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Boundary Layer Flow and Heat Transfer Flow past Stretching Sheet with Emperature Gradient Dependent Heat Sink and Internal Heat Eneration

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ABSTRACT
Free convection viscous fluid flow and heat transfer flow past a stretching sheet with temperature gradient dependent heat sink and internal heat generation is considered for the study. The exact solutions of momentum equation and energy equation are obtained in terms of Kummer's function and the effects of various parameters like magnetic parameter (Mn), permeability parameter (k_2) and suction parameter (S) are analyzed by drawing various graphs for tangential and vertical velocity and temperature field.

NOMENCLATURE

c	stretching rate
C_p	specific heat
E	elastics parameter
G	dimensionless stream function
k-	thermal conductivity
k_0	coefficient of viscosity
k_1	visco-elastic parameter
k_2	permeability parameter
Mn	magnetic parameter
Pr	prandtl number
Q	heat source/sink parameter
q_w	local wall heat flux
S	suction parameter
T	temperature
T_w	temperature at wall
T_∞	temperature away from wall
u,v	velocity components along x- and y- directions
v_0	suction velocity

Greek symbols

γ	coefficient of kinematic viscosity
ρ	fluid density
σ	electrical conductivity
β	heat generation parameter
η	dimensionless similarity variable
ξ	dimensionless transformed variable
θ	dimensionless concentration variable
ε	small variable

Subscripts

w	condition at the wall
∞	condition at infinity
η	derivative with respect to η

I. INTRODUCTION

The study of boundary layer behaviours over a stretching sheet occurring in several engineering applications and manufacturing processes in industry. The practical applications of continuous flat surfaces are in aerodynamic extrusion of plastic sheets, rolling and manufacturing of plastic films, cooling of metallic plates and boundary layer flow over heat treated materials between feed roll and a windup roll.

Sakiadis [1] initiated the study of boundary layer over a continuous solid surface, flat surface and the cylindrical surfaces, Mc-Carmack and Crane [2] presented an analysis on boundary layer flow caused by stretching of elastic flat surfaces and between two surfaces under various physical situations.

The investigations have a definite bearing on the problem of polymer sheet extruded continuously from a dye. It is generally assumed that the sheet is inextensible. But many cases arises in polymer industry in which it is necessary to deal with a stretching sheet as noted by Crane [3]. Gupta and Gupta [4] presented the heat and mass transfer over a stretching sheet with blowing or suction. Grubka and Bobba [5] have worked on the heat transfer occurring on a linearly stretching surface under variable temperature. Bujurke et. al. [6] made an investigation on the heat transfer analysis in a second order fluid flow past a stretching surface with heat transfer. Datta et. al. [7] have studied the distribution of temperature in a continuous stretching sheet with uniform wall heat flux. Chen and Char [8] analyzed the heat transfer on continuous stretching sheet with two different cases viz power law surface temperature (PST) and power law surface heat flux (PHF). Further flow and heat transfer from a linearly stretching sheet gained more importance due to practical applications in industrial processes. Abel and Veena [9] have analyzed visco-elastic fluid flow and heat transfer in a porous medium over a stretching sheet, Vajravelu and Rollins [10] studied heat transfer properties in a visco-elastic fluid over a continuous stretching sheet with power law surface temperature (PST) and power law surface heat flux (PHF) by considering heat transfer characteristics in a fluid initially at rest and at a uniform temperature. Further Vajravelu and Nayefh [11] presented the flow and heat transfer by introducing temperature dependent heat source or sink. They considered heat transfer in a saturated porous medium over a continuous impermeable stretching surface with power law surface temperature (PST) and power law surface heat flux (PHF) including the effects of fractional heat and internal heat generation or absorption.

Many authors including Anjalidevi and Thiyagarajn [12], Ranjagopal et. al. [13], Sanyal and Das gupta [14], Sujit Kumar Khan et. al. [15], Takhar and Soundalgekar [16], Mahapatra et. al. [17], Bhargava et. al.[18], Idress and Abel [19], Takhar et. al.[20], have analyzed the problem on boundary layer flow due to the stretching sheet/continuous moving sheet for different flow models and boundary conditions.

Motivated by these analyses, in the present paper we studied the free convection flow past a non-isothermal stretching surface in the presence of porous medium and temperature gradient dependent heat sink including the internal heat generation. The exact solutions of momentum equation and energy equation in terms of Kummer's function are developed. The result of the study are discussed for different numerical values of the parameters like magnetic parameter (Mn), permeability parameter (k_2), suction parameter (S), Prandtl number (Pr) through graphs.

II. MATHEMATICAL FORMULATION

We consider a steady two dimensional boundary layer free convection viscous fluid flow past a stretching sheet in porous medium in the presence of temperature gradient dependent heat sink for the study.

The flow is produced due to stretching of the sheet by applying two equal and opposite forces along x-axis, keeping origin fixed. The flow is assumed in a region y > 0, in order to get the effect of temperature difference between the surface and the ambient fluid. Hence the governing boundary layer equations for momentum and heat transfer are of the following form.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \ldots 1$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\gamma\frac{\partial^2 u}{\partial y^2}-k_0\left[u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right]-\frac{\sigma\beta_0^2 u}{\rho}-\frac{\gamma}{k^1}u \qquad \ldots 2$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho Cp}\frac{\partial^2 T}{\partial y^2}+Q^1\frac{\partial T}{\partial y}+\frac{Q}{\rho Cp}(T-T_\infty) \qquad \ldots 3$$

Where u and v are the velocity components in x- and y- directions respectively. k-the thermal conductivity of the fluid medium. K_0- coefficient of visco-elasticity,

Q-the heat source/sink parameter, γ -the coefficient of kinematic viscosity.

The appropriate boundary conditions for velocity are of the form.

$$u = c\,x \quad v = -v_0 \quad \text{at } y = 0 \qquad \ldots 4$$
$$u = 0 \quad \text{as } y \to \infty$$

where c is a constant known as stretching rate and v_0 is the suction velocity.

III. VELOCITY TRANSFER ANALYSIS

To solve the equations (1) and (2) we assume a solution for velocity field as below

$$u = c\,x\,G'(\eta), \quad v = -\sqrt{c\gamma}\,G(\eta) \text{ and } \eta = \sqrt{\frac{c}{\gamma}}y \qquad \ldots 5$$

Obviously for the above u and v, continuity equation (1) is identically satisfied. Substituting (5) in momentum equation (2), it reduces to fourth order non-linear ordinary differential equation of the form

$$G'^2(\eta)-G(\eta)G''(\eta)=G'''(\eta)-k_1\left[2G'(\eta)G'''(\eta)-G(\eta)G''''(\eta)-G''^2(\eta)\right]-(Mn+k_2)G'(\eta) \qquad \ldots 6$$

where $k_1 = \frac{k_o c}{\gamma}-visco-elastic\ parameter$

$Mn = \frac{\sigma\beta_0^2}{\rho c}-\text{magnetic parameter}$

$k_2 = \frac{\gamma}{k'c}-permeability\ parameter$

and the corresponding boundary conditions (4) reduces to

$$G'(\eta)=1 \qquad G(\eta)=-\frac{v_0}{\sqrt{c\gamma}}\ at\ \eta=0$$

$$G'(\eta)=0 \qquad G''(\eta)=0\ as\ \eta\to\infty \qquad \ldots 7$$

Here $S=\frac{v_O}{\sqrt{c\gamma}}$ - Suction parameter.

The exact solution of momentum equation (6) subjected to the boundary conditions (7) is of the type,

$$G(\eta) = A + Be^{-E\eta} \qquad \ldots 8$$

where A = $\dfrac{E^2 - (k_2 + Mn)}{E}$

B = $-\dfrac{1}{E}$

and

E = $\frac{1}{2}\left[S + \sqrt{S^2 + 4(Mn + k_2 + 1)}\right]$

Then exact solution equation (8) can be expressed as

$$G(\eta) = \frac{1}{E}\left[E^2 - (k_2 + Mn) - e^{-E\eta}\right] \qquad \ldots 9$$

Hence we have

$$G'(\eta) = e^{-E\eta} \qquad \ldots 10$$

IV. HEAT TRANSFER ANALYSIS

In the energy equation (3) where k - the thermal conductivity varies approximately with temperature and the temperature gradient dependent heat sink is a linear function of the temperature. The appropriate boundary conditions for heat transfer boundary flow are

$T = T_w = T_\infty + b\,x^p$ at $y = 0$ …11

and $T = T_\infty$ as $y \to \infty$

where $T = T_w$ and $T = T_\infty$ are the temperature at wall and the temperature far away from the wall. b is a constant value which depends upon the property of the fluid.

To solve energy equation (3) we introduce the following non-dimensional quantity for temperature T as

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \qquad \ldots 12$$

and expression for temperature T as

$T = T_\infty + b\,x^p\,\theta(\eta)$ …13

By using the transformation given in equations (4), (12) and (13), energy equation (3) reduces to the following form.

$$\theta'' + \Pr G(1 + Q^*)\theta' - \Pr(G' + \beta)\theta = 0 \qquad \ldots 14$$

Where Q* = $-\sqrt{\dfrac{C}{\gamma}}Q$ - volumetric rate of heat generation

Pr = $\dfrac{\mu Cp}{k}$ - Prandtl number

β = $\dfrac{Q}{\rho c Cp}$ - heat generation parameter

and the corresponding boundary conditions are

$\theta(\eta) = 1$ at $\eta = 0$ …15

$\theta(\eta) \to 0$ as $\eta \to \infty$

To obtain the solution of equation (14) we define a new change of variable ξ as

$$\xi = -\frac{\Pr(1 + Q^*)\,e^{-E\eta}}{E^2} \qquad \ldots 16$$

By using equations (13) and (16), equation (14) reduce to the form

$$\xi\,\theta_{\xi\xi} + \left[1 - \Pr(1 + Q^*)\left(1 - \frac{k^{-1}}{E^2}\right) - \xi\right]\theta_\xi - \Pr\left[\frac{\beta}{E^2\xi} - \frac{1}{\Pr(1 + Q^*)}\right]\theta = O \qquad \ldots 17$$

and the corresponding conditions (15) converts to

$$\theta\left(\xi = -\Pr\frac{(1 + Q^*)}{E^2}\right) = 1,\ \ \theta(\xi = 0) = 0 \qquad \ldots 18$$

The solution of the equation (17) subjected to the boundary conditions (18) is obtained and expressed in the following form of confluent hyper-geometric function (i.e. Kummer's function) of the similarity variable η as

$$\theta(\eta) = e^{-E\delta_1\eta}\,\frac{M\left(2S_1 - k_1,\ 1 + b_o,\ -k_2 e^{-E\eta}\right)}{M\left(2\delta_1 - k,\ 1 + b_o,\ -k_2\right)} \qquad \ldots 19$$

Where $\delta_1 = \dfrac{a_o + b_o}{2}$

$a_o = \Pr(1 + Q^*)\left(1 - \dfrac{k^{-1}}{E^2}\right)$

$$b_o = \frac{1}{E}\sqrt{\Pr^2(1+Q^*)^2\left(E^2-k^{-1}\right)^2+4\Pr p\beta}$$

$$k_1 = \frac{p}{1+Q^*} \quad and \quad k_2 = -\frac{\Pr(1+Q^*)}{E^2}$$

Non-dimensional wall temperature gradient from equation (19) is derived as

$$\theta'(0) = -E\delta_1 M(2\delta_1-k_1,1+b_o,-k_2)+\frac{\left(\frac{2\delta_1-k_1}{1+b_o}\right)M(2\delta_1-k_1+1,2+b_o,-k_2)}{M(2\delta_1-k_1,1+b_o,-k_2)} \quad \ldots 20$$

The local wall heat flux can be expressed as

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{\eta=o} = -kbx^p\sqrt{\frac{c}{v}}\,\theta'(O) \quad \ldots 21$$

SKIN FRICTION

After velocity transfer skin friction co-efficient in non-dimensional form is obtained as

$$T_o = -\mu\left(\frac{\partial u}{\partial y}\right)_{\eta=o} = u_o x, \text{ where } u_o = \mu\beta E^2 e^{-E\eta} \quad \ldots 22$$

V. RESULTS AND DISCUSSION

The equations for free convection of incompressible viscous fluid flow and heat transfer past a stretching sheet with temperature gradient dependent heat sink and internal heat generation are examined. Energy and momentum equations are solved analytically and expressed in terms of Kummer's function. The effect of various physical parameters like Magnetic parameter (Mn), Permeability (k_2), Suction parameter (S) are examined on velocity profiles and temperature profiles. The value of Prandtl number is considered to be Pr = 0.7 which refers to air. Variation in logitudinal and transverse velocities are depicted for different values of Mn, k_2 and S. The longitudinal velocity is calculated for S at Mn = 25, k_2 = 5 and c x =1.0.The transverse velocity is calculated for various values of S at Mn = 25,k_2 = 5 and $\sqrt{c\gamma} = -1.0$.

Fig.1(a) shows the variation of u versus η for various values of suction parameter S = 0,0, 0.5,1.0,1.5. We observed from the figure that there is a steady decrease in u with increase in S. The longitudinal velocity is maximum at the wall for all cases.

Fig.1(b)shows the graph of transverse velocity v versus η for S = 0. 0.5, 1.0, 1.5. We observed that there is steady increase in v with increase in η from η = 0 to η = 0.7.The transverse velocity steadly increases with the increase in the value of η.

Fig.2(a) is depicted to study the variation of G(η) versus η for different values of visco-elastic parameter (k_1). It is noticed from the figure that G(η) increases with increase of visco-elastic parameter (k_1).

Fig.2(b) presents the variation of the function G(η) versus η for different values of magnetic parameter (Mn) . From the figure it is observed that G(η) increases with increase in the value of magnetic parameter (Mn).

In fig.2 (c) the variation of the function G(η) versus η for different values of Suction parameter (S) is shown. From the figure it follows that G(η) increases with increase in Suction parameter (S).

Fig.3(a) is drawn to show the variation of temperature θ(η) versus η for different values of Mn along with k_2 = 20, Pr = 0.7, β = 2 and for different combinations of p, Q^* and S. We observe from the figure that the maximum temperature corresponds to the curve I for which Mn = 20, p = 0.5, S = 0.5 and Q^* = 0.5. The temperature lowered for the rise in Mn for different combinations of p, Q^* and S. Wall temperature parameter plays an important rule for lowering the temperature. The boundary layer thickness decreases as the magnetic parameter (Mn) increases which results in high temperature distribution at the wall.

Fig.3(b) shows the variation of temperature θ(η) versus η for effect of different values of permeability parameter (k_2) along with Mn = 20, Pr = 0.7, β = 2 and for different combinations of p, Q^* and S. We observe from the figure that the temperature profile decreases on the boundary layer with increase in k_2 for different combinations of p, Q^* and S. The temperature is unchanged at the wall with the change in physical parameters.

Fig.4 (a) is depicted in regard of wall temperature gradient $\theta'(0)$ versus Prandtl number (Pr) for different values of k_2, Mn and β with the combinations of different values p, Q^* and S. It is observed from the figure that temperature gradient increases with increase of values of Prandtl number (Pr).

Fig.4(b) shows the variation of wall temperature gradient $\theta'(0)$ versus magnetic parameter (Mn) for different values of k_2, Pr and β with different combinations values of p, Q^* and S. We observe from the figure that wall temperature gradient increases with the increase of values of magnetic parameter (Mn).

Fig.4 (c) is presented to show the variation of wall temperature gradient $\theta'(0)$ versus visco-elastic parameter (k_1) for different values of Mn, Pr and β with different combinations values of p, Q^* and S. It is noticed from the figure that wall temperature gradient increases with increase in values of visco-elastic parameter (k_1)..

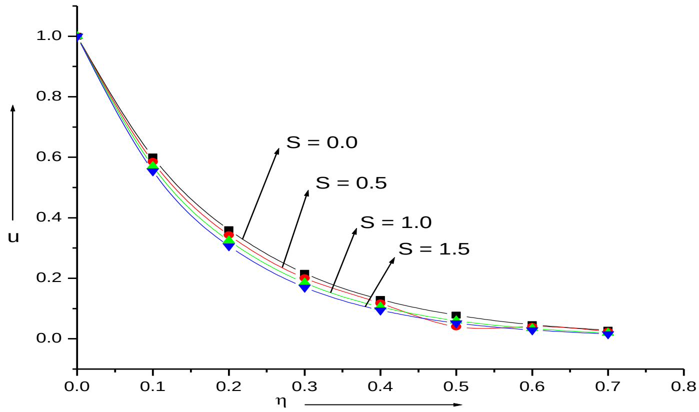

Figure1(a). Effects of S on longitudinal velocity.

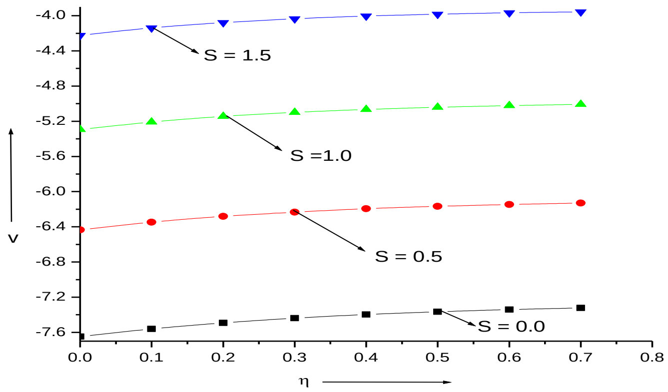

Figure 1(b).Effects of S on transverse velocity.

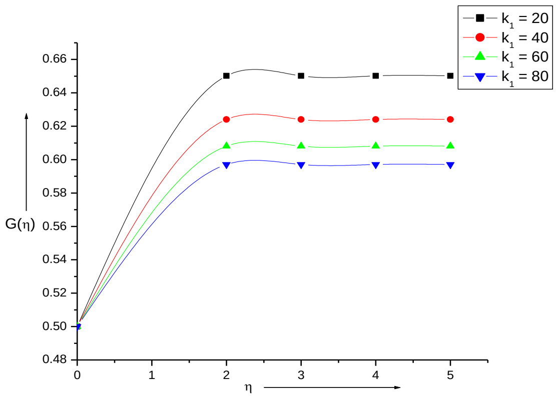

Fig.2 (a). Variation of G(η) Vs η for Mn=20, S=0.5 with different values of k_1

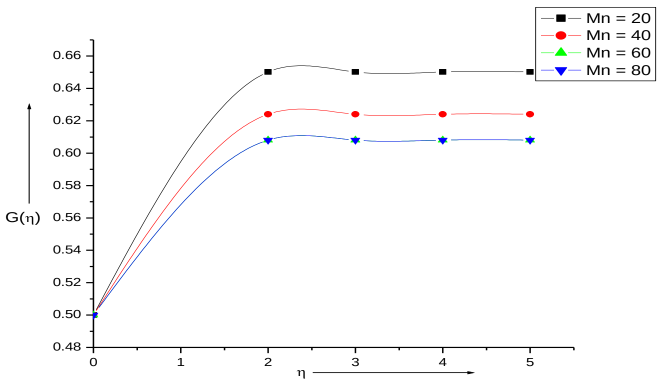

Fig.2(b). Variation of G(η) Vs η for k_2=20,S=0.5 with different values of Mn

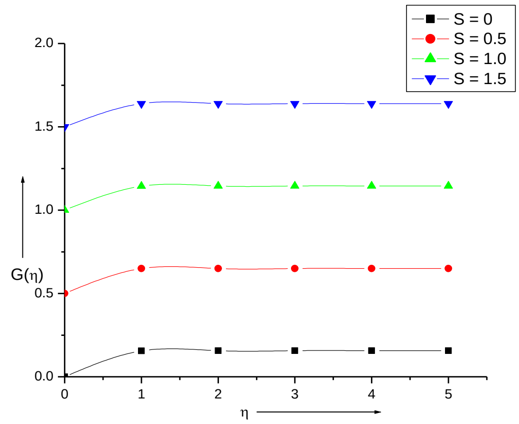

Figure 2(c). Varation ofG(η) versus η for different values of suction parameter-S

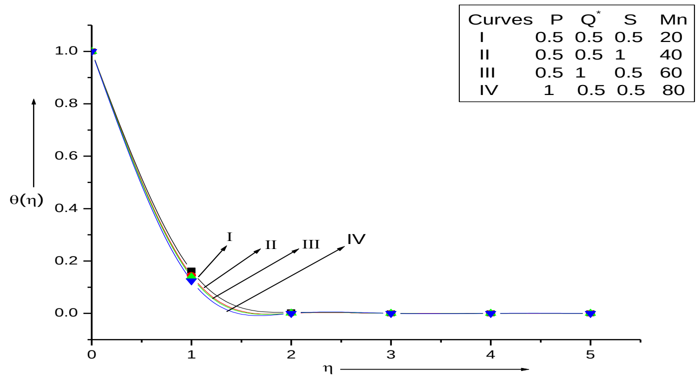

Fig. 3(a). Variation of $\theta(\eta)$ Vs η for k_2=20,Pr=0.7,β=2 with different values of Mn

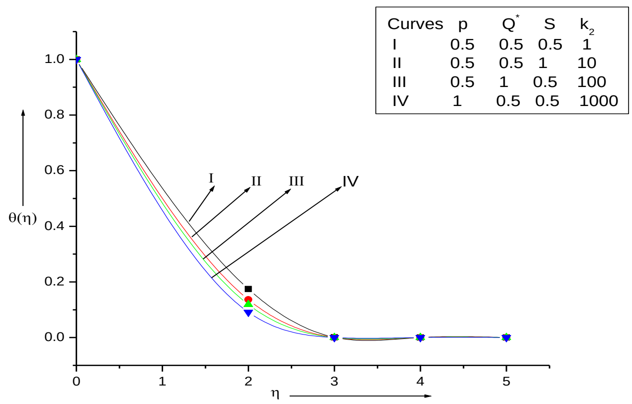

Fig.3(b). Variation of $\theta(\eta)$ Vs η for Mn=20,Pr=0.7,β=2 with different values of k_2

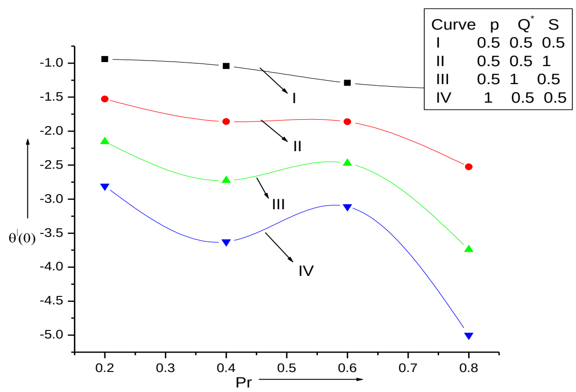

Fig.4(a).Variation of $\theta^{I}(0)$ Vs Pr for k_2=20,Mn=20 ,β=2 with different values of P,Q^*,S

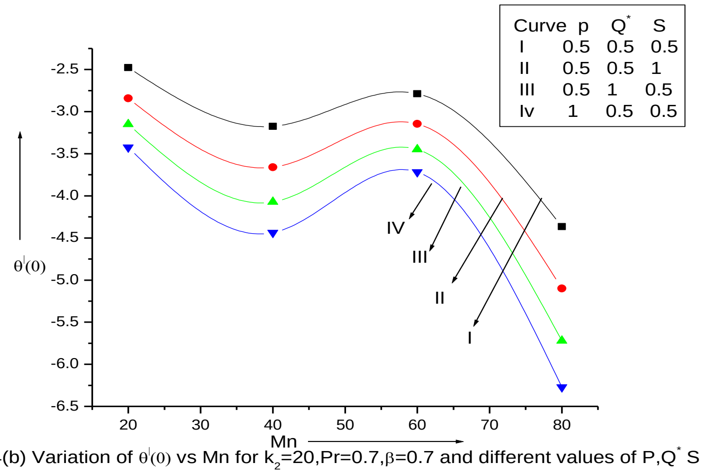

Fig.4(b) Variation of $\theta'(0)$ vs Mn for k_2=20,Pr=0.7,β=0.7 and different values of P,Q* S

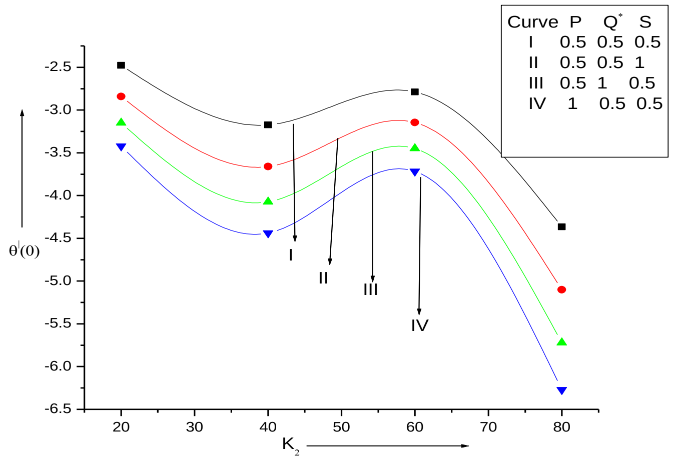

Fig.4(c).Variation of $\theta'(0)$ Vs K_2 for Mn=20,Pr=0.7,β=2

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