Variable Step Variable Weights Lms (Vsvwlms) Algorithm By Modified Leaky Lms

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Abstract: *This paper proposes a modified leaky least-mean-square (LLMS) algorithm by variable step and variable weight factor (VSVWLMS) that used magnification of error signal. This algorithm uses a variant rate of convergence (ROF) and a variant weight factor for searching in performance surface. In each step the algorithm chooses a new rate of convergence for searching and run computations according to this new rate of convergence. This algorithm at the beginning of convergence uses large value of rate of convergence for increasing speed of convergence. After that by approaching to optimum point the algorithm uses small rate of convergence to give a more accurate response. Also the algorithm uses variant parameter to multiply to the weight vector and new method of varying this parameter in each step. Here the gist of matter refers to using the features of two algorithm, Variable Step Least-Mean-Square (VSLMS) and Leaky LMS which helps to increase the speed of convergence and combining these feature.*

Keywords: *adaptive filter , Leaky LMS, rate of convergence , VSLMS*

I. Introduction

In recent years a growing field of research in adaptive systems has resulted in a variety of adaptive automatons whose characteristics in limited ways resemble certain characteristics of living systems and biological adaptive processes [1].

The essential and principal property of the adaptive system is its time-varying self-adjusting performance. In many instances, however, the complete range of input conditions may not be known exactly, or even statistically; or the condition changes from time to time. In such circumstances, an adaptive system that continually seeks the optimum within an allowed class of possibilities, using an orderly search process, would give superior performance compared with a system of fixed design.

We begin by representing the performance feedback process In the Fig.1. where the input $x_k \epsilon \mathfrak{R}^N$ is a stationary zero-mean vector random process with autocorrelation matrix R $\triangleq$ E [$x_k x_k^T$] for all k, the desired output d_k is a stationary zero-mean scalar random process, $W_k \epsilon \mathfrak{R}^N$ is the weight vector, and k is the time index. The system output at time k is given by $y_k = W_k^T x_k$, and the error ϵ_k is computed via $\epsilon_k = d_k - y_k$.

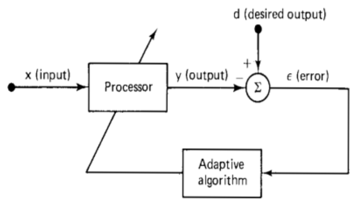

Figure.1 Block diagram of an adaptive filter.

Assume that x_k and d_k are jointly stationary with crosscorrelation vector P $\triangleq$ E $[d_k\, x_k]$ for all k. Define a convex cost function

$$\zeta \triangleq E\,[\,\epsilon_k^2\,] = E\,[d_k^2] - 2P^T W + W^T R W \qquad (1)$$

This cost function is the mean square error (MSE). It can easily be shown that, if R is full rank, the unique optimal fixed weight vector which minimizes ζ is given by

$$W^* = R^{-1}P \qquad (2)$$

This is called the Wiener solution [2].

It is clear from this expression that the mean square error ζ is precisely a quadratic function of the component of the weight vector W when the input component and desired response input are stationary stochastic variables. A portion of a typical two-dimensional mean square error function is illustrated in Fig. 2.

The vertical axis represent the mean square error and the horizontal axes the value of the two weights. The bowl-shaped quadratic error function, or performance surface for in this manner is a paraboloid.

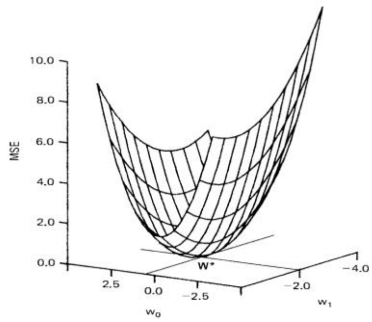

Figure.2 portion of a two dimensional quadratic performance surface. the mean square error is plotted vertically, and the optimum weights vector is $W^* = (0.65,-2.10)$. the minimum MSE is 0.0 in this example.

1.2. Gradient and minimum mean-square error

Many useful adaptive process that cause the weight vector to seek the minimum of the performance surface so by gradient method [1]. The gradient of the mean square error surface, designated ∇(ζ) or simply ∇ , can be obtained by differentiating (1) to obtain the column vector

$$\nabla \triangleq \frac{\partial\zeta}{\partial W} = \left[\frac{\partial\zeta}{\partial W_0}\ \frac{\partial\zeta}{\partial W_1} \cdots \frac{\partial\zeta}{\partial W_L}\right]^T = 2RW - 2P \qquad (3)$$

where R is autocorrelation and P is crosscorrelation. To obtain the minimum square error the weight vector W is set at its optimum value W^*, where the gradient is zero :

$\nabla = 0 = 2RW^* - 2P$ (4)

Assuming that R is nonsingular, the optimum weight vector W^*, sometimes called the wiener weight vector, is found from (9) to be [3-5]

$W^* = R^{-1}P$

II. VSVWLMS algorithm

In fact a filter consists of an electrical circuit or software program that chooses the desired signal from unwanted noise and signals. In an adaptive filter (self-improved filter), the filter modified itself after a while (a few iteration) and learn how to optimize weights to achieve the optimum response called wiener response. Leaky LMS algorithm is one of the algorithms used in adaptive filters

$W(k+1) = \beta W(k) + 2\mu\epsilon(k)X(k)$ (5)

where β is a small number near one and W(k) is weight vector in nth iteration. ε(n) is the error signal and x(n) is the input signal and μ is rate of convergence.

What is discussed in this algorithm represents a new method according to Leaky LMS and VSLMS algorithm to accelerating the speed of the convergence. In this algorithm β and μ are variant and defined like below

$W(k+1) = \beta(k)W(k) + 2\mu(k)\epsilon'(k)X(k)$ (6)

$\beta(k) = e^{-\left(\frac{k}{k+1}\right)}$ (7)

$\mu(k) = \mu(k-1) + 0.01^k\left(A(k)A(k-1)\right)$ (8)

$A(k) = \epsilon'(k)X(k)$ (9)

where ϵ' is the magnificated error sifnal, i.e.

$\epsilon'(k) = 5\epsilon(k)$ (10)

$y(k) = X^T(k)W(k)$ (11)

III. Convergence proof

This section examines the process of mathematical algorithms and its convergence explains. After that by transforming coordinate system to principle coordinate, convergence condition is examined. In the following gradient estimate convergence examined.

$$E[\nabla'(k)] = E\,[-2\epsilon'(k)X(k)]$$
$$= -10\,E\,[d(k)\,X(k) - X(k)\,X^T(k)\,W(k)]$$
$$=10\,(RW-p) = 5\nabla(k) \quad (12)$$

now the convergence of the weights to the optimum weight is discussed.

$$E[w(k+1)] = E[\beta(k)w(k)] + 10\mu E[\epsilon(k)X(k)]$$
$$E[w(k+1)] = e^{-\left(\frac{k}{k+1}\right)}E[w(k)] + 10\mu E[\epsilon(k)X(k)] \quad (13)$$

by substituting ε(n) in the above equation and considering $W^* = R^{-1}P$ it will be

$E[w(k+1)] = (\beta(k) - 10\mu R)E[w(k)] + 10\mu Rw^*$ (14)

by transforming the coordinates to the principle coordinate and using eigen value matrix Λ it will be

$$\Lambda = \begin{pmatrix} \lambda_0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_L \end{pmatrix} \quad (15)$$

$$E[v'] = (\beta(k) - 10\mu\Lambda)^k v'_0 \quad (16)$$

r=geometric ratio=β(k)-10$\mu\Lambda$

v'_0 is the initial weight vector in the principle-axis system. we also note that [4-7]

where v' is the weight vector, W, in the principle-axis system, Λ is the diagonal eigenvalue matrix of R, and

$r \simeq e^{-\frac{1}{\tau}} = 1 - \frac{1}{\tau} + \frac{1}{2\tau^2} - \ldots$ (17)

we have

$\beta(k) - 10\mu\lambda_k \simeq 1 - \frac{1}{\tau}$

therefore

$\tau_k = \frac{1}{10\mu(k)(\lambda_k + \frac{1-\beta(k)}{10\mu(k)})}$ (18)

The geometric ratio of the learning curve , on the other hand [8], [10]

$r_{mse} = r^2$ (19)

$exp(-\frac{1}{\tau_{mse}}) = exp\left(-\frac{2}{\tau}\right) = r^2 = r_{mse}$ (20)

$\tau_{mse} = \frac{\tau}{2}$ (21)

and know that

$M = \frac{excess\ \ MSE}{\zeta_{min}} \simeq \mu\ tr[R]$ (22)

$$tr\,[R] = \sum_{n=0}^{L}\lambda_n$$
$$= \frac{1}{4\mu}\sum_{n=0}^{L}\frac{1}{\tau_{mse}} = \frac{L+1}{4\mu}\left(\frac{1}{\tau_{mse}}\right)_{av}$$

$M \simeq \frac{L+1}{4\tau_{mse}}$ (23)

where L+1 is the total weights, Therefore misadjustment in the VSVWLMS is

$M \simeq (L+1)(5\mu(k))(\lambda_k + \frac{1-\beta(k)}{10\mu(k)})$ (24)

We also note that λmax cannot be greater than the trace of R, which is the sum of the diagonal elements of R, that is [1], [8-12]

$\lambda max \leq tr[\Lambda] = \sum(diagonal\ elements\ of\ \Lambda) = \sum(diagonal\ elements\ of\ R) = tr[R]$

(25)

here the condition of convergence is bellow

$|r|<1$ which yields below equation

$\frac{\beta(k)-1}{10\lambda min} < \mu < \frac{\beta(k)+1}{10\lambda max}$

This disparity increases as the eigenvalue spread $\lambda max/_{\lambda min}$ increases – corresponding to an equivalent increase in the eccentricity of the elliptical contours of constant MSE [13-18]. Thus, the key step in improving the transient performance of LMS lies in decreasing the input eigenvalue spread.

IV. Simulation results

In this section an example which compares the proposed algorithm with the Leaky LMS and VSLMS algorithm is represented. The block diagram of the example is shown

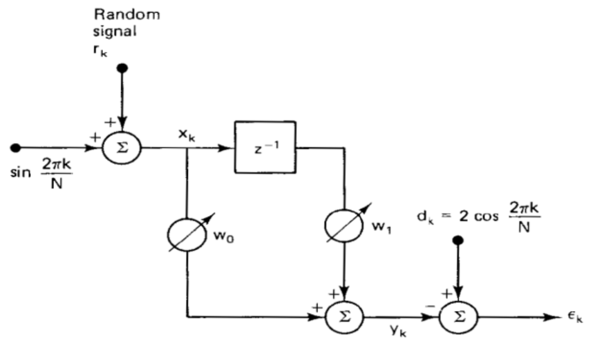

Figure.3Blockdiagramforafilterwithtwoweights.

The simulation result of this example for the same starting point and the identical stage (250 iteration) for the proposed algorithm, Leaky LMS and VSLMS algorithm and the comparison between them is presented in Fig. 4 and Fig. 5.

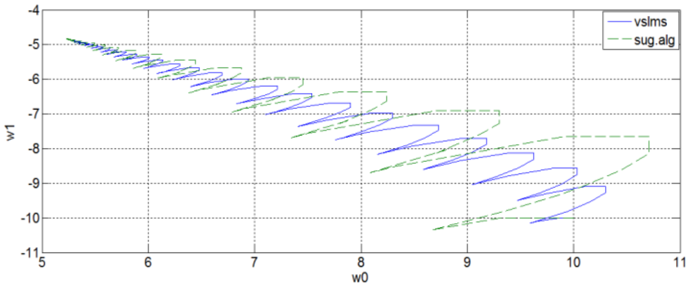

Figure.4 suggested algorithm versus VSLMS.

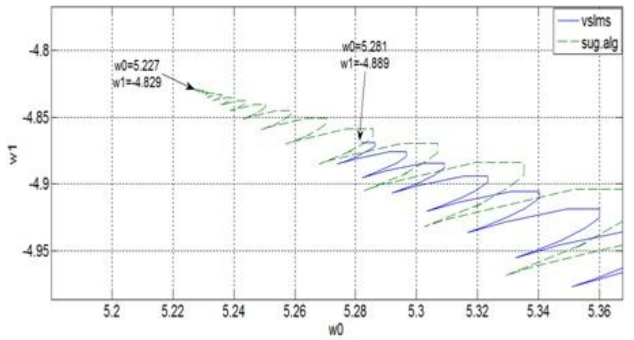

Figure.5 Comparison between the suggested algorithm and the VSLMS in seeking the optimum point with the same iteration

In Fig. 4 it is shown that the suggested algorithm converge to the optimum point with a large steps. It shows that the convergence occur faster. In Fig. 5 which enlarge the last point of the convergence, shown with the same iteration, the response of the proposed algorithm is closer than to the optimum point. In this figure the last point of Leaky LMS is (5.281, -4.889) and the last point of suggested algorithm is (5.227, -4.829) in the same iteration while the optimum point is (4.828, -5.226).

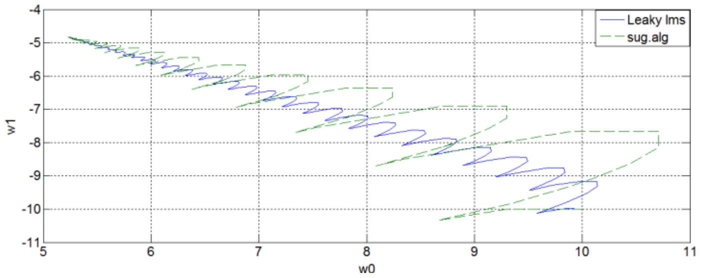

Figure.6 Suggested algorithm versus Leaky LMS.

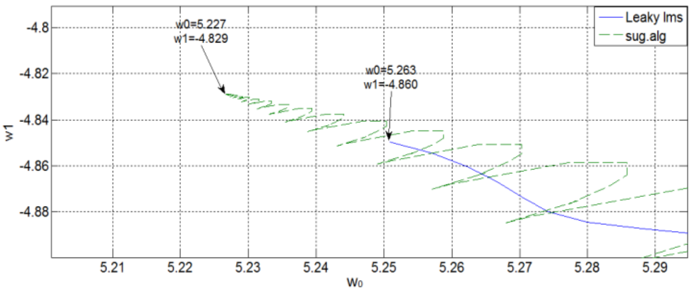

Figure.7 Comparison between the suggested algorithm and the Leaky LMS in seeking the optimum point with the same iteration.

In Fig. 6 it is shown that the suggested algorithm converge to the optimum point with a bigger stage. It shows that the convergence occur faster. In Fig. 7 which enlarge the last point of the convergence, it is shown that with the same iteration, the response of the proposed algorithm is closer than to the optimum point. In this figure the last point of VSLMS is (5.263, -4.860) and the last point of suggested algorithm is (5.227, -4.829) in the same iteration while the optimum point is (4.828, -5.226).

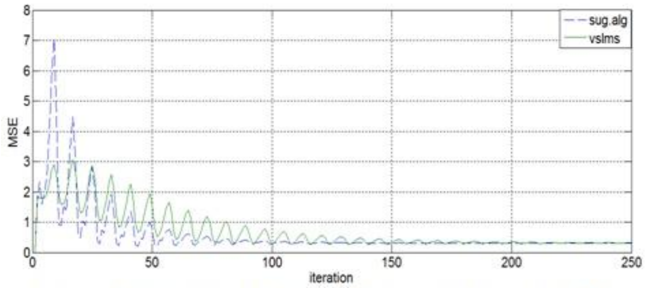

Figure.8 Comparison between the learning curves of the suggested algorithm and VSLMS.

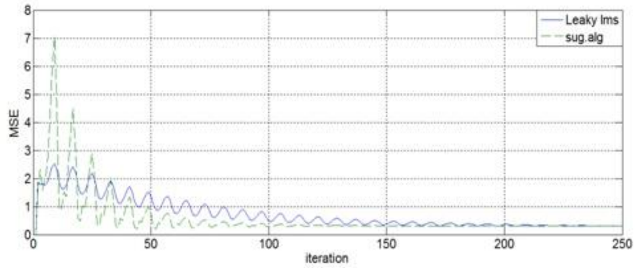

Figure.9 Comparison between learning curves of suggested algorithm and Leaky LMS

In the Figures 8 and 9 the mean-square-error (MSE) versus iteration are illustrated. It is clear that the suggested algorithm at first has a bigger MSE, but after some iteration it will be lower than Leaky LMS and VSLMS algorithm's MSE. It shows that the algorithm in fewer iteration reachs to the desirable accuracy and give us a better response.

V. Conclusion

In this paper a new algorithm for adaptive filter has been introduced which was called VSVWLMS. This algorithm uses the features of Leaky LMS and VSLMS and combined them to accelerate the speed of convergence. In the simulation result it has been observed that in the same iteration VSVWLMS algorithm converge to the optimum point with more speed.

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