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Estimation of Pit Excavation Volume by Fifth Degree Polynomial

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Abstract: In this paper, we derived a new formula using a high degree polynomial (fifth degree curve) in representation ground surface profile. Also, we show the result of calculation of volume by this new formula give a better accuracy because of using high degree curve in representing the ground profile which provides more smoothness in the representation.

Keywords: Pit excavation volume, fifth degree polynomial, ground surface profile

I. Introduction:

How to estimate the volume of earth work in a different engineering and industrial project such as foundation of water tank, underground reservoir and bringing a filled materials from a borrow pit in road construction project is a major task [1, 3]. The standard method has been to construct a grid by dividing the area into rectangles, the elevation at the intersections of the grid lines are measured before and after excavation and used for depth computation at the lattice of the grid then using these lattice values to estimate the volume[2,4] , Fig.1 .

In the past this estimation was done based on that the excavation depth between the grid points is linear .In real situations , this assumption may not be valid and formulas that consider a noun linear excavation depth should be used [4].

In [4] improved the estimation by assuming the excavation depth function follows (approximately) second – or – third – degree polynomials along the two grid directions and using Simpson's rule twice to estimate the integration of this function. This was improvement, since clearly the ground profile (curve representing the excavation depth a long a grid lines) need not be linear [2].

Here in this paper a trial is done to obtain a new formula based on a higher polynomial degree such as fifth degree curve which can represent the excavation depth a long a grid line in both directions better than the linear and second or third degree polynomials representation which were used before.

Materials and methods

Consider Fig.1, in which the ground profile is assumed to follow fifth degree polynomial,

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 \qquad (1)$$

In which $f(x)$ is the excavation depth at point x , (f_0, f_1, f_2 , f_3 , f_4, and f_5 are six excavation depths at the points x_0, x_1, x_2 , x_3 , x_4, and x_5)

The area under the ground profile can be find out as follow ,

$$A = \int_{x_0}^{x_5} (a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5)dx \qquad (2)$$

$$A = [a_0x + \frac{a_1x^2}{2} + \frac{a_2x^3}{3} + \frac{a_3x^4}{4} + \frac{a_4x^5}{5} + \frac{a_5x^6}{6}]_{x_0}^{x_5} \qquad (3)$$

The values of the coefficients of the polynomial $a_0, a_1, a_2, a_3, a_4, a_5$ can be determined by substituting the known coordinates of these points into Eq.1 and solving the resulting six equations.

At $x = x_0 = 0 \rightarrow y = y_0 = f_0 \rightarrow a_0 = f_0$ (4)

At $x = x_1 \rightarrow y_1 = f_1 = f_0 + a_1x_1 + a_2x_1^2 + a_3x_1^3 + a_4x_1^4 + a_5x_1^5$ (5)

At $x = x_2 \rightarrow y_2 = f_2 = f_0 + a_1x_2 + a_2x_2^2 + a_3x_2^3 + a_4x_2^4 + a_5x_2^5$ (6)

At $x = x_3 \rightarrow y_3 = f_3 = f_0 + a_1x_3 + a_2x_3^2 + a_3x_3^3 + a_4x_3^4 + a_5x_3^5$ (7)

At $x = x_4 \rightarrow y_4 = f_4 = f_0 + a_1 x_4 + a_2 x_4^2 + a_3 x_4^3 + a_4 x_4^4 + a_5 x_4^5$ (8)

At $x = x_5 \rightarrow y_5 = f_5 = f_0 + a_1 x_5 + a_2 x_5^2 + a_3 x_5^3 + a_4 x_5^4 + a_5 x_5^5$ (9)

Because of using equal intervals, this led to say that

$h = x_1 - x_0 : 2h = x_2 - x_0 : 3h = x_3 - x_0 : 4h = x_4 - x_0 : 5h = x_5 - x_0$

Solving equation 5,6,7,8 and 9 give the values of coefficients

a_1 , a_2 , a_3 , a_4, and a_5 as follow ,

$$a_1 = \frac{1}{60h}[-137f_0 + 300f_1 - 300f_2 + 200f_3 - 75f_4 + 12f_5]$$

$$a_2 = \frac{1}{24h^2}[45f_0 - 154f_1 + 214f_2 - 156f_3 + 61f_4 - 10f_5]$$

$$a_3 = \frac{1}{24h^3}[-17f_0 + 71f_1 - 118f_2 + 98f_3 - 41f_4 + 7f_5]$$

$$a_4 = \frac{1}{24h^4}[3f_0 - 14f_1 + 26f_2 - 24f_3 + 11f_4 - 2f_5]$$

$$a_5 = \frac{1}{120h^5}[-f_0 + 5f_1 - 10f_2 + 10f_3 - 5f_4 + f_5]$$

Substituting for $a_0, a_1, a_2, a_3, a_4, a_5$ in equation 3 and noting that $x_0 = 0$ and $x_5 = 5h$, this yields the following formula for calculating area under the curve,

$$Area = \frac{5h}{288}[19 f_0 + 75 f_1 + 50 f_2 + 50 f_3 + 75 f_4 + 19 f_5] \quad (10)$$

Now to calculate volume take a unit grid of 5*5 of equal intervals in both direction as in Fig.2 , the volume of this unit grid is given by

$$volume = \int_{x0}^{x5} \int_{y0}^{y5} f(x,y)dydx \quad (11)$$

Where $f_{(x,y)}$ is a depth of excavation at grid nodes

First, the inner integral is calculated using the results in Eq. 10,

$$v = \frac{5h}{288}[\int_{x0}^{x5} 19 f(x,y_0) + 75 f(x,y_1) + 50 f(x,y_2) + 50 f(x,y_3) + 75 f(x,y_4) + 19 f(x,y_5)]dx \quad (12)$$

$$v = \frac{5h}{288}[19 \int_{x0}^{x5} f(x,y_0)dx + 75 \int_{x0}^{x5} f(x,y_1)dx + 50 \int_{x0}^{x5} f(x,y_2)dx + 50 \int_{x0}^{x5} f(x,y_3)dx + 75 \int_{x0}^{x5} f(x,y_4)dx + 19 \int_{x0}^{x5} f(x,y_5)dx] \quad (13)$$

use the results in equation number 10 again to each integral term in Eq.13 to obtained ,

$$v = (\frac{25h}{1440})^2 \gamma_{ij} * f_{ij} \quad (14)$$

Where γ_{ij} are the corresponding elements of the following matrix :

$$C = \begin{pmatrix} 361 & 1425 & 950 & 950 & 1425 & 361 \\ 1425 & 5625 & 3750 & 3750 & 5625 & 1425 \\ 950 & 3750 & 2500 & 2500 & 3750 & 950 \\ 950 & 3750 & 2500 & 2500 & 3750 & 950 \\ 1425 & 5625 & 3750 & 3750 & 5625 & 1425 \\ 361 & 1425 & 950 & 950 & 1425 & 361 \end{pmatrix}$$

And f_{ij} are the corresponding elements of the following matrix which represent depth of excavation at Grid nodes

$$F = \begin{pmatrix} f_{0,5} & f_{1,5} & f_{2,5} & f_{3,5} & f_{4,5} & f_{5,5} \\ f_{0,4} & f_{1,4} & f_{2,4} & f_{3,4} & f_{4,4} & f_{5,4} \\ f_{0,3} & f_{1,3} & f_{2,3} & f_{3,3} & f_{4,3} & f_{5,3} \\ f_{0,2} & f_{1,2} & f_{2,2} & f_{3,2} & f_{4,2} & f_{5,2} \\ f_{0,1} & f_{1,1} & f_{2,1} & f_{3,1} & f_{4,1} & f_{5,1} \\ f_{0,0} & f_{1,0} & f_{2,0} & f_{3,0} & f_{4,0} & f_{5,0} \end{pmatrix}$$

Equation 14 is a single formula for volume calculation for a grid unit of 5*5 of equal intervals in both directions in terms of the intersection points.

To calculate the total volume of any grid capacity which must be multiple of 5 in both direction is the sum of volumes of the unit grids . To calculate total volume let,

$x_i = x_0 + ih$; i=0,1 , …….. m.

$y_j = y_0 + jh$; j=0,1,……….n.

Then the composite formula for calculating the volume of the total grid, V, is given by ,

$$V = \int_{x_0}^{x_m} \int_{y_0}^{y_n} f(x, y) dy dx$$

$$V = \left(\frac{25h}{1440}\right)^2 \sum_{i=0}^{m} \sum_{j=0}^{n} \gamma_{ij} f_{ij} \quad \text{(Composite formula)} \qquad (15)$$

In which γ_{ij} =the corresponding elements of the following matrix

$$C = \begin{pmatrix}
361 & 1425 & 950 & 950 & 1425 & 722 & 1425 & . & . & 1425 & 722 & 1425 & 950 & 950 & 1425 & 361 \\
1425 & 5626 & 3750 & 3750 & 5625 & 2850 & 5625 & . & . & 5625 & 2850 & 5625 & 3750 & 3750 & 5625 & 1425 \\
950 & 3750 & 2500 & 2500 & 3750 & 1900 & 3750 & . & . & 3750 & 1900 & 3750 & 2500 & 2500 & 3750 & 950 \\
950 & 3750 & 2500 & 2500 & 3750 & 1900 & 3750 & . & . & 3750 & 1900 & 3750 & 2500 & 2500 & 3750 & 950 \\
1425 & 5625 & 3750 & 3750 & 5625 & 2850 & 5625 & . & . & 5625 & 2850 & 5625 & 3750 & 3750 & 5625 & 1425 \\
722 & 2850 & 1900 & 1900 & 2850 & 1444 & 2850 & . & . & 2850 & 1444 & 2850 & 1900 & 1900 & 2850 & 722 \\
1425 & 5625 & 3750 & 3750 & 5625 & 2850 & 5625 & . & . & 5625 & 5625 & 5625 & 3750 & 3750 & 5625 & 1425 \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
1425 & 5625 & 3750 & 3750 & 5625 & 2850 & 5625 & . & . & 5625 & 5625 & 5625 & 3750 & 3750 & 5625 & 1425 \\
722 & 2850 & 1900 & 1900 & 2850 & 1444 & 2850 & . & . & 2850 & 2850 & 2850 & 1900 & 1900 & 2850 & 722 \\
1425 & 5625 & 3750 & 3750 & 5625 & 2850 & 5625 & . & . & 5625 & 5625 & 5625 & 3750 & 3750 & 5625 & 1425 \\
950 & 3750 & 2500 & 2500 & 3750 & 1900 & 3750 & . & . & 3750 & 3750 & 3750 & 2500 & 2500 & 3750 & 950 \\
950 & 3750 & 2500 & 2500 & 3750 & 1900 & 3750 & . & . & 3750 & 3750 & 3750 & 2500 & 2500 & 3750 & 950 \\
1425 & 5625 & 3750 & 3750 & 5625 & 2850 & 5625 & . & . & 5625 & 5625 & 5625 & 3750 & 3750 & 5625 & 1425 \\
361 & 1425 & 950 & 950 & 1425 & 722 & 1425 & . & . & 1425 & 1425 & 1425 & 950 & 950 & 1425 & 361
\end{pmatrix} ..$$

$f_{i,j}$ = Depth of grid point with coordinates ($x_{i,} y_i$)

The matrix of Eq.15 corresponds to the grid points in Fig.3. This with the note that the second , third , fourth and fifth columns are to be repeated , as well as the second , third , fourth and fifth rows) . This, to calculate V, we only need to multiply each depth $f_{i,j}$ by the corresponding elements and sum the results for all points. The elements of the matrix C can be implemented easily in a computer program because they exhibit a specific pattern.
Eq.15 is for square grid with equal interval in both direction, if the gird is rectangular grid with h distance between grid nodes in x direction and with k distance in y direction, the equation became,

$$V = \left(\frac{25}{1440}\right)^2 * h * k \sum_{i=0}^{m} \sum_{j=0}^{n} \gamma_{ij} f_{ij}$$ Composite equations for rectangle grid (16)

NUMERICAL EXAMPLE:

Consider the unit grid of Fig.4, in which m=1, n=1, h = 10 m. The excavation depths (in meter) are shown beside the grid intersection points, let to calculate the volume as follow;-

A) Classical method which is based on a linear relationship between the depth ends which represent the earth surface in both direction,

$$V[3] = \frac{A_s}{4}\left(\sum h_1 + 2\sum h_2 + 3\sum h_3 + 4\sum h_4\right)$$

Where $\sum h_1$ = sum of depth used once

$2\sum h_2$ = sum of depth used twice

$3\sum h_3$ = sum of depth used thrice

$4\sum h_4$ = sum of depth used four times

A_s= area of one square in a grid

$\sum h_1$ = 0.70 + 0.14 + 1.30 + 1.0 = 3.14 m.

$2\sum h_2$ = 2(1.10+1.32+0.87+0.25 + 1.14+2.14+1.69+0.45 + 1.4+1.35+0.89+0.75 +1.25+2.09+0.79+0.75)=36.46 m.

$3\sum h_3$ =0

$4\sum h_4$ =4(2.17+1.25+0.98+0.18+3.12+2.92+1.75+1.55+1.75+2.03+0.99+1.35+1.69+1.10+1.62+2.11)= 106.24m.

Volume = $\frac{100}{4}$ (3.14+36.46+106.24)= 3646 m^3

B) Volume calculation based on the obtained equation 15,

$$volume = \left(\frac{25h}{1440}\right)^2 \sum_{i=0}^{m} \sum_{j=0}^{n} \gamma_{ij} f_{ij}$$

$$V = \left(\frac{25*10}{1440}\right)^2 \begin{pmatrix} 361 & 1425 & 950 & 950 & 1425 & 361 \\ 1425 & 5625 & 3750 & 3750 & 5625 & 1425 \\ 950 & 3750 & 2500 & 2500 & 3750 & 950 \\ 950 & 3750 & 2500 & 2500 & 3750 & 950 \\ 1425 & 5625 & 3750 & 3750 & 5625 & 1425 \\ 361 & 1425 & 950 & 950 & 1425 & 361 \end{pmatrix} * \begin{pmatrix} 0.70 & 1.10 & 1.32 & 0.87 & 0.25 & 0.14 \\ 1.14 & 2.17 & 1.25 & 0.98 & 0.18 & 1.25 \\ 2.14 & 3.12 & 2.92 & 1.75 & 1.55 & 2.09 \\ 1.69 & 1.75 & 2.03 & 0.99 & 1.35 & 0.79 \\ 0.45 & 1.65 & 1.10 & 1.62 & 2.11 & 0.75 \\ 1.00 & 1.40 & 1.35 & 0.89 & 0.75 & 1.30 \end{pmatrix}$$

Volume = 3710.743 m^3

II. Conclusions

1) the obtained formula for volume computation here on this paper is based on the assumption that the ground profile is nonlinear with fifth –degree polynomials
2) the obtained formula of course provide a better accuracy than formula that assumes a linear or second, or third-degree polynomials
3) the limitation here is that the grid must be equal to the interval in both direction and total number of interval must be equal to 5 or multiple of five
4) using this formula need computer programming

Notation:-

A = area of region with irregular boundary
As = area of a single square in a used grid
$a_0, a_1, a_2, a_3, a_4, a_5$ = coefficients of used fifth –degree polynomial
C = matrix of elements
dx = width of increment in x- direction
dy = width of increment in y-direction
f_0, f_1, f_2, f_3, f_4, and f_5 = six excavation depths at the points x_0, x_1, x_2, x_3, x_4, and x_5)
$f_{i,j}$ = depth of grid point with coordinates (x_i, y_j)
$f(x)$ = depth of point at horizontal distance x
$f(x,y)$ = depth of point with coordinates (x , y)
h = distance between grid points in both x and y direction in the case of square grid

h_1 = depth used once

h_2 = depth used twice
h_3 = depth used thrice
h_4 = depth used four times
i = index for grid points in x-direction
j = index for grid points in y-direction
k = distance between grid points in y direction in the case of rectangle grid
m = number of grid intervals in x- direction
n = number of grid intervals in y-direction
V = volume of total grid
v = volume of unit grid of 5*5 equal intervals
γ_{ij} = the corresponding elements of the following matrix C

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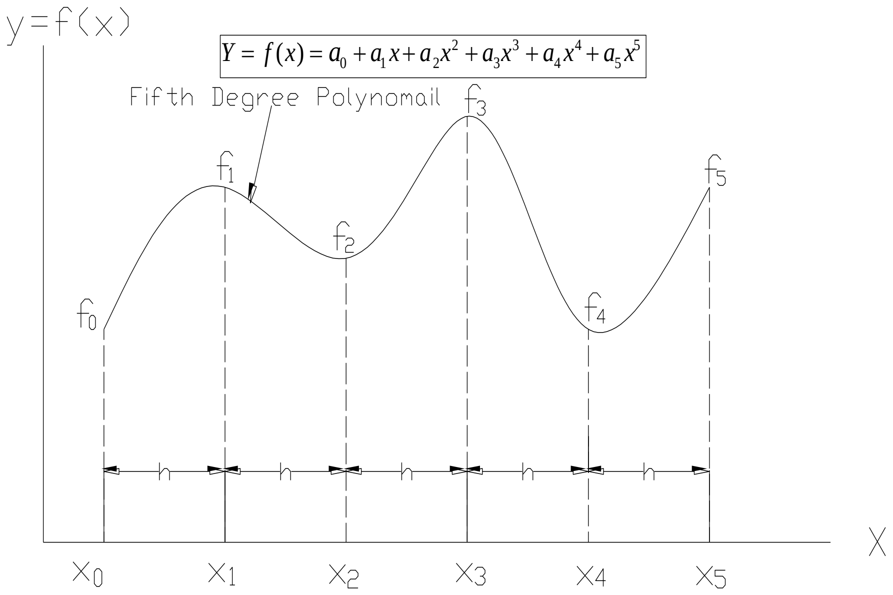

Fig-1-

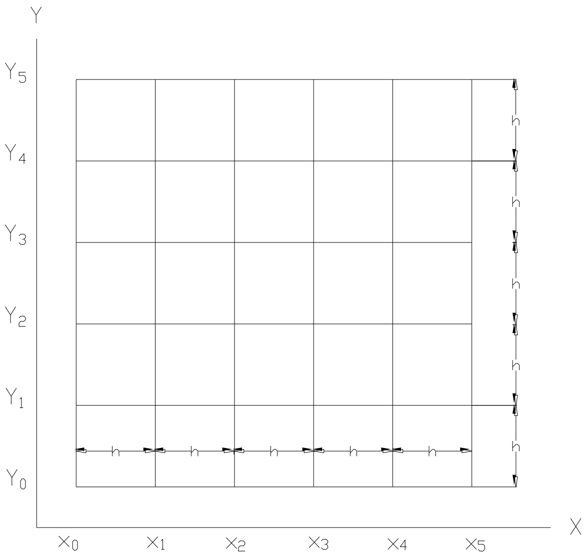

Fig-2-: 5*5 Square Grid with Equal Intervals

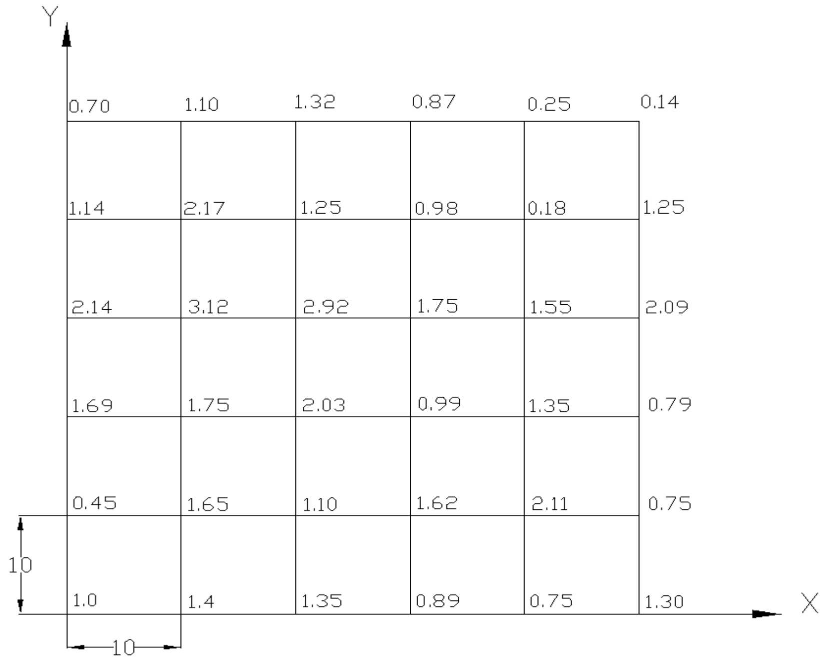

Fig-3-:Grid for Numerical Example,
Depth are in Meters