Study on the Effect of Stress Concentration on Cutout Orientation of Plates with Various Cutouts and Bluntness

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Abstract: *Plates with variously shaped cut-out are often used in engineering structures. The understanding of the effect of cut-out on the load bearing capacity and stress concentration of panels is very important in designing of structures. Different cut-out shapes in structural elements are needed to reduce the weight of the structure or provide access to other parts of the structure. Extensive studies have been carried out on stress concentration in perforated panels which consider cut-out shapes, boundary conditions and bluntness of cut-outs. This study focuses on the stress concentration analysis of perforated panels with not only various cut-outs and bluntness but also different cut-out orientations. Therefore, at the design stage, once the direction of a major tensile force is known, the cut-outs can be aligned properly based on the findings of the work to reduce the stress concentration at the cut-outs thereby increasing the load bearing capacity of the panel.*

Keywords: *Bluntness, Cut-out, Load Bearing capacity, Orientation, Stress concentration factor, etc*

1. Introduction

Plates and shells of various constructions find wide uses as primary structural elements in aerospace, mechanical and civil engineering structures. In recent years, the increasing need for lightweight efficient structures has led to structural shape optimization. Different cut-out shapes in structural elements are needed to reduce the weight of the system and provide access to other parts of the structure. It is well known that the presence of a cut-out or hole in a stressed member creates highly localized stresses at the vicinity of the cut-out. The ratio of the maximum stress at the cut-out edge to the nominal stress is called the stress concentration factor (SCF). The understanding of the effects of cut-out on the load bearing capacity and stress concentration of such plates is very important in designing of structures.

The study of the importance of SCF in isotropic plates is well established. Previous works on stress concentration presented a series solution for stress field around circular holes in plates with arbitrary thickness [1]. A wide range of holes diameters to plate thickness was presented. Also Schwarz–Christoffel transformation was used to evaluate the stress concentration factor for an infinite plate with central triangular cut-out [2]. Stress and strain distributions along the boundary of rectangular cut-out in an infinite elastic plate were presented [2]. The relaxation element method was used to determine the stress fields in a plate with three circular cut-outs subjected to uni-axial tensile load [3]. And numerical results based on generalized work–energy method for rectangular plates with circular cut-out and circular plates with a rectangular cut-out was presented [4]. Ultimate strength of metallic plates with central circular cut-out under shear loading was also investigated [5]. The bluntness effects on stress concentration in perforated composite plates were also presented [6]. Optimum design of holes and notches by considering fatigue life were presented [7]. For a variety of materials, for various geometry of notches and fillets, stress concentration factor was presented [8].

However, it seems to be difficult to locate a work that quantifies the rotation effect of polygonal cut-outs on stress concentration. Therefore, this study mainly focuses on stress concentration analyses of aluminium plates according to cut-out orientation. Therefore, this study mainly focuses on stress concentration analyses of perforated aluminium plates with not only various cut outs (circle, triangle, and square) and bluntness (a counter measure of radius ratio, *r/R*) but also for different cut-out orientation (θ = 15, 30, 45). For the analyses, first, we select three different cut-outs: circle, triangle, and square; secondly, we identify a number of degrees of bluntness to describe the radius ratio; and finally, we consider the rotation of cut-outs. In the paper, stress concentration analyses are performed by, a general using MSC Patran & MSC Nastran, a general purpose finite element program. From the the analysis we estimate the stress concentration of plates with various cut-out shapes, bluntness and orientation.

2. Finite Element Model

Finite element analyses are conducted for the stress concentration analyses of perforated aluminium plates. The structural aluminium plates have dimensions 200 mm (x-direction), 200 mm (y-direction), and 5 mm (z-direction) as shown in Fig.1. Material properties are shown in Table 1 and the location of cut-out is the centre of the plates. To clearly observe the concentration effect, the plate size is modeled as rather large for the cut-out size. MSC NASTRAN, a general purpose finite element program, is used for the analysis. A 4-node shell element is used for modelling. To investigate stress concentration in an elastic range, the plates are modeled as a linear elastic material. The loading condition is a uni-axial tensile force at the left and right sides as shown in Fig.1. Based on Rezaeepazhand and Jafari (2005), stress concentration reaches up to eleven times, depending on cut-out shapes; hence, in the study, to limit the maximum stress to the elastic range, 20 MPa is loaded as the tensile loading condition. Since element size is critical for precise analysis, in the study, the size is 2 mm in most parts and 0.5 mm near the cut-out areas.

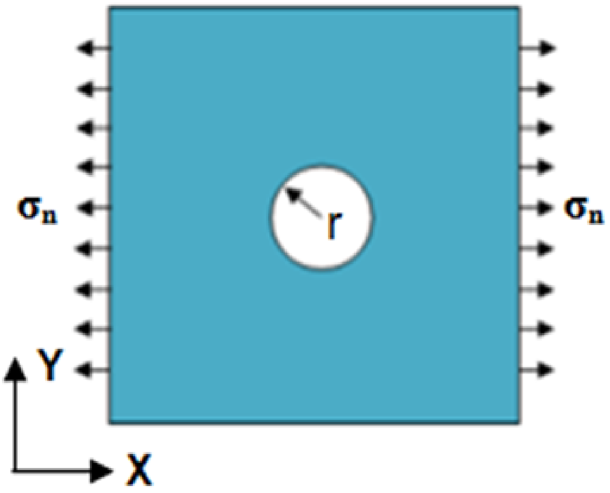

Figure 1: Loading condition: uni-axial tensile force

2.1 Material Properties

Table 2: Material properties of Aluminium 2024-T3

Material Properties	Values
Young's Modulus, (GPa)	73.1
Poisson ratio, μ	0.33
Tensile yield strength (MPa)	345
Tensile Ultimate strength (MPa)	483

3. Cut-Out Shapes, Bluntness and Rotation

We consider three cut-out shapes – circle, square, and regular triangle. For the square and triangle cut-outs the concept of inscribing circle is used, as shown in Figs.2 and 3, to compare with the corresponding circular cut-out. In the figures, the solid-lined circles are the inscribing circles in the polygons. The radius size of the circular cut-out is 10 mm. In general, to reduce the stress concentration at the edges of cut-outs, the edges are fabricated to be rounded. In the study, rather than 'roundness', we use 'bluntness' as a physical terminology to effectively describe stress concentration. As shown in Fig.4, a term 'radius ratio' is defined as the ratio of the edge radius (r) to the inscribing circle radius (R).

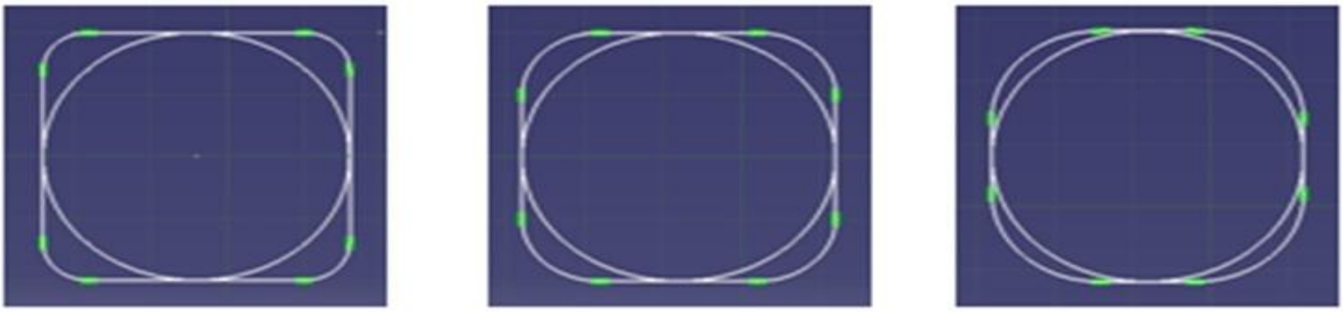

Figure 2: Square cut-out with *r/R* = 0.3(left), 0.5(centre), 0.7(right)

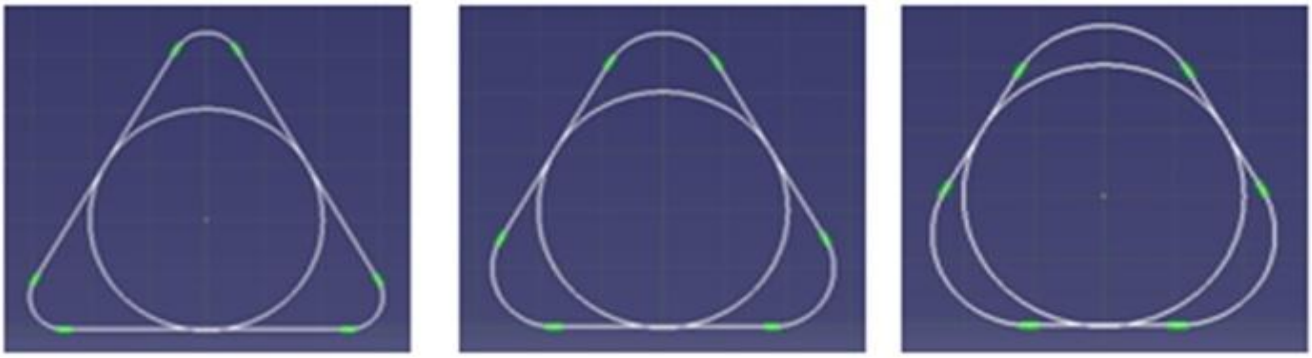

Figure 3: Triangle cut-out wit *r/R* = 0.3(left), 0.5(centre), 0.7(right)

Accordingly, bluntness is a counter measure to the radius ratio (r/R) because bluntness decreases as the radius ratio increases. For an extreme example, a circular cut-out has a unit radius ratio but it has zero bluntness. In other words, the degree of bluntness decreases as r/R increases. Here, again, we emphasize that the term 'bluntness' is used to describe that the edges of polygons are blunt. We consider a total of six different degrees of bluntness, including 0.1, 0.3, 0.5, 0.7, 0.9, and 1.0 for the polygon cut-outs. Figs.2 and 3 only show three of the six cases for the square and triangle cut-outs.

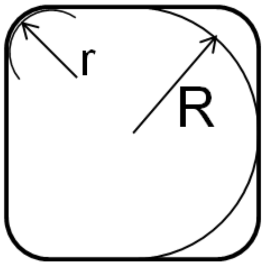

Figure 4: Radius ratio (r/R) defined by edge radius (r) and Inscribing circle radius (R)

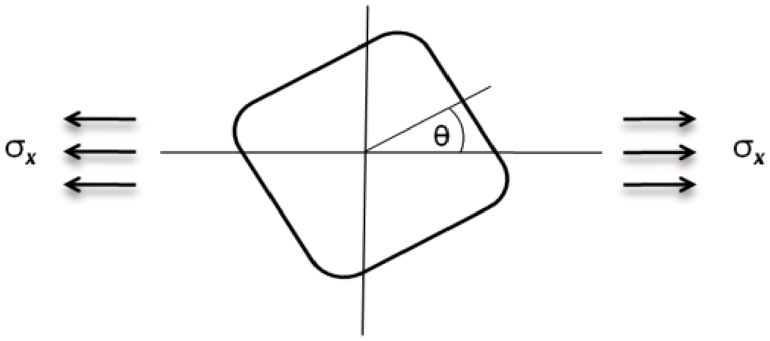

Figure 5: Rotation of cut-out

In addition to the shapes and bluntness, the last design consideration for cut-out patterns is orientation. Fig.5 shows the definition of orientation. The rotation angle θ represents how the cut-outs are oriented from the baseline (+x axis). As shown in the figure, the loading directions are fixed as they are. Fig.6 shows a number of parts of the rotated cut-outs for each case. By considering the symmetry of the polygonal cut-outs, the angle increment 15° is applied; hence, a total of three cases are considered (15°, 30° and 45°) for the square cut-outs and three cases (15°, 30° and 45°) for the triangular cut-outs.

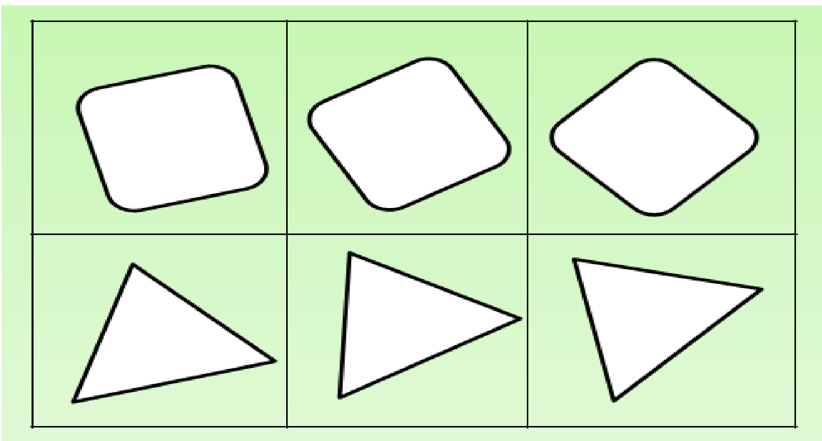

Figure 6: Square and Triangular cut-out with □ = 15° (left), 30° (centre), and □=45° (right

4. Results

By considering the design variables or factors – cut-out shape, the degree of bluntness, and cut-out rotation – the stress concentration pattern, the maximum von-Mises stress and the stress concentration factor are obtained. These results are as shown in the following sections.

4.1 Cut-Out Shapes and Bluntness

As mentioned previously, there are three different cut-out shapes – circle, square, and triangle. In addition, for considering bluntness (a counter measure of r/R), a total of six radius ratios are considered: r/R = 0.1, 0.3, 0.5, 0.7, 0.9 and 1.0 respectively. This section discusses the variation of stress concentration with respect to the cut-out shapes and bluntness. All of the other factors remain the same, for example the uni-axial tensile forces are fixed at 20 MPa.

Table 2: The maximum von-Mises stress and stress concentration factor

r/R	Triangle		Square	
	σ_{max}, MPa	SCF	σ_{max}, MPa	SCF
0.1	178	8.9	89.6	4.48
0.3	115	5.75	70	3.5
0.5	93.1	4.65	60.1	3
0.7	78.7	3.9	61	3.05
0.9	68	3.4	58.7	2.93
1	64.3	3.2	64.3	3.2

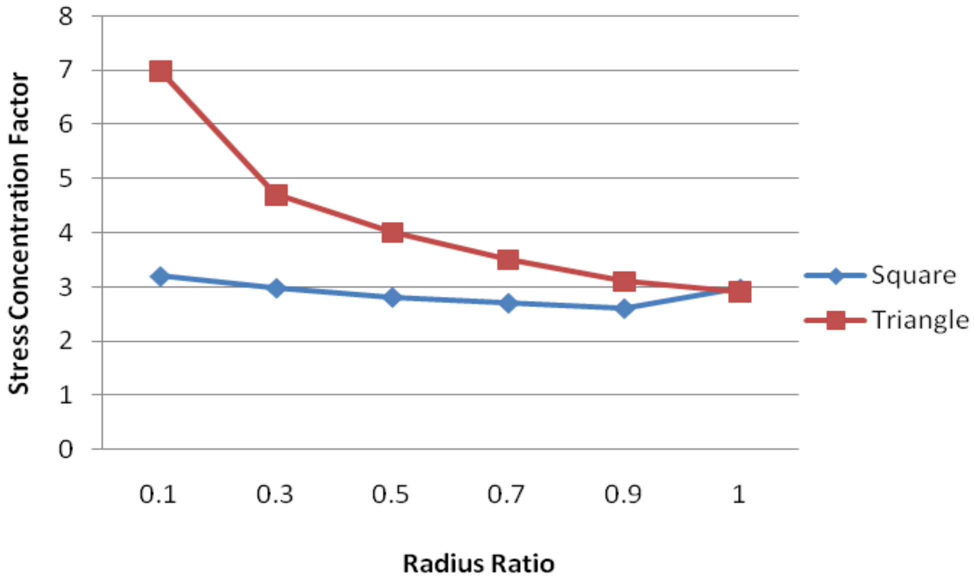

Figure 7: SCF with respect to radius ratio

It should be noted here that the zero bluntness (r/R = 1) actually means that the cut-out shape is a circle; hence, from the Table 2, we can see how the shapes and the degrees of bluntness vary the maximum von-mises stress and stress concentration factor. Fig.7 shows how the stress concentration factor (SCF) varies with respect to cut-out shapes and the radius ratio (a counter measure of degree of bluntness).

In the case of the circular cut-out, the maximum stress is 64.3 MPa and the stress concentration factor is 3.2. According to previous studies, the maximum stress is about three times the tensile force [8]. Since our tensile force is 20 MPa, the magnitude of 64.3 MPa exactly concurs with the previous observation. As shown in Table 2, the maximum von-Mises stresses and accordingly stress concentration factors change, depending on the cut-out shapes and bluntness.

In the case of the square cut-outs, although the quantities range between 89.6 and 64.3 MPa, they do not significantly differ from 60.26 MPa, which is the maximum von-Mises stress that occurred in the circularly-perforated aluminium plate. It is interesting to note that: (1) the stresses for r/R = 0.5, 0.7, and 0.9 are smaller than that of r/R = 1.0 which is the circular cut-out case, and (2) the maximum stress (89.6 MPa) occurs in the case of r/R = 0.1.

In the case of the triangular cut-outs, the results are quite consistent because: (1) all the stresses exceed that of the circular cut-out case, and (2) unlike the square cases, starting from the maximum stress (178 MPa) the stresses decrease as the degrees of radius ratio increases. In other words, the stresses increase as the degree of bluntness increases.

To visualize the stress patterns, two stress contours are shown in Figs.9 and 10. Fig.9 shows the stress contour in the case of the square cut-out with r/R = 0.1. The circle on the contour indicates the area having the maximum von-Mises stress. In addition, the left and right balloon shapes represent the areas under 11 MPa. Fig.10 shows the stress contour in the case of the triangle cut-out with r/R =0.1. The circle on the contour shows the area having the maximum von-Mises stress. Similarly, the left and right balloon shapes represent the area under 16 MPa. It is interesting to note that stress concentration occurs in the broad range of the top and bottom sides in the case of the square cut-out while stress concentration occurs in the narrow range of the top and bottom edges in the case of the triangle cut-out. From the observation, we can conclude that the bluntness effect on the stress concentration patterns is also dependent on cut-out shapes. However, in general, as bluntness increases, stress concentration increases.

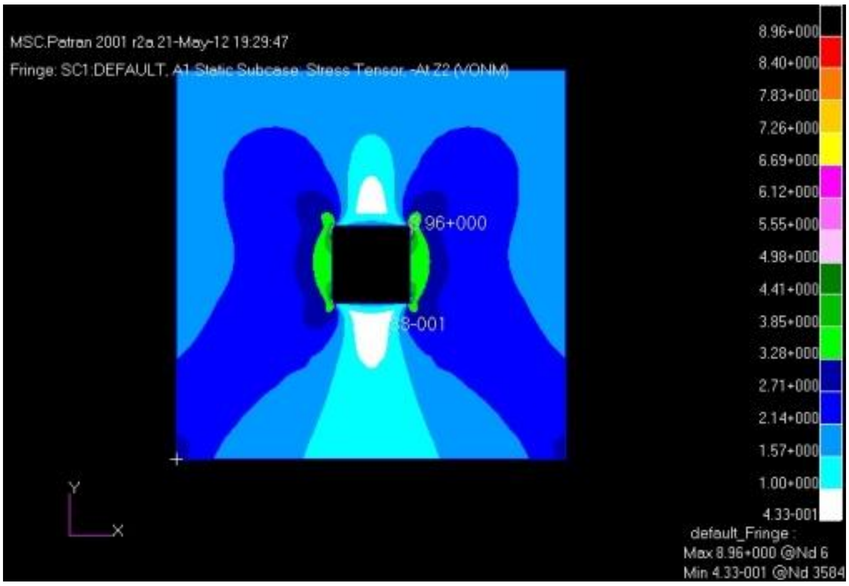

Figure 9: Stress contour of plate with square cut-out (r/R = 0.1)

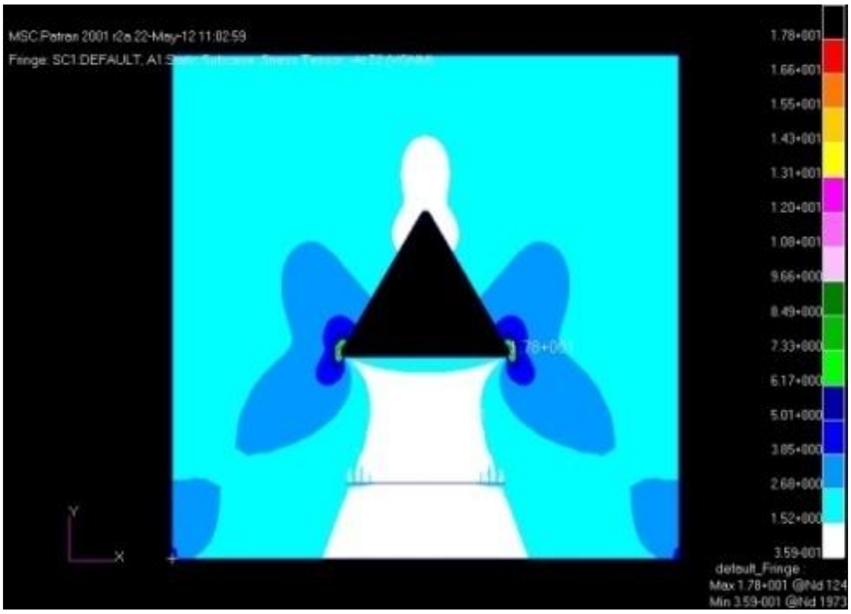

Figure 10: Stress contour of plate with triangle cut-out (r/R = 0.1)

4.2 Rotation of cut-outs

This section discusses the stress analysis results by considering the rotation of the cut-outs. In the cases of the square cut-out, three rotation angles are considered 15°, 30° and 45°, while three angles 15°, 30° and 45°, are considered in the case of the triangle cut-out. Table 3 shows the maximum von-Mises stresses and stress concentration factors for the aluminium plates with square cut-outs, which have the four rotations. As a result, we can see that many differences occur in the maximum stresses, depending on the rotation angle. However, for all of the cases consistently, the stresses increase as the rotation angles increase. By combining the rotation effect with the bluntness effect, the maximum stress (141 MPa) occurs in the case of the r/R = 0.1 (maximum bluntness) and the rotation of 45° (maximum rotation). In addition, we can see that with the exception of the zero rotation case, all the cases show that the maximum stress increases as the bluntness increases, as shown in Fig.11. Table 4 shows the maximum von-Mises stresses and stress concentration factors for the aluminium plates with triangle cut-outs, which have the three rotations. For all of the cases, the stresses increase as the rotation angles increase, as clearly shown in Fig.12. With both effects of the rotation angle and bluntness, the maximum stress (166 MPa) occurs in the case of the bluntness of r/R = 0.1 (maximum bluntness) and the rotation of 30°. Fig.13 shows the stress contour in the case of the square cut-out with r/R = 0.1 and rotation 45°, which gives the maximum stress. This figure represents different patterns from that of Fig. 9 showing the case of 0°. The maximum stress concentration occurs in the top and bottom edges. Fig.14 shows the stress contour in the case of the triangle cut-out with r/R = 0.1 and rotation 30°, which also gives the maximum stress. The maximum stress concentration occurs in the top edge.

Table 3: Maximum von-Mises stress and stress concentration factor (SCF) of square cut-outs with rotation angle

r/R	0^o (MPa)	15^o (MPa)	30^o (MPa)	45^o (MPa)
0.1	74	110	134	141
0.3	59.1	86.4	99.1	103
0.5	56.3	75	82.5	84.5
0.7	54	68	72	73.2
0.9	51.9	63.6	64.8	65.1
1	59.5	59.5	59.5	59.5

r/R	0^o (SCF)	15^o (SCF)	30^o (SCF)	45^o (SCF)
0.1	3.7	5.5	6.7	7
0.3	2.9	4.3	4.95	5.1
0.5	2.8	3.7	4.1	4.2
0.7	2.7	3.4	3.6	3.6
0.9	2.6	3.1	3.2	3.25
1	2.97	2.97	2.97	2.97

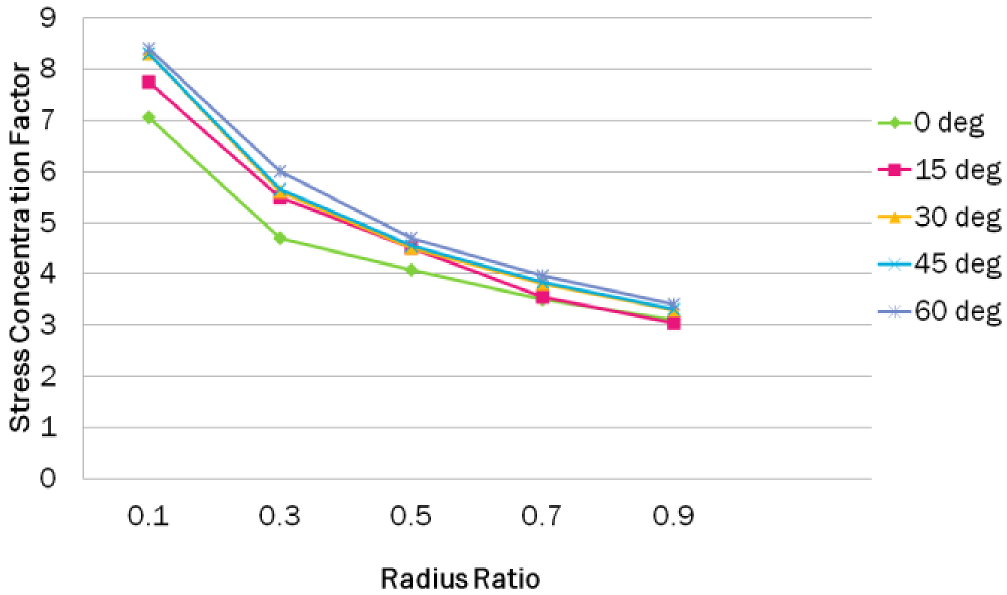

Figure 11: SCF with respect to rotation for square cut-outs

Table 4: Maximum von-Mises stress and stress concentration factor (SCF) of triangular cut-outs with rotation angle

r/R	0^o (SCF)	15^o (SCF)	30^o (SCF)	45^o (SCF)
0.1	141	155	166	166
0.3	95.5	110	112	113
0.5	81.5	89.7	90.5	91.1
0.7	69.8	71.1	76.5	76.8
0.9	62.1	60.8	66.3	66.3
1	59.5	59.5	59.5	59.5
r/R	**0^o (SCF)**	**15^o (SCF)**	**30^o (SCF)**	**45^o (SCF)**
0.1	7	7.7	8.3	8.3
0.3	4.7	5.5	5.6	5.65
0.5	4	4.5	4.5	4.55
0.7	3.5	3.5	3.8	3.84
0.9	3.1	3	3.3	3.31
1	2.9	2.9	2.9	2.9

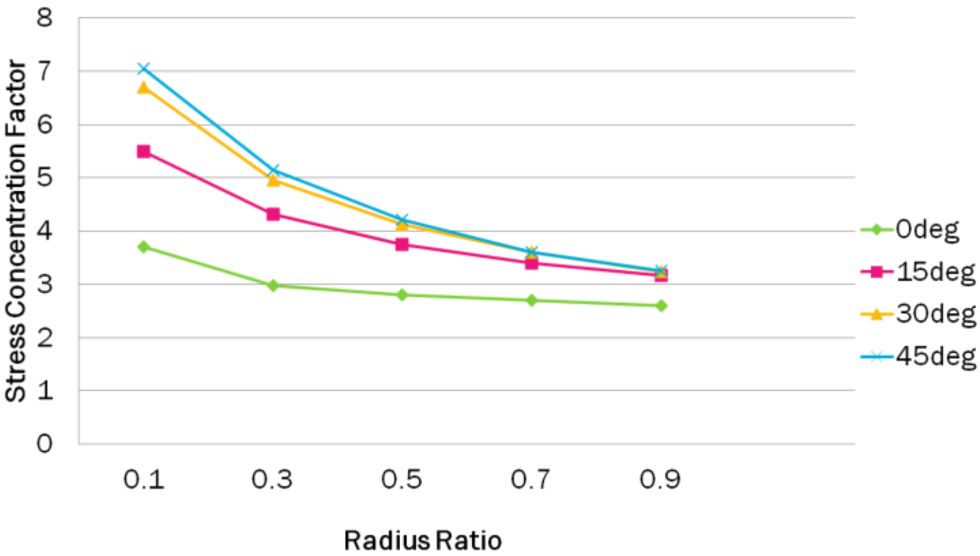

Figure 12: SCF with respect to rotation for triangle cut-outs

From the results (see Figs.9, 10, 13, and 14), in the case of the square cut-out, it is more advantageous to orient two sides of the square cut-out to be perpendicular to the applied tensile force because this reduces the maximum stress. For example, in the case of square cut-outs with r/R = 0.1, the maximum stress decreases from 141 (θ = 45°) to 74 MPa (θ = 0°), which is a 67 MPa or 47% decrease. Similarly, in the case of the triangle cut-out, it is also preferable to orient one side of the triangle cut-out to be perpendicular to the applied tensile forces because of stress reduction. For example, in the case of triangle cut-outs with r/R = 0.1, the maximum stress decreases from 166 (θ = 45°) to 141 MPa (θ = 0°), which is a 25 MPa or 15% decrease. Accordingly, at the design stage, determining the direction of a major tensile force is required. By aligning these polygon cut-outs as observed here, we can then reduce stress concentration.

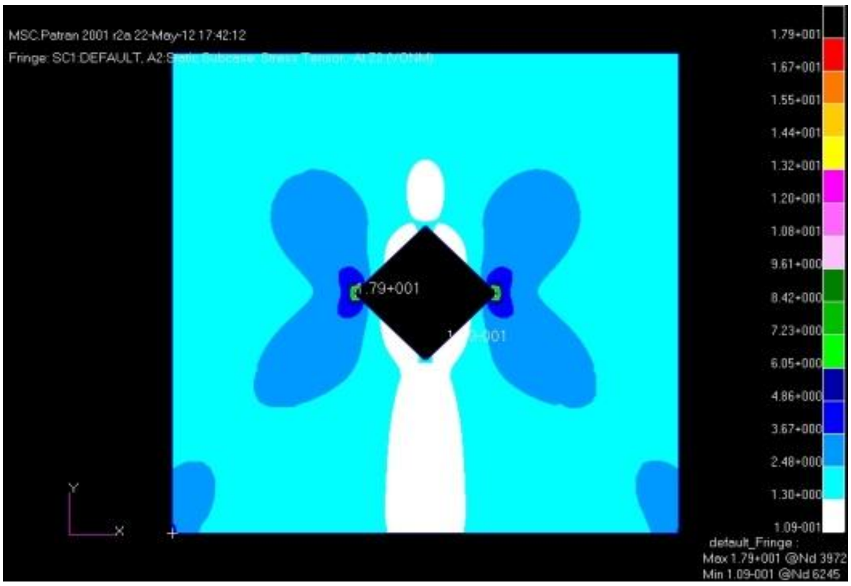

Figure 13: Stress contour for square cut-out (r/R = 0.1, □ = 45°)

In addition to rotation, similarly to the previous section, for all the degrees of orientation, it is also observed that the stress concentration decreases as the bluntness of the cut-outs decreases. For example, in the case of square cut-outs with 45° rotation, the maximum stress decreases from 141 (r/R = 0.1) to 65 MPa (r/R = 0.9), which is a 76 MPa or 54% decrease. Similarly, in the case of triangle cut-outs with 30° rotation, the maximum stress decreases from 166 (r/R = 0.1) to 66.3 MPa (r/R= 0.9) with a 100 MPa or 60% decrease.

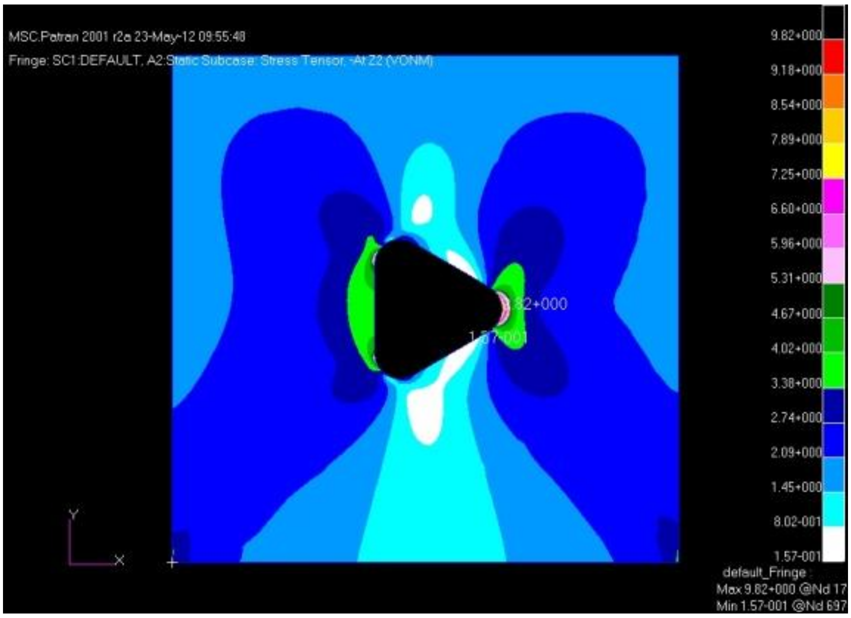

Figure 14: Stress contour for triangle cut-out (r/R = 0.1, □ = 30°)

Therefore, the next question among these two factors (rotation and bluntness) is which factor should preferably be controlled to minimize the stress concentration. Based on Tables 3 and 4 and Figs.11 and 12, we can clearly see that bluntness is a more effective factor. For example, as the bluntness approaches zero (r/R approached to unit), the maximum stresses tend to converge to 64.3 MPa (the maximum stress in the case of circular cut-out) and naturally the rotation effect vanishes. However, this does not reflect the manufacturing ease and cost. In a sense, it may be preferable to control orientation to reduce the manufacturing costs and cumbersome manufacturing. Therefore, at this analysis stage, the judgment should be handed over.

In summary, to minimize the stress concentration of the aluminium plates with polygon cut-outs, the cut-outs should have smooth edges and proper rotations. In other words, by controlling the smoothness (or bluntness) and rotation, we can minimize the stress concentration of the perforated aluminium plates. Among bluntness and rotation, controlling bluntness is analytically preferable to minimize the stress concentration.

5. Conclusions

This study presents stress concentration analyses of perforated aluminium plates with various shapes, bluntness, and rotation of polygonal cut-outs. For the analysis we intentionally limit resulting stresses in an elastic range by controlling the applied uniaxial tensile forces. We observed that the maximum stress in the perforated aluminium plate with the circular cut-out is about three times the applied force; hence, the previous observation performed by Pilkey et al. (2008) is verified. From the finite element analyses, the following findings are reported. Depending on cut-out shapes, bluntness and rotation effects on stress concentration vary. However, in general, as bluntness increases, the stress concentration increases, regardless of the shape and rotation. A more important finding is that the stress concentration increases as the cut-outs become more oriented from the baseline, which is the positive horizontal axis (+x) and one of the directions of the applied tensile forces. This fact demonstrates that the orientation is also a relatively significant design factor to reduce stress concentration. In general, in the case of the triangle cut-out, it is preferable to orient one side of the triangle cut-out to be perpendicular to the applied tensile forces. Similarly, in the case of the square cut-out, it is more advantageous to orient two sides of square cut-out to be perpendicular to the applied tensile force. Therefore, at the design stage, determining the direction of a major tensile force is required. By aligning these polygon cut-outs properly, we can then reduce stress concentration. This finding is mainly for uni-axial tensile forces in an elastic range. Other cases such as uni-axial compressive forces and bi-axial tensile and/or compressive forces should be considered for the future work. In addition, stress concentration analyses in a non-elastic range could be an interesting topic for future work.

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