.
(ii) ⇒(i): If $A\neq \varphi\in RO(X)$. We want to show that $\alpha g(f(A))^{o}\neq \varphi$. Suppose $\alpha g(f(A))^{o} = \varphi$. Then, $\alpha g cl\{(f(A))\} = Y$. Then by (ii), $f^{-1}(Y - f(A))$ is dense in X. But $f^{-1}(Y\text{-} f(A)) \subset X\text{-}A$. Now, $X\text{-}A\in RC(X)$. Therefore, $f^{-1}(Y\text{-} f(A)) \subset X\text{-}A$ gives $X = cl\{(f^{-1}(Y\text{-}f(A)))\} \subset X\text{-}A$. Thus $A = \varphi$, which contradicts $A \neq \varphi$. Therefore, $\alpha g(f(A))^{o}\neq \varphi$. Hence f is somewhat almost αg-open.

Theorem 5.4: Let f be somewhat almost αg-open and $A\in RO(X)$. Then $f_{/A}$ is somewhat almost αg-open.
Proof: Let $U\neq \varphi\in RO(\tau_{/A})$. Since $U\in RO(A)$ and $A\in RO(X)$, $U\in RO(X)$ and since f is somewhat almost αg-open, there exists $V\in \alpha GO(Y)$, such that $V\subset f(U)$. Thus $f_{/A}$ is somewhat almost αg-open.

Theorem 5.5: Let f be a function and $X = A\cup B$, where $A,B\in RO(X)$. If $f_{/A}$ and $f_{/B}$ are somewhat almost αg-open, then f is somewhat almost αg-open.
Proof: Let $U\neq \varphi\in RO(X)$. Since $X = A\cup B$, either $A\cap U \neq \varphi$ or $B\cap U \neq \varphi$ or both $A\cap U \neq \varphi$ and $B\cap U \neq \varphi$. Since U is regular open in X, U is regular open in both A and B.
Case (i): If $A\cap U \neq \varphi\in RO(A)$. Since $f_{/A}$ is somewhat almost αg-open, there exists a αg-open set V of Y such that $V\subset f(U\cap A) \subset f(U)$, which implies that f is a somewhat almost αg-open.
Case (ii): If $B\cap U \neq \varphi\in RO(B)$. Since $f_{/B}$ is somewhat almost αg-open, there exists a αg-open set V in Y such that $V\subset f(U\cap B) \subset f(U)$, which implies that f is somewhat almost αg-open.
Case (iii): Suppose that both $A\cap U \neq \varphi$ and $B\cap U \neq \varphi$. Then by case (i) and (ii) f is somewhat almost αg-open.

Remark 1: Two topologies τ and σ for X are said to be αg-equivalent iff the identity function f is somewhat almost αg-open in both directions.

Theorem 5.6: If $f:(X, \tau)\to(Y, \sigma)$ is somewhat almost open. Let τ^* and σ^* be topologies for X and Y, respectively such that τ^* is equivalent to τ and σ^* is αg-equivalent to σ. Then f: $(X; \tau^*)\to (Y; \sigma^*)$ is somewhat almost αg-open.

VI. Somewhat M-αg-open function

Definition 6.1: A function f is said to be somewhat M-αg-open provided that if $U\in \alpha GO(\tau)$ and $U\neq \varphi$, then there exists a non-empty αg-open set V in Y such that $V\subset f(U)$.

Example 7: f as in Example 6 is somewhat M-αg-open.

Theorem 6.1: Let f be r-open and g be somewhat M-αg-open. Then $g\bullet f$ is somewhat M-αg-open.

Theorem 6.2: For a bijective function f, the following are equivalent:
(i) f is somewhat M-αg-open.
(ii) If $C\in \alpha GC(X)$, such that $f(C) \neq Y$, then there is a $D\in \alpha GC(Y)$ such that $D\neq Y$ and $D\supset f(C)$.
Proof: (i) $\Rightarrow$(ii): Let $C\in \alpha GC(X)$ such that $f(C) \neq Y$. Then $X\text{-}C\neq \varphi\in \alpha GO(X)$. Since f is somewhat M-αg-open, there exists $V\neq \varphi\in \alpha GO(Y)$ such that $V\subset f(X\text{-}C)$. Put $D = Y\text{-}V$. Clearly $D\neq \varphi\in \alpha GC(Y)$. If $D = Y$, then $V = \varphi$, which is a contradiction. Since $V\subset f(X\text{-}C)$, $D = Y\text{-}V \supset (Y\text{-} f(X\text{-}C)) = f(C)$.
(ii) $\Rightarrow$(i): Let $U\neq \varphi\in RO(X)$. Then $C = X\text{-}U\in \alpha GC(X)$ and $f(X\text{-}U) = f(C) = Y\text{-} f(U)$ implies $f(C) \neq Y$. Then by (ii), there is $D\in \alpha GC(Y)$ such that $D\neq Y$ and $f(C) \subset D$. Clearly $V = Y\text{-}D\neq \varphi\in \alpha GO(Y)$. Also, $V = Y\text{-}D \subset Y\text{-} f(C) = Y\text{-} f(X\text{-}U) = f(U)$.

Theorem 6.3: The following statements are equivalent:
(i) f is somewhat M-αg-open.
(ii)If A is a αg-dense subset of Y, then $f^{-1}(A)$ is a αg-dense subset of X.
Proof: (i) $\Rightarrow$(ii): Let A be a αg-dense set in Y. If $f^{-1}(A)$ is not αg-dense in X, then there exists $B\in \alpha GC(X)$ in X such that $f^{-1}(A) \subset B\subset X$. Since f is somewhat M-αg-open and X-B is αg-open, there exists a $C\neq \varphi\in \alpha GO(Y)$ such that $C\subset f(X\text{-}B)$. Therefore, $C\subset f(X\text{-}B) \subset f(f^{-1}(Y\text{-}A)) \subset Y\text{-}A$. That is, $A\subset Y\text{-}C \subset Y$. Now, Y-C is a αg-closed set and $A\subset Y\text{-}C \subset Y$. This implies that A is not a αg-dense set in Y, which is a contradiction. Therefore, $f^{-1}(A)$ is a αg-dense set in X.
(ii) $\Rightarrow$(i): Let $A\neq \varphi\in \alpha GO(X)$. To prove $\alpha g(f(A))^{o}\neq \varphi$. Assume $\alpha g(f(A))^{o} = \varphi$. Then, $\alpha g cl\{(f(A))\} = Y$. Then by (ii), $f^{-1}(Y\text{-}f(A))$ is αg-dense in X. But $f^{-1}(Y\text{-} f(A)) \subset X\text{-}A$. Now, $X\text{-}A\in \alpha GC(X)$. Therefore, $f^{-1}(Y\text{-} f(A)) \subset X\text{-}A$ gives $X = cl\{(f^{-1}(Y\text{-}f(A)))\} \subset X\text{-}A$. Thus $A = \varphi$, which contradicts $A\neq \varphi$. Therefore, $\alpha g(f(A))^{o}\neq \varphi$. Hence f is somewhat M-αg-open.

Theorem 6.4: If f is somewhat M-αg-open and A is r-open in X. Then $f_{/A}:(A; \tau_{/A}) \to (Y, \sigma)$ is somewhat M-αg-open.
Proof: Let $U \neq \varphi\in \alpha GO(\tau_{/A})$. Since $U\in \alpha GO(A)$ and $A\in RO(X)$, $U\in RO(X)$ and since f is somewhat M-αg-open, there exists $V\in \alpha GO(Y)$, such that $V\subset f(U)$. Thus $f_{/A}$ is somewhat M-αg-open.

Theorem 6.5: Let f be a function and $X = A\cup B$, where $A,B\in \alpha GO(X)$. If $f_{/A}$ and $f_{/B}$ are somewhat M-αg-open, then f is somewhat M-αg-open.
Proof: Let $U\neq \varphi\in RO(X)$. Since $X = A\cup B$, either $A\cap U \neq \varphi$ or $B\cap U \neq \varphi$ or both $A\cap U \neq \varphi$ and $B\cap U \neq \varphi$. Since U is regular open in X, U is regular open in both A and B.
Case (i): If $A\cap U \neq \varphi\in RO(A)$. Since $f_{/A}$ is somewhat M-αg-open, there exists a αg-open set V of Y such that $V\subset f(U\cap A) \subset f(U)$, which implies that f is a somewhat M-αg-open.
Case (ii): If $B\cap U \neq\varphi\in RO(B)$. Since $f_{/B}$ is somewhat M-αg-open, there exists a αg-open set V in Y such that $V\subset f(U\cap B) \subset f(U)$, which implies that f is somewhat M-αg-open.

Case (iii): If both $A \cap U \neq \varphi$ and $B \cap U \neq \varphi$. Then by case (i) and (ii) f is somewhat M-αg-open.

Remark 2: Two topologies τ and σ for X are said to be αg-equivalent iff the identity function f is somewhat M-αg-open in both directions.

Theorem 6.6: If f:(X, τ)→(Y, σ) is somewhat M-open. Let τ^* and σ^* be topologies for X and Y, respectively such that τ^* is equivalent to τ and σ^* is αg-equivalent to σ. Then f: (X; τ^*)→ (Y; σ^*) is somewhat M-αg-open.

VII. CONCLUSION

In this paper we defined Somewhat-αg-continuous functions, studied its properties and their interrelations with other types of Somewhat-continuous functions.

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