Cutting Speed and Feed Rate Optimization for Minimizing Production Time of Turning Process

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ABSTRACT: *Optimum selection of cutting conditions importantly contribute to the increase of productivity by minimization of production time and the associated costs, therefore utmost attention is paid to this problem in this contribution. Time is the most important parameter in any operation and all the manufacturing firms aim at producing a product in minimum time to reach the customer quickly and enhance the customer satisfaction. This can be achieved by using optimization techniques. The success of an optimization technique does not lie in its complexity but the time in which it provides a solution to the manufacturing firms. In this research paper, a geometric programming based approach to optimize the production time of the turning process within in some operating constraints is proposed. It involves mathematical modeling for production time of turning process, which is expressed as a function of the cutting parameters which include the cutting speed and feed rate. Then, the developed mathematical model was optimized with in some operating constraints like the maximum cutting speed, maximum feed rate, power constraint and the surface roughness constraint. The results of the model reveal that the proposed method provides a systematic and efficient technique to obtain the optimal cutting parameters that will minimize the production time of turning process. Thus, it is possible to optimize the production time of turning process effectively by geometric programming. The approach is suitable for fast determination of optimum cutting parameters during machining, where there is not enough time for deep analysis.*

Keywords - *cutting parameters, geometric programming, model.*

Nomenclature:

C_{01} = constant = $\left(\frac{\pi dl}{1000}\right)$

C_{02} = constant = $\left(\frac{\pi dl\, t_c}{1000\, Z^{\frac{1}{n}}}\right)$

C_{11} = constant

C_0 = machine cost per unit time ($/min.)

C_m = machining cost per piece ($/piece)

C_t = tool cost ($/cutting edge)

d = diameter of the work piece (mm.)

f = feed rate (mm/revolution)

F = cutting Force (N)

l = length of the work piece (mm.)

n and p are constants.

P_t = production time per piece (min./piece)

R= nose radius of the tool (mm)

R_a =average surface roughness (µm)

t_c = tool changing time (min.)

t_h = tool handling time (min.)

t_m = time required to machine a work piece = $\frac{\pi dl}{1000\, vf}$ (min.)

T = tool life (min.)

v = the cutting speed (m/min.)

Z = constant

η = efficiency of cutting

$\lambda_{01}, \lambda_{02}$ *and* λ_{11} are lagrange multipliers.

I. Introduction

Turning is a widely used machining process in manufacturing. Therefore, an optimal selection of cutting parameters to satisfy an economic objective within the constraints of turning operation plays a very important role [1]. To determine the optimal cutting parameters, reliable mathematical models have to be formulated to associate the cutting parameters with the cutting performance. However, it is also well known that reliable mathematical models are not easy to obtain [2-5]. In any optimization problem, it is very crucial to identify the prime objective called as the objective function or optimization criterion. In manufacturing processes, the most commonly used objective function is the specific cost [6]. Walvekar and Lambert [7] used geometric programming for the selection of machining variables. The optimum values of both cutting speed and feed rate were found out as a function of depth of cut in multi-pass turning operations [8]. Wu et. al. [9] analyzed the problem of optimum cutting parameters selection by finding out the optimal cutting speed which satisfies the basic manufacturing criterion. Basically, this optimization procedure, whenever carried out, involves partial differentiation for the minimization of unit cost, maximization of production rate or maximization of profit rate. These manufacturing conditions are expressed as a function of cutting speed. Then, the optimum cutting speed is determined by equating the partial differentiation of the expressed function to zero. This is not an ideal approach to the problem of obtaining an economical metal cutting. The other cutting variables, particularly the feed rate also have an important effect on cutting economics. Therefore, it is necessary to optimize the cutting speed and feed rate simultaneously in order to obtain an economical metal cutting operation. The process of the metal cutting depends upon the features of tools, input work materials and machine parameter settings influencing process efficiency and output quality characteristics or responses. A significant improvement in process efficiency may be obtained by process parameters optimization that identifies and determines the regions of critical process control factors leading to desired outputs or responses with acceptable variations ensuring a lower cost of manufacturing [10].

The aim of this research paper is the construction of a mathematical model describing the objective function in terms of the cutting parameters with some operating constraints, then; the mathematical model was optimized by using geometric programming approach. The developed model and program can be used to determine the optimal cutting parameters to satisfy the objective of obtaining minimum production time of turning process under different operating constraints. The results of the mathematical model are obtained by using suitable software. This research paper proposes a very simple, effective and efficient way of optimizing the production time of the turning process with in some operating constraints such as the maximum cutting speed, maximum feed rate, power requirement, surface roughness. This paper also highlights the merits of geometric programming optimization over other optimization approaches

II. Mathematical Modeling for Optimization

The production time to produce a part by turning operation can be expressed as follows:

P_t = Machining Time + Tool Changing Time + Set-up Time (1)

$$P_t = t_m + (t_c)\frac{t_m}{T} + t_h \quad (2)$$

The Taylor's tool life (T) used in Eq. (2) is given by:

$$T = \left(\frac{Z}{f^p\, v}\right)^{\frac{1}{n}} \quad (3)$$

Where, n, p and Z depend on the many factors like tool geometry, tool material, work piece material, etc.

On substituting Eq. (3) in Eq. (2), we get

$$P_t(v,f) = C_{01}v^{-1}f^{-1} + C_{02}v^{\left(\frac{1}{n}\right)-1}f^{\left(\frac{p}{n}\right)-1} + t_h \quad (4)$$

t_h does not depend on cutting speed or feed rate. So, the modified objective function from Eq. (4) can be written as:

$$P_t(v,f) = C_{01}v^{-1}f^{-1} + C_{02}v^{\left(\frac{1}{n}\right)-1}f^{\left(\frac{p}{n}\right)-1} \quad (5)$$

2.1 Machining constraints:

There are many constraints which affect the selection of the cutting parameters. These constraints arise due to various considerations like the maximum cutting speed, maximum feed rate, power limitations, surface finish, surface roughness, etc.

2.1.1 Maximum cutting speed:

The increasing of cutting speed also increases the tool wear, therefore, the cutting speed has to be kept below a certain limit called the maximum cutting speed.

$$v \le v_{max\,.} \quad (6)$$

$$C_{11}v \le 1 \quad (7)$$

$$\text{Where } C_{11} = \frac{1}{v_{max\,.}} \quad (8)$$

By the method of primal and dual programming of geometric programming, the maximum value of dual function or the minimum value of primal function is given by:

$$v(\lambda) = \left[\frac{C_{01}}{\lambda_{01}}(\lambda_{01}+\lambda_{02})\right]^{\lambda_{01}}\left[\frac{C_{02}}{\lambda_{02}}(\lambda_{01}+\lambda_{02})\right]^{\lambda_{02}}\left[\frac{C_{11}}{\lambda_{11}}\right]^{\lambda_{11}} \quad (9)$$

Subject to the following constraints:

$$\lambda_{01} + \lambda_{02} = 1 \quad (10)$$

$$-\lambda_{01} + \left\{\left(\frac{1}{n}\right)-1\right\}\lambda_{02} + \lambda_{11} = 0 \quad (11)$$

$$-\lambda_{01} + \left\{\left(\frac{p}{n}\right)-1\right\}\lambda_{02} = 0 \quad (12)$$

And the non-negativity constraints are:

$$\lambda_{01} \ge 0, \lambda_{02} \ge 0 \text{ and } \lambda_{11} \ge 0 \quad (13)$$

On adding Eq. (10) and Eq. (12), we get

$$\lambda_{02} = n \quad (14)$$

From Eq. (10) and Eq. (14), we get

$$\lambda_{01} = 1-n \quad (15)$$

From Eq. (11), (14) and (15), we get

$$\lambda_{11} = 1-p \quad (16)$$

Therefore, the maximum value of dual function and the minimum value of primal function is given by:

$$v(\lambda) = \left(\frac{C_{01}}{1-n}\right)^{1-n}\left(\frac{C_{02}}{n}\right)^{n}\{C_{11}(1-p)\}^{1-p} \quad (17)$$

$$\text{Now, } \lambda_{11} = \frac{C_{11}v}{v(\lambda)} \quad (18)$$

From Eq. (17) and Eq. (18), we get the optimum values of cutting speed as:

$$v = \frac{\lambda_{11}\left(\frac{C_{01}}{1-n}\right)^{1-n}\left(\frac{C_{02}}{n}\right)^{n}(C_{11})^{1-p}}{C_{11}} \quad (19)$$

$$\text{And, } \lambda_{01} = \frac{C_{01}v^{-1}f^{-1}}{v(\lambda)} \quad (20)$$

Therefore, from (19) and (20), we get optimum feed rate as:

$$f = \frac{(C_{01}\times C_{11})}{(1-n)\times\lambda_1\times\left(\frac{C_{01}}{1-n}\right)^{1-n}\left(\frac{C_{02}}{n}\right)^{n}(C_{11})^{1-p}} \quad (21)$$

2.1.2 Maximum feed rate:

In rough machining operations, feed rate is taken as a constraint to achieve the maximum production rate.

$$f \le f_{max\,.} \quad (22)$$

$$C_{11}f \le 1 \quad (23)$$

$$\text{Where } C_{11} = \frac{1}{f_{max\,.}} \quad (24)$$

Following the same procedure as described for the first constraint, we get the following values:

$$\lambda_{01} = 1-n \quad (25)$$

$$\lambda_{02} = n \quad (26)$$

$$\lambda_{11} = 1-p \quad (27)$$

$$f = \frac{\lambda_{11}\left(\frac{C_{01}}{1-n}\right)^{1-n}\left(\frac{C_{02}}{n}\right)^{n}(C_{11})^{1-p}}{C_{11}} \quad (28)$$

$$v = \frac{(C_{01}\times C_{11})}{(1-n)\times\lambda_1\times\left(\frac{C_{01}}{1-n}\right)^{1-n}\left(\frac{C_{02}}{n}\right)^{n}(C_{11})^{1-p}} \quad (29)$$

2.1.3 Power constraint

The maximum power available for the turning operation will be a constraint in the turning operation, which has to be taken in to consideration. The power available for the turning operation is given by:

$$P = \frac{F\times v}{6120\eta} \leq P_{max\,.} \tag{30}$$

$$C_{11}v \leq 1 \tag{31}$$

Where $C_{11} = \frac{F}{6120\eta P_{max\,.}}$ (32)

Following the same procedure as described for the first constraint, we get the following values:

$$\lambda_{01} = 1 - n \tag{33}$$

$$\lambda_{02} = n \tag{34}$$

$$\lambda_{11} = 1 - p \tag{35}$$

$$v = \frac{\lambda_{11}\left(\frac{C_{01}}{1-n}\right)^{1-n}\left(\frac{C_{02}}{n}\right)^{n}(C_{11})^{1-p}}{C_{11}} \tag{36}$$

$$f = \frac{(C_{01}\times C_{11})}{(1-n)\times\lambda_1\times\left(\frac{C_{01}}{1-n}\right)^{1-n}\left(\frac{C_{02}}{n}\right)^{n}(C_{11})^{1-p}} \tag{37}$$

2.1.4 Surface roughness

Surface roughness can be used as a constraint in finishing operations. Therefore, it becomes a very important factor in determining finish cutting conditions. Surface roughness can be expressed in terms of feed as follows:

$$R_a = \frac{f^2}{32R} \tag{38}$$

$$R_a C_{11} \leq \frac{f^2}{32R} \tag{39}$$

Following the same procedure as described for the first constraint, we get the following values:

$$\lambda_{01} = 1 - n \tag{40}$$

$$\lambda_{02} = n \tag{41}$$

$$\lambda_{11} = \frac{(1-p)}{2} \tag{42}$$

$$v = \frac{\left[\frac{(1-p)}{2}\right]\left[\frac{C_{01}}{1-n}\right]^{1-n}\left[\frac{C_{02}}{n}\right]^{n}[C_{11}]^{1-p}}{C_{11}} \tag{43}$$

$$f = \frac{C_{01}\times C_{11}}{(1-n)\left\{\frac{(1-p)}{2}\right\}\left\{\left[\frac{C_{01}}{1-n}\right]^{1-n}\left[\frac{C_{02}}{n}\right]^{n}[C_{11}]^{1-p}\right\}^{2}} \tag{44}$$

III. Results and Discussion

3.1 Figures:

The figures obtained from the implementation of the mathematical model are as follows:

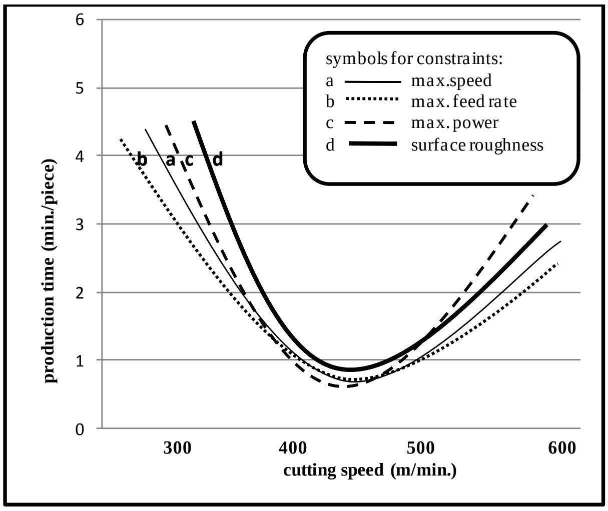

Fig 1: Production time versus cutting speed curve for different constraints.

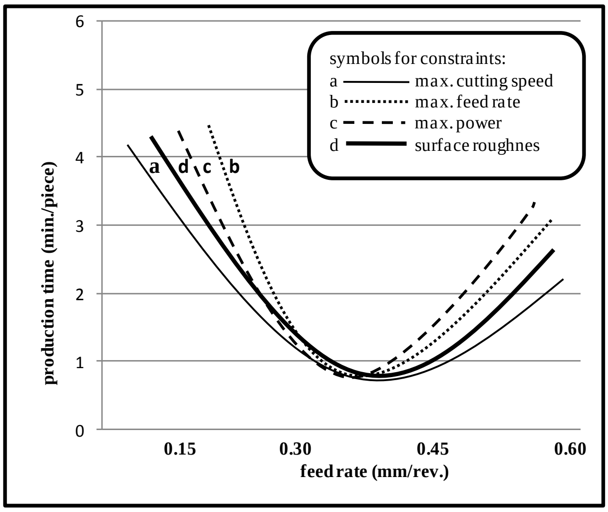

Fig 2: Production time versus feed rate curve for different constraints.

3.2 Analysis of results:

The curves obtained between production time and cutting speed reveal that a smaller value of cutting speed results in a high production time. It is due to the fact that a smaller cutting speed increases the production time of parts. Also, it will decrease the profit rate due to the production of a lesser number of parts. However, if the cutting speed is too high, it will also lead to a high production time due to excessive tool wear and increased machine downtime. The optimum cutting speed is somewhere between “too slow” and “too fast” which will yield the minimum production time.

The curves between the production time and the feed rate indicate that a small feed rate will result in high production time. A smaller feed rate means the number of revolutions should be increased. The more the number of revolutions, the more will be the production time. Even a very high feed rate is not advisable as it will increase the tool wear and surface roughness resulting in increased machining time and machine downtime resulting in high production time. So, the optimum feed rate is somewhere between “too small” and “too high” which will result in the minimum production time.

IV. Conclusion

In this research paper, the cutting speed and feed rate were modeled for the minimum production time of a turning operation. The maximum cutting speed, the maximum feed rate, maximum power available and the surface roughness was taken as constraints. The results of the model show that the proposed method provides a systematic and efficient method to obtain the minimum production time for turning. This approach helps in quick analysis of the optimal region which will yield a small production time rather than focusing too much on a particular point of optimization. It saves a lot of time and can be easily implemented by manufacturing firms. The developed model will provide with the optimal values of the cutting speed and feed rate that will satisfy the objective of production time minimization within the given operating constraints. The coefficients n, p and Z of the extended Taylor's tool life equation are not described in depth for all cutting tool and work piece combinations. Obtaining these coefficients experimentally requires lot of time, resources and then, the analysis of the obtained values increases the complexity of the process.

It can be concluded from this study that the obtained model can be used effectively to determine the optimum values of cutting speed and feed rate that will result in minimum production time. The developed model saves a considerable time in finding the optimum values of the cutting parameters. It has been shown that the method of geometric programming can be applied successfully to optimize the production cost of turning process.

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