Static Response of a Involute Pairs Of Gears In Application Scenario (Validation Using AGMA Standards) By Using FEM Cutting Edge Technology

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Abstract: *This paper investigates the characteristics of an involutes gear system including contact stresses, bending stresses, and the transmission errors of gears in mesh. Gearing is one of the most critical components in mechanical power transmission systems. Transmission error is considered to be one of the main contributors to noise and vibration in a gear set. Transmission error measurement has become popular as an area of research on gears and is possible method for quality control. To estimate transmission error in a gear system, the characteristics of involute spur gears were analyzed by using the finite element method. The contact stresses were examined using 2-D FEM models. The bending stresses in the tooth root were examined using a 3-DFEM model. Current methods of calculating gear contact stresses use Hertz's equations, which were originally derived for contact between two cylinders. To enable the investigation of contact problems with FEM, the stiffness relationship between the two contact areas is usually established through a spring placed between the two contacting areas. This can be achieved by inserting a contact element placed in between the two areas where contact occurs. The results of the two dimensional FEM analyses from ANSYS are presented. These stresses were compared with the theoretical values. Both results agree very well.*

This indicates that the FEM model is accurate. The finite element method is very often used to analyze the stress state of an elastic body with complicated geometry, such as a gear. There have been numerous research studies in the area [1],[2]

I. INTRODUCTION

1.1 Background

When one investigates actual gears in service, the conditions of the surface and bending failure are two of the most important features to be considered. The finite element method is very often used to analyze the stress states of elastic bodies with complicated geometries, such as gears. There are published papers, which have calculated the elastic stress distributions in gears. In these works, various calculation methods for the analysis of elastic contact problems have been presented. The finite element method for two-dimensional analysis is used very often. It is essential to use a three-dimensional analysis if gear pairs are under partial and non uniform contact. However, in the three dimensional calculation, a problem is created due to the large computer memory space that is necessary. In this chapter to get the gear contact stress a 2-D model was used Because it is a nonlinear problem it is better to keep the number of nodes and elements a slow as possible. In the bending stress analysis the 3-D model and 2-D models are used for simulation

1.2 Analytical Procedure

From the results obtained in earlier the present method is an effective and accurate method, which is proposed to estimate the tooth contact stresses of a gear pair. Special techniques of the finite element method were used to solve contact problems. Using the present method, the tooth contact stresses and the tooth deflections of a pair of spur gears analyzed by ANSYS 13. Since the present method is a general one, it is applicable to many types of gears. In early works, the following conditions were assumed in advance:

• There is no sliding in the contact zone between the two bodies

• The contact surface is continuous and smooth

Using the present method ANSYS can solve the contact problem and not be limited by the above two conditions. A two-dimensional and an asymmetric contact model were built. First, parameter definitions were given and then many points of thin involute profile of the pinion and gear were calculated to plot an involute profile using a cylindrical system. The equations of an involute curve below were taken from Buckingham [3]:

$$r = r_b * (1+\beta^2)^{½} \qquad 1.2.1$$
$$\psi = \theta + \pi / 2n_1 - \xi \qquad 1.2.2$$
$$\theta = \tan \varphi_1 - \varphi_1 \qquad 1.2.3$$

where *r* = radius to the involute form, *b r* = radius of the base circle

$\beta = \xi + \varphi_1$

θ = vectorial angle at the pitch circle

ξ = vectorial angle at the top of the tooth

φ_1 = pressure angle at the pitch circle

pressure angle at radius *r*

One spur tooth profile was created using above equations , shown in Figure 1, as are the outside diameter circle, the dedendum circle, and base circle of the gear. Secondly, in ANSYS from the tool bars using "CREATE", "COPY", "MOVE", and "MESH" and so on, any number of teeth can be created and then kept as the pair of gear teeth in contact along the line of the action. The contact conditions of gear teeth are sensitive to the geometry of the contacting surfaces, which means that the element near the contact zone needs to be refined. It is not recommended to have a fine mesh everywhere in the model, in order to reduce the computational requirements. There are

wo ways to build the fine mesh near the contact surfaces. One is the same method as presented in later, a fine mesh of rectangular shapes were constructed only in the contact areas. The other one, "SMART SIZE" in ANSYS, was chosen and the fine mesh near the contact area was automatically created. A FEM gear contact model was generated as shown in Figure .

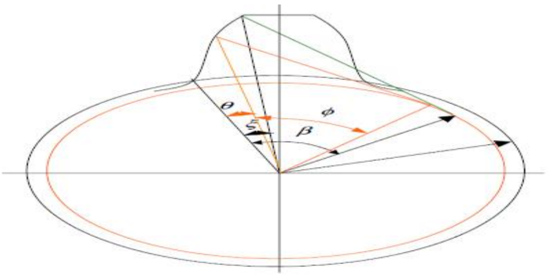

Figure 1 Involumetry of a spur gear

Thirdly, proper constraints on the nodes were given. The contact pair was inserted between the involute profiles, the external loads were applied on the model from ANSYS "SOLUTION > DEFINE LOAD > FORCE / MOMENT", and finally, ANSYS was run to get the solution

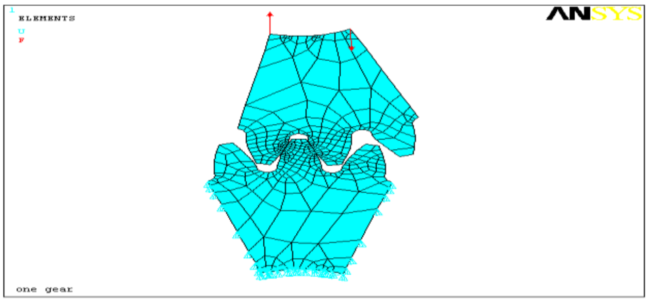

Figure 2 Gear contact stress model

II. Rotation Compatibility of the Gear Body

In order to know how much load is applied on the contact stress model and the bending stress model, evaluating load sharing between meshing gears is necessary. It is also an important concept for transmission error. It is a complex process when more than one-tooth pair is simultaneously in contact taking into account the composite tooth deflections due to bending, shearing and contact deformation. This section presents a general approach as to how the load is shared between the meshing teeth in spur gear pairs. When the gears are put into mesh, the line tangent to both base circles is defined as the line of action for involute gears. In one complete tooth mesh circle, the contact starts at points A shown in Figure 3 where the outside diameter circle, the addendum circle of the gear intersects the line of action. The mesh cycle ends at point E as shown in Figure 4 where the outside diameter of the pinion intersects the line of action.

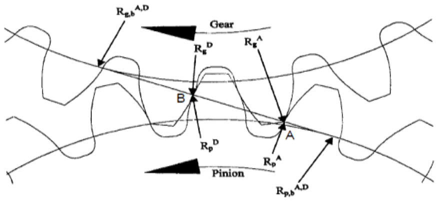

Figure 3 Illustration of one complete tooth meshing cycle

Consider two identical spur gears in mesh. When the first tooth pair is in contact at point A it is between the tooth tip of the output gear and the tooth root of the input gear(pinion). At the same time a second tooth pair is already in contact at point D in Figure3. As the gear rotates, the point of contact will move along the line of action APE. When the first tooth pair reaches point B shown in Figure 4, the second tooth pair disengage at point E leaving only the first tooth pair in the single contact zone. After this time there is one pair of gear in contact until the third tooth pair achieves in contact at point A again. When this tooth pair rotates to point D, the another tooth pair begins engagement at point A which starts another mesh cycle.

After this time there are two pairs of gear in contact until the first tooth pair disengage at point E. Finally, one complete tooth meshing cycle is completed when this tooth pair rotates to point E. To simplify the complexity of the problem, the load sharing compatibility condition is based on the assumption that the sum of the torque contributions of each meshing tooth pair must equal the total applied torque.

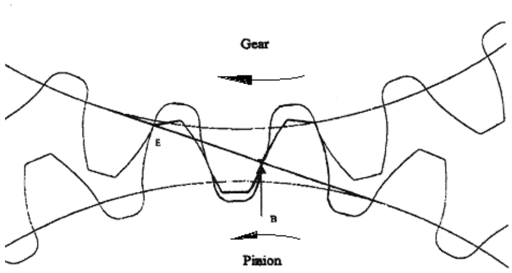

Figure 4 Different positions for one complete tooth meshing cycle

Analytical equations can also be developed for the rotation of the gear and pinion hubs, including the effects of tooth bending deflection and shearing displacement and contact deformation . In the pinion reference frame, it is assumed that the pinion hub remains stationary, while the gear rotates due to an applied torque. Considering the single pair contact zone at point B, the condition of angular rotation of the gear body will then be given by for the pinion

$$\theta_p^B = (B^B_P + H_P^B)/R_P^B \qquad 2.1$$

and for the gear

$$\theta_g^B = (B^B_g + H_g^B)/R_g^B \qquad 2.2$$

where B^B_P and B^B_g are the tooth displacement vectors caused by bending and shearing for pairs B of the pinion and gear respectively, H_P^B and H_g^B are the contact deformation vectors of tooth pair B of the pinion and gear respectively. θ_p^B denotes the transverse plane angular rotation of the pinion body caused by bending deflection, shearing displacement and contact deformation of the tooth pair B while the gear is stationary. Conversely, for the gear rotation while the pinion is stationary, θ_g^B gives the transverse plane angular rotations of the gear body.

III. Gear Contact Stress

One of the predominant modes of gear tooth failure is pitting. Pitting is a surface fatigue failure due to many repetitions of high contact stress occurring on the gear tooth surface while a pair of teeth is transmitting power. In other words, contact stress exceeding surface endurance strength with no endurance limits or a finite life causes this kind of failure. The AGMA has prediction methods in common use. Contact failure in gears is currently predicted by comparing the calculated Hertz stress to experimentally determined allowable values for the given material. The details of the subsurface stress field usually are ignored. This approach is used because the contact stress field is complex and its interaction with subsurface discontinuities are difficult to predict. However, all of this information can be obtained from the ANSYS model. Since a spur gear can be considered as a two-dimensional component, without loss of generality, a plane strain analysis can be used. The nodes in the model were used for the analysis. The nodes on the bottom surface of the gear were fixed. A total load is applied on the model. It was assumed to act on the two points shown in Figure 1 and three points in Figure 4.

There are two ways to get the contact stress from ANSYS. Figure 4 shows the first one, which is the same method as we use earlier to create the contact element COCNTA 48 and the rectangular shape fine mesh beneath the contact surfaces between the contact areas. Figure 5 shows the enlarged-area with a fine mesh which is composedof rectangular shapes.

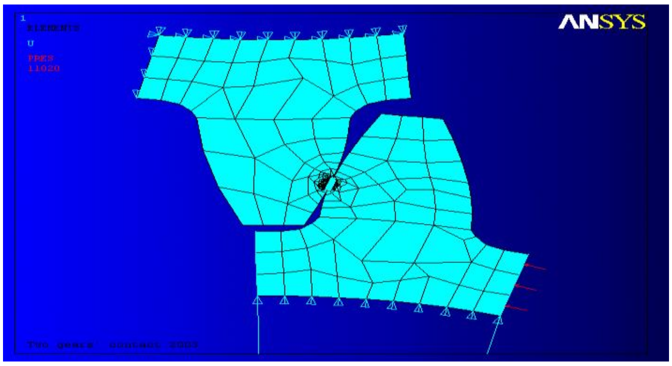

Figure 5 FEM Model of the gear tooth pair in contact

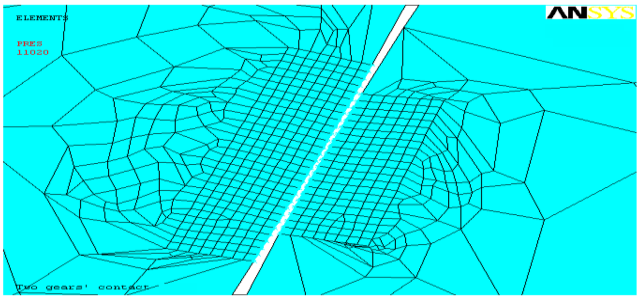

Figure 6 Fine meshing of contact areas

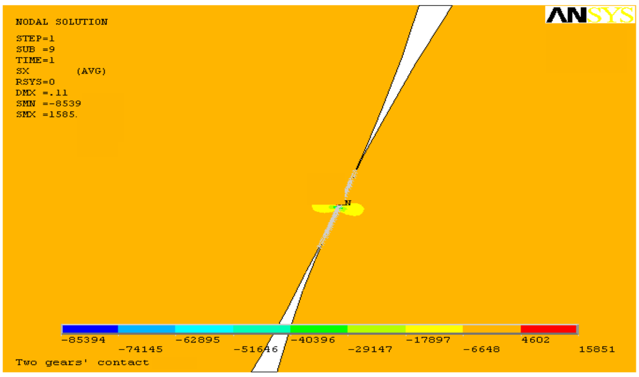

Figure 7 Contact stress along contact areas

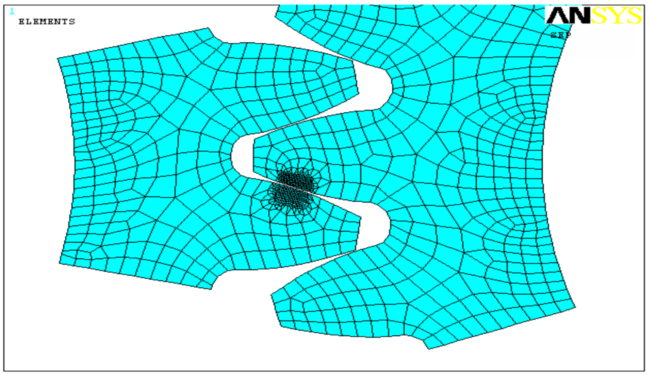

Figure 8 A fine mesh near contact areas

Fig. 6 shows the normal contact stress along the contact areas. The results are very similar to the results in the two cylinders in our solution. Fig. 7 presents how to mesh using a second method. Different methods should show the close results of maximum contact stress if the same dimension of model and the same external loads are applied on the model. If there is a small difference it is likely because of the different mesh patterns and restricted conditions in the finite element analysis and the assumed distribution form of the contact stresses in the contact zone.

IV. The Lewis Formula

There are several failure mechanisms for spur gears. Bending failure and pitting of the teeth are the two main failure modes in a transmission gearbox. Pitting of the teeth is usually called a surface failure. This was already discussed in the last section. The bending stresses in a spur gear are another interesting problem. When loads are too large, bending failure will occur. Bending failure in gears is predicted by comparing the calculated bending stress to experimentally-determined allowable fatigue values for the given material. This bending stress equation was derived from the Lewis formula. Wilfred Lewis (1892) [4] was the first person to give the formula for bending stress in gear teeth using the bending of a cantilevered beam to simulate stresses acting on a gear tooth shown in Figure 9 are Cross-section $=b * t$, length = l, load = $t\ F$, uniform across the face. For a rectangular section, the area moment of inertia is

$$I = bh^3/12 \qquad 4.1$$

$M = F_t l$ and $c = t/2$, stress then is

$$\sigma = \frac{M}{I/c} = \frac{F_t l(t/2)}{bt^3/12} = \frac{6F_t l}{bt^2} \qquad 4.2$$

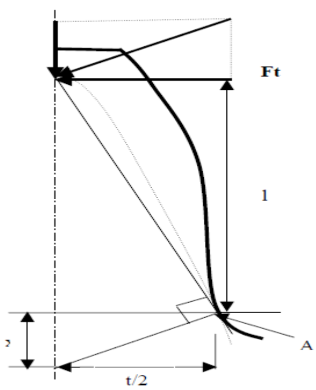

Figure 9 Length dimension used in determining bending tooth stress

Where b = the face width of the gear. For a gear tooth, the maximum stress is expected at point A, which is a tangential point where the parabola curve is tangent to the curve of the tooth root fillet called parabola tangential method. Two points can be found at each side of the tooth root fillet. The stress on the area connecting those two points is thought to be the worst case. The crack will likely start from the point A.

From similar triangles $\tan\alpha = \frac{t/2}{x} = \frac{l}{t/2}$ where $l = t^2/4x$ 4.3

Substituting (4.7) into (4.6):

$$\sigma = \frac{F_t\, t^2/4x}{bt^2} = \frac{3F_t}{2bx} = \frac{3F_t P_d}{2bp_d x} = \frac{F_t P_d}{bY} \quad 4.4$$

where = p_d diametral pitch

$$Y = 2xP_d = \text{Lewis form factor} \quad 4.5$$

$$\sigma_t = \frac{F_t P_d K_a K_s K_m}{b Y_j K_v} \quad 4.6$$

where K_a = application factor , K_s = size factor ,

K_m = load distribution factor, K_v = dynamic factor,
F_t = normal tangential load, Y_j= Geometry factor.

V. The Two Dimensional Model.

Fatigue or yielding of a gear tooth due to excessive bending stresses are two important gear design considerations. In order to predict fatigue and yielding, the maximum stresses on the tensile and compressive sides of the tooth, respectively, are required. In the past, the bending stress sensitivity of a gear tooth has been calculated using photo elasticity or relatively coarse FEM meshes. However, with present computer driving gear, the iterative procedures were used to solve the static equilibrium of the gear pair and to calculate the load distribution on the contact lines and the static transmission error. However the contact deformation was excluded in those models. This section considers a FEA model, which was used to predict static transmission error of a pair of spur gears in mesh including the contact deformation. The model involves the use of 2-D elements, coupled with contact elements near the points of contact between the meshing teeth. When one pair of teeth is meshing one set of contact elements was established between the two contact surfaces, while when two pairs of teeth are meshing two sets of contact elements were established between the two contact bodies. When gears are unloaded, a pinion and gear with perfect involutes profiles should theoretically run with zero transmission error. However, when gears with involute profiles are loaded, the individual torsional mesh stiffness of each gear changes throughout the mesh cycle, causing variations in angular rotation of the gear body and subsequent transmission error. The theoretical changes in the torsional mesh stiffness throughout the mesh cycle match the developed static transmission error using finite element analysis shown in Figure 8.

The literature available on the contact stress problems is extensive. But that available on the gear tooth contact stress problem is small, especially for transmission error including the contact problem. Klenz [5] examined the spur gear contact and bending stresses using two dimensional FEM. Coy and Chao [6] studied the effect of the finite element grid size on Hertzian deflection in order to obtain the optimum aspect ratio

at the loading point for the finite element grid. Gatcombe and Prowell [7] studied the Hertzian contact stresses and duration of contact for a very specific case, namely a particular rocket motor gear tooth. T say [8] has studied the bending and contact stresses in helical gears using the finite element method with the tooth contact analysis technique. However, the details of the techniques used to evaluate the transmission error including contact stresses were not presented.

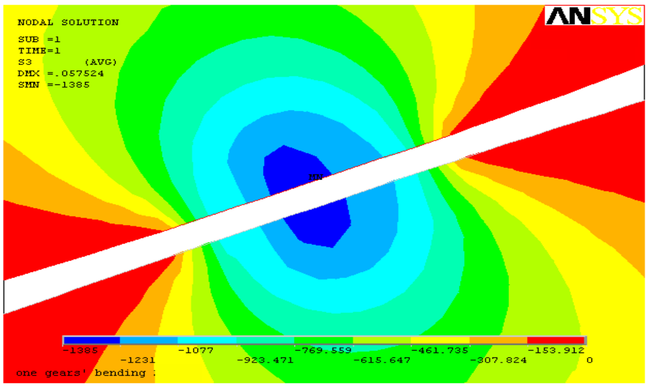

Figure 10 The distribution of contact stresses between two teeth

6 The Transmission Error. The static transmission error is expressed as a linear displacement at the pitch point. A kinematic analysis of the gear mesh allows determining the location of contact line for each loaded tooth pair. These contact lines were discretized. The total length of lines of contact grows with the applied load. For each position of the

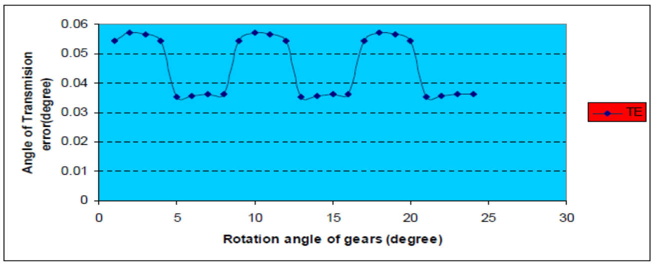

Figure 11 static transmission error from ansys

VI. Conclusion

Mesh stiffness variation as the number of teeth in contact changes is a primary cause of excitation of gear vibration and noise. This excitation exists even when the gears are perfectly machined and assembled. Numerical methods using 2-D FEM modelling of toothed bodies including contact elements have been developed to analyze the main static transmission error for spur gear pairs. Numerous simulations allowed validating this method and showed that a correct prediction of transmission error needed an accurate modelling of the whole toothed bodies.

The elasticity of those bodies modifies the contact between loaded tooth pairs and the transmission error variations. The developed numerical method allows one to optimize the static transmission error characteristics by introducing the suitable tooth modifications. These offer interesting possibilities as first steps of the development of a transmission system and can be also successfully used to improve to control the noise and vibration generated in the transmission system.

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