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On The Transcendental Equation

$$\sqrt[3]{X^2+Y^2}+\sqrt[3]{Z^2+W^2}=2(k^2+s^2)R^5$$

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Abstract: *The transcendental equation with five unknowns given by,*
$\sqrt[3]{X^2+Y^2}+\sqrt[3]{Z^2+W^2}=2(k^2+s^2)R^5$ *is analyzed for its infinitely many non-zero integral solutions.*

Keywords: *Transcendental equations, Integral solutions.*

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I. Introduction

Diophantine equations have an unlimited field of research by reason of their variety. Most of the Diophantine problems are algebraic equations [1-3].It seems that much work has not been done to obtain integral solutions of transcendental equations. In this context, one may refer [4-10].This communication analyzes a transcendental equation given by $\sqrt[3]{x^2+y^2}+\sqrt[3]{z^2+w^2}=2(k^2+s^2)R^5$ for its infinitely many non-zero integer quintuples (x,y,z,w,R).

II. Method Of Analysis

The transcendental equation to be solved is

$$\sqrt[3]{x^2+y^2}+\sqrt[3]{z^2+w^2}=2(k^2+s^2)R^5 \qquad (1)$$

Where k and s are non-zero integer constants.
To start with, the substitution

$$\left.\begin{aligned} x&=m(m^2+n^2)\\ y&=n(m^2+n^2)\\ z&=m^3-3mn^2\\ w&=3m^2n-n^3 \end{aligned}\right\rangle \qquad (2)$$

In (1) ,lead to

$$m^2+n^2=(k^2+s^2)R^5 \qquad (3)$$

Which is analyzed for its distinct integral solutions when

i) k^2+s^2 is not perfect square

ii) k^2+s^2 is a perfect square.

Case:1 k^2+s^2 is not a perfect square.

Assume

$$R=a^2+b^2 \qquad (4)$$

Using (4) in (3) and employing the method of factorization, define,

$$(m+in)=(k+is)(a+ib)^5$$

Equating real and imaginary parts, we get,

$$\left.\begin{aligned} m&=kf(a,b)-sg(a,b)\\ n&=sf(a,b)+kg(a,b) \end{aligned}\right| \qquad (5)$$

Where $f(a,b)=a^5-10a^3b^2+5ab^4$

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$$g(a,b)=5a^4b-10a^2b^3+b^5$$

Using (5) in (2), the non-zero distinct integer values of x,y,z,w are,

$$\left.\begin{aligned}x&=[kf(a,b)-sg(a,b)](k^2+s^2)[f^2(a,b)+g^2(a,b)]\\y&=[sf(a,b)+kg(a,b)](k^2+s^2)[f^2(a,b)+g^2(a,b)]\\z&=[kf(a,b)-sg(a,b)][(k^2-3s^2)f^2(a,b)+(s^2-3k^2)g^2(a,b)-8ksf(a,b)g(a,b)]\\w&=[sf(a,b)+kg(a,b)][(3k^2-s^2)f^2(a,b)+(3s^2-k^2)g^2(a,b)-8ksf(a,b)g(a,b)]\end{aligned}\right\}\quad(6)$$

Thus (4) and (6) represents the non-zero integral solutions of (1).
A few numerical examples are given in the table I below.

Table I Numerical examples:

x	y	Z	w	R
640	-1920	-1664	1152	2
493750	18750	490906	56142	5
-1875000	-546875	-1287000	-1469125	5
-1828125	687500	-922077	1721764	5

Case II: k^2+s^2 ia a perfect square

Let $k^2+s^2=d^2$ (7)

Using (4) and (7) in (3) and employing the method of factorization define

$$(m+in)(m-in)=(id)(-id)(a+ib)^5$$

Equating real and imaginary parts, we get,

$$\left.\begin{aligned}m&=-dg(a,b)\\n&=df(a,b)\end{aligned}\right\}\quad(8)$$

Using (8) in (2), non-zero integral solutions of x,y,z,w are given by,

$$\left.\begin{aligned}x&=(-d^3)[g^3(a,b)+f^2(a,b)g(a,b)]\\y&=d^3[f(a,b)g^2(a,b)+f^3(a,b)]\\z&=d^3[3f^2(a,b)g(a,b)-g^3(a,b)]\\w&=d^3[3f(a,b)g^2(a,b)-f^3(a,b)]\end{aligned}\right\}\quad(9)$$

Thus (4) and (9) represents the non-zero integral solutions of (1)
Numerical examples are given in the Table 2 below

Table II Numerical examples:

x	y	z	W	R
16000	-16000	-16000	-16000	2
281216	-281216	-281216	-281216	2
-281490625	-260893750	238794127	-300466114	5
-124417736704	249036300000	227941548032	-246810701824	8

From the above table, we see that each of the expressions $\pm 2(3x+z)$ and $\pm 2(3y-w)$ is a cubical integer.

III. Conclusion

One may search for other patterns of solutions.

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