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Thermal Stress Analysis of Orthotropic Graded Rotating Discs

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Abstract: *The present study deals with stress analysis on orthotropic graded rotating annular discs subjected to temperature distributions parabolically decreasing with radius. I have used infinitesimal deformation theory of elasticity and for graded parameters power law functions in the solution procedure. With the increasing temperature, the tangential stress component decreased at the inner surface whereas increased at the outer surface, and the radial stress component reduced gradually for all the temperature distributions. The magnitude of the tangential stress component was higher than ones of the radial stress component under the room temperatures for both discs. But, the tangential stress component decreased more at the inner surface whereas it increased at the outer surface when the temperature increased further. Finally, the radial displacement at the outer surface had higher value than that of the inner surface with the increasing temperature.*

keyword: *ANSYS,FGM,FGD,*

I. Introduction

Composites are gradually being used as structural materials in many aerospace and automobile applications. The reinforcement in these composites is generally distributed uniformly. Functionally graded composite materials (FGMs) have been the subject of intense researches and attracted considerable attention in resent years. FGMs are being used as interfacial zone to improve the bonding strength of layered composites, to reduce the residual and thermal stresses in bonded dissimilar materials and as wear resistant layers in machine and engine components (Pindera et al., 1995; Erdoğan, 1995). One of the advantages of FGMs over laminates is that, due to continuous material property variation, there is no stress build-up at sharp material boundaries thus eliminating potential structural integrity issues such as delamination. Analysis of rotating discs is an important issue in mechanics and engineering applications.

An analytical solution for the stress analysis in the isotropic rotating disc or a disc under pressure can be found in literature (Timoshenko and Goodier, 1951). Çallioğlu et al. (2006) investigated elastic-plastic stress analysis of the curvilinearly rotating discs.

Sayman (2006) studied elastic-plastic stress analysis of a thermoplastic composite disc under uniform temperature distribution analytically and using a finite element commercial code (ANSYS).

Singh and Ray (2002) investigated creep in orthotropic aluminum-silicon carbide composite rotating disc by using Hill's anisotropic yield. In that study, the results obtained have been compared with the results obtained using von Mises yield criterion for the isotropic composites. In all of these studies, tangential and radial elasticity moduli are constant, that is, the orthotropy degree is a constant. Since the mathematical problems arising are complicated, much of the work on FGMs has been carried out numerically. Nevertheless, the mechanical and mathematical modeling of FGMs is currently an active research area.

Horgan and Chan (1999a, b) investigated the stress response in rotating disks and pressurized hollow cylinder or disk made of functionally graded isotropic linearly elastic materials. They investigated a body with Young's modulus varying radially only.

Horgan and Baxter (1996) examined the externally pressurized hollow **FGD,** stresses in functionally graded discs.

thermal stresses in the basic structural components of FGMs.

Jahed H, Dubey RN [1] has presented a modification of the Tsai-Wu criterion, needed in the case of the multi-criterion optimal design of thin-walled composite structure and a proposal of the evaluation of the load carrying capacity of multi-layered composites with respect to their failure mode.

Jahed H, Sherkatti S., [2] have presented a semi-analytical three-dimensional elasticity solution for rotating functionally graded disks. Their solution includes the responses of both of the hollow and solid disks and is a generalization of the two-dimensional plane-stress solution.

variation along the radius.

Leopold WR [5] has examined the stress analysis on orthotropic rotating annular disks subjected to various temperature distributions, such as uniform, linearly increasing and decreasing with radius temperatures.

Hosseini Kordkheili and Naghdabadi [6] have presented a semi-analytical thermoelasticity solution for hollow and solid rotating axisymmetric disks made of functionally graded cylinder (or disk) with radially orthotropic material.

Durodola and Attia (2000) investigated deformation and stresses in functionally graded rotating disks. They compared two methods, finite element method (ABAQUS) and direct numerical integration of governing differential equations, with each other.

Zenkour (2007) dealt with a solution for a rotating annular disk which is assumed to be graded in the radial direction according to a simple exponential-law distribution.

Chen et al. (2007) presented three-dimensional

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analytical solution for a rotating disc of functionally graded materials with transverse isotropic. A significant amount of these studies has been done in order to see the effects of the FGMs (variation of the elasticity modulus only) on the isotropic discs and cylinders.

You et al. (2007) investigated stress analysis on the FG rotating circular discs under uniform temperature.

Zhang et al. (2003) provided an exact thermal stress solution for a functionally graded plate that has a circular hole, with the material properties and applied temperature varying arbitrarily in the radial direction.

Mohammadi and Dryden (2008) examined the role of nonhomogeneous stiffness on the thermoelastic stress field. In their contribution, Young's modulus and thermal expansion were considered to change symmetrically across the radius representing coatings on inner and outer radii of the beam.

Çallioğlu (2008) studied the stress analysis of the rotating hollow discs made of functionally graded materials under internal and external pressures. In that study, elasticity modulus and density change in the radial direction. In the study it is assumed that the isotropic material is of radially varying elasticity modulus *E*, density *ρ* and thermal expansion coefficient *α* and Poisson's ratio ν as a constant.

Closed-form solutions for stresses and displacements in functionally graded annular discs rotating at a constant angular velocity and subjected to temperature varying parabolically along the radial direction is obtained using the infinitesimal theory of elasticity and for functional graded case power-law function.

II. THERMAL STRESS ANALYSIS

Due to the fact that this problem is axisymmetric, the equilibrium equation for a rotating thin disc is:

$$\frac{r d\sigma}{dr} + \sigma_r - \sigma_\theta + \rho(r)\omega^2 r^2 = 0 \qquad (2.1)$$

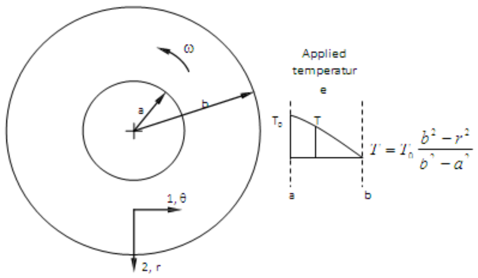


Figure 1. a functionally graded rotating disk under parabolic temperature distribution

Where, σ*r*, σ_θ, ω and ρ*(r)* are respectively, the radial stress, tangential stress, angular velocity and the radially varying material mass density. *r* is the radial distance, r ≠0 and a < r <.b. Here *a* and *b* are inner and outer radii of disc illustrated in Figure 1. The solution can be efficiently handled by using a special stress function that automatically satisfies the equilibrium in Equation 2.1. The particular stress-stress function relation with this property is given by

$$\sigma_r = \frac{F}{r}, \quad \sigma_\theta = \frac{dF}{dr} + \rho(r)\omega^2 r^2 \qquad (2.2)$$

Where, *F = F(r)* is the stress function.

The governing equation for the stress function is determined from the compatibility statement. For this axisymmetric case, the displacement field is of the form *u*= u_r = $u_r(r)$ and u_θ = 0. Therefore, the strain field is given by:

$$\varepsilon_r = \frac{du}{dr}, \quad \varepsilon_\theta = \frac{u}{r}, \quad \varepsilon_{r\theta} = 0 \qquad (2.3)$$

Where, ε*r*, ε_θ and *u* are the strains in radial and tangential directions and displacement component in the radial direction. Eliminating *u* from these equations develops the simple compatibility statement:

$$\varepsilon_r = \varepsilon_\theta + r\frac{d\varepsilon_\theta}{dr} \qquad (2.4)$$

Using Hooke's law for plane stress case, the strains are given by:

$$\varepsilon_r = \frac{1}{E(r)}(\sigma_r - \nu\sigma_\theta) + \alpha(r)T(r) \qquad (2.5)$$

$$\varepsilon_\theta = \frac{1}{E(r)}(\sigma_\theta - \nu\sigma_r) + \alpha(r)T(r)$$

Where, *E*, *α* and *T* are respectively, elasticity modulus, thermal expansion coefficient and temperature change, and it is assumed that the disc is of material properties (*E*,*ρ* and α) and thermal change (*T*) varying through the radial section. Poisson's ratio υ is assumed that a constant due to the fact that its variation has much less practical significance than that in the other material properties. Using this result in the compatibility relation to, Equation 2.4 generates the governing equation:

$$r^2 F'' + rF'\left(1 - r\frac{E'(r)}{E(r)}\right) + F\left(\nu r\frac{E'(r)}{E(r)} - 1\right) =$$
$$-\rho(r)\omega^2 r^3\left(3 + \nu - r\frac{E'(r)}{E(r)}\right) - \rho'(r)\omega^2 r^4 - E(r)r^2(\alpha'(r)T(r) + \alpha(r)T'(r)) \qquad (2.6)$$

The superscript "n" represents derivatives with respect to *r*. Let us assume for the sake of argument that:

$$E(r) = E\left(\frac{r}{b}\right)^{n_1} \qquad (2.7)a$$

$$\rho(r) = \rho\left(\frac{r}{b}\right)^{n_2} \qquad (2.7)b$$

$$\alpha(r) = \alpha\left(\frac{r}{b}\right)^{n_3} \qquad (2.7)c$$

$$T(r) = T_0\left(\frac{b^2 - r^2}{b^2 - a^2}\right) \qquad (2.8)d$$

Where, n_1, n_2 and n_3 are dimensionless arbitrary constants (gradient parameters) and T_0 is the initial temperature at the inner surface of the functionally graded disc. The differential Equation 2.6 reduces to:

$$r^2 F^n + rF'(1 - n_1) + F(\nu n_1 - 1) =$$
$$-\rho\omega^2 \frac{(3 + \nu - n1 + n2)}{b^{n_2}} r^{n_2+3} - \frac{E\alpha T_0 n_3}{b^{n_1+n_2-2}(b^2 - a^2)} r^{n_1+n_2+1} + \frac{E\alpha}{b^{n_1+}} \qquad (2.9)$$

The stress function *F* can be written as:

$$F = C_1 r^{((n_1+m)/2)} + C_2 r^{((n_1-m)/2)} + A r^{n_2+3} + B r^{n_1+n_3+1} + C r^{n_1+n3+3} \qquad (2.10)$$

where $m = \sqrt{(n_1^2 - 4\nu n_1 + 4)}$, C_1 and C_2 are the integration constant

$$A = -\frac{\rho\omega^2(3 + v - n_1 + n_2)}{b^{n_2}(n_2^2 + 6n_2 - n_1 n_2 - 3n_1 + \upsilon n_1 + 8)}$$

$$B = \frac{E\alpha T_0 n_3}{b^{n_1+n_2-2}(b^2-a^2)(n_3^2 + 2n_3 - n_1 n_3 + n_1 + \upsilon n_1)}$$

$$C = \frac{E\alpha T_0 (n_3+2)}{b^{n_1+n_2}(b^2-a^2)(n_3^2 + 6n_3 - n_1 n_3 - 3n_1 + \upsilon n_1 + 8)}$$

As $n_1 = n_2 = n_3 = 0$ and $T_0 = 0$, B and C terms are equal to zero and, the disc becomes the isotropic rotating disc and the stress function F is

$$F = C_1 r + \frac{C_2}{r} - \frac{3 + v}{8}\rho\omega^2 r^3 \qquad (2.11)$$

The stress components can be obtained from the stress function in Equation 9 as:

$$\sigma_r = C_1 r^{((n_1+m-2)/2)} + C_2 r^{((n_1-m-2)/2)} + A r^{n_2+3} + B r^{n_1+n} + C r^{n_1+n_3+2} \qquad (2.12)a$$

$$\sigma_\theta = \frac{(n_1+m)}{2} C_1 r^{((n_1+m-2)/2)} + \frac{(n_1-m)}{2} C_2 r^{((n_1-m-2)/2)} + (n_2+3) A r^{n_2+3} + (n_1+n_3+1) B r^{n_1+n} + (n_1+n_3+3) C r^{n_1+n_3+2} + \rho(r)\omega^2 r^2 \qquad (2.12)b$$

$$C_1 = \frac{D_2 b^{\frac{-n_1+m+2}{2}} - D_1 a^{\frac{-n_1+m+2}{2}}}{b^m - a^m}$$

$$C_2 = \frac{D_1 b^m a^{\frac{-n_1+m+2}{2}} - D_2 a^m b^{\frac{-n_1+m+2}{2}}}{b^m - a^m}$$

Where,

$$D_1 = -A a^{n_2+2} - B a^{n_1+n_3} - C a^{n_1+n_3+2}$$
$$D_2 = -A b^{n_2+2} - B b^{n_1+n_3} - C b^{n_1+n_3+2} \qquad (2.13)$$

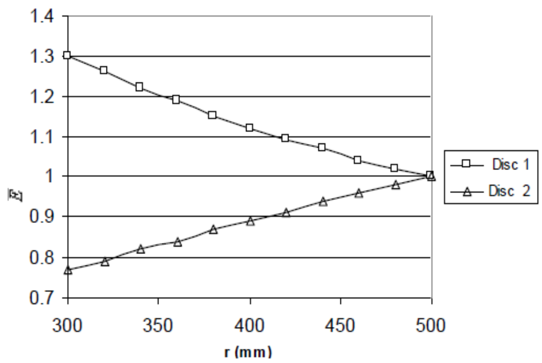


Figure 2. The normalized elasticity modulus distributions along the radial direction of discs.

Radial displacement component

Radial displacement by using the infinitesimal deformation theory of elasticity can be determined as:

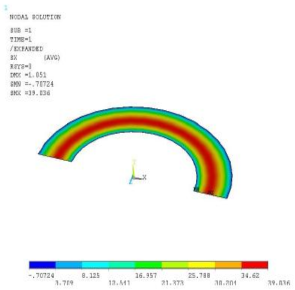
$$u = \frac{r}{E(r)}(\sigma_\theta - \nu\sigma_r) + r\alpha(r)T(r) \qquad (2.14)$$

III. Results and Discussion

In this paper, a thermal stress analysis is carried out on functionally graded rotating annular discs by using an analytical solution including small deformation of theory of elasticity. The results are presented for Poisson's ratio v= 0.3 and angular velocity ω = 650 *rad / s* . The inner and outer radii of the discs are a = 300 mm and *b* = 500 mm, respectively. Mechanical properties of the discs, such as elasticity modulus, density and thermal expansion coefficient, and temperature applied are assumed to be varying along the radial direction. The material coefficients are taken to be elasticity modulus E= 150 GPa, density ρ = 5600 kg/m^3 and thermal expansion coefficient α = 23 × 10^{-6} 1/°C, gradient parameters n_1 = -0.5194, n_2 = -0.4873 and n_3 = 0.55236 for Disc 1 and n_1 = 0.5194, n_2 = 0.4873 and n_3 = -0.55236 for Disc 2 (You et al., 2007). Temperature change is set to T_0 = 0, 300 and 600°C, respectively. If room temperature is considered as reference temperature, the room temperature should be added to the initial temperature, T_0. Elasticity modulus, density and thermal expansion coefficient variations are given as normalized values along the radial direction of the discs in order to demonstrate the effects of FGMs on the discs. For *E*, ρ and α, the following formal normalized variables are used:

Figure 2 illustrates the variations of the normalized elasticity modulus *E* along the radial direction for Discs 1 and 2. As seen in this figure, the elasticity modulus is equal to elasticity modulus of the isotropic, homogeny disc at the outer surface. In the inner surface of the disc, elasticity modulus value increases gradually for Disc 1 while decreases for Disc 2. Figure 3 shows the variations of the normalized density ρ along the radial direction for Discs 1 and 2. As seen from this figure, the density value is equal to density of the isotropic, homogeny disc at the outer surface. Density value in the inner surface of the disc increases gradually for Disc 1 whilst decreases for Disc 2. Figure 4 depicts the variations of the normalized thermal expansion coefficient α along the radial direction for Discs 1 and 2. As seen from this figure, the thermal expansion coefficient value is equal to thermal expansion coefficient of the isotropic, homogeny disc at the outer surface. But, in the inner surface of the disc, contrary to *E* and ρ thermal expansion coefficient value increases gradually for Disc 2 whilst decreases for Disc 1. As seen from the last three figures, when the normalized elasticity modulus and density values for Disc 1 increase about 1.3 times of isotropic disc, the normalized thermal expansion coefficient value decreases about 0.23 times of isotropic disc. These values for Disc 2 are the opposite of Disc 1 approximately. That is to say, thermal expansion coefficient decreases when elasticity modulus and density increase, or this situation is diametrically opposite. Variations of the temperature applied along the radial section of the discs are depicted in Figure 5. The temperature applied is of a variation decreasing parabolically from inner surface to outer surface along the radial direction. Figure 6 shows radial stress distributions along the radial section of both discs. Due to the boundary conditions, the radial stress is equal to zero at the inner and outer surfaces. For both discs, it has positive value at the reference temperature (T_0 = 0°C). However, with the increasing temperature it decreases gradually. The reduction is also very much in Disc 2 when it is small for the other. Figure 7 illustrates tangential stress distributions along the radial direction of both discs. For both discs, the magnitudes of the tangential stresses are of lower values in the outer surface when they have higher values in the inner surface at the reference temperature, and after the middle section of the discs the stresses values come close to each other. But, with the increasing temperature the tangential stresses decrease at the inner surface whereas they increase at the outer surface for both discs. Figure 8 depicts radial displacement distributions along the radial section of both discs. The radial displacements increase less in the inner edge whereas increase more in the outer edge with the increasing temperature when they in the reference temperature adjacent to each other, approximately. If the both discs are compared with each other, it can be seen from the figure that the radial displacement values in Disc 2 are higher than those in Disc 1.

3.1 Radial Stress Plot



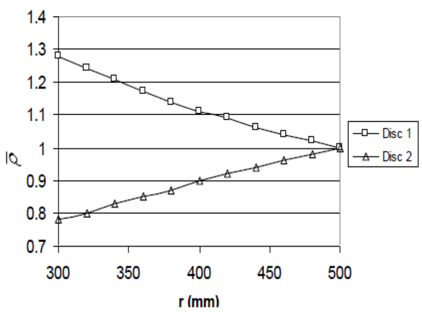


Figure 3. The normalized density distributions along the radial section of the discs.

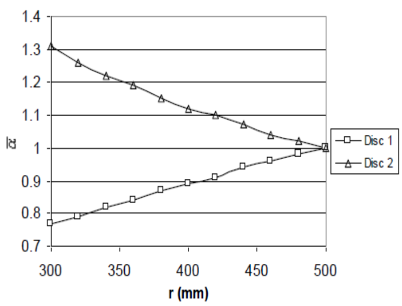
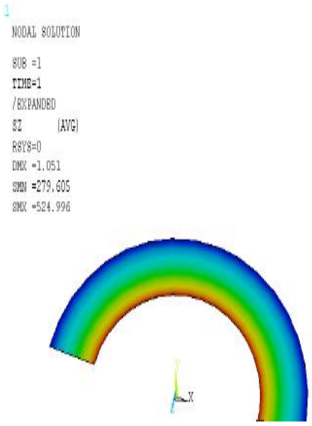


Figure 4. The normalized temperature expansion coefficient distributions along the radial section of the discs.

3.2 Tangential Stress Plot



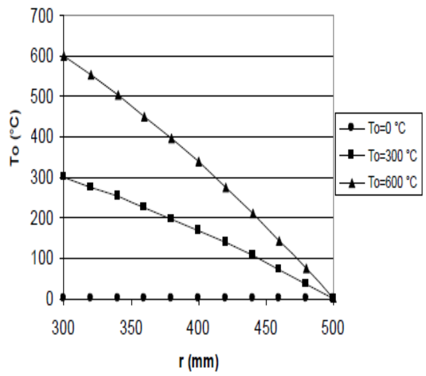


Figure 5. The temperature applied distributions along the radial section of the discs.

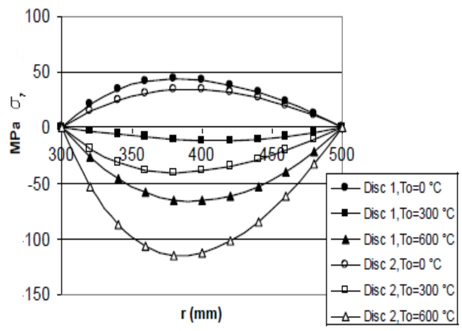


Figure 6. Radial stress distributions along the radial section of the discs.

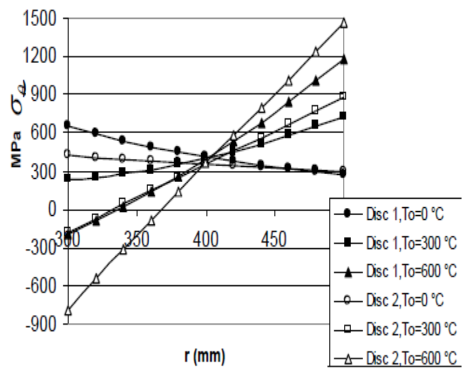


Figure 7. Tangential stress distributions along the radial section of the discs.

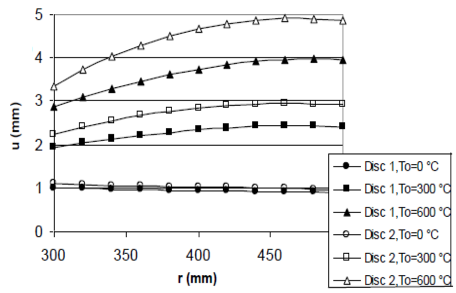
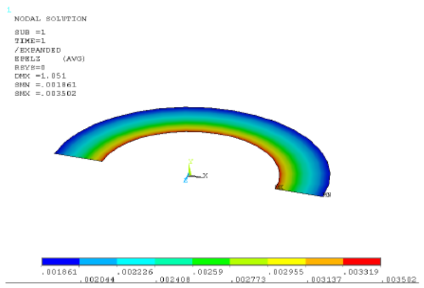
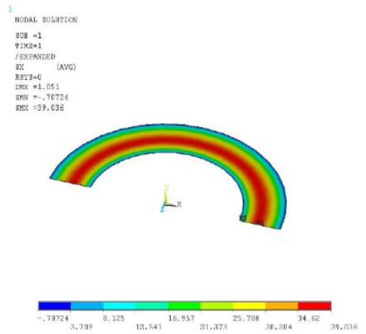


Figure 8. Radial displacement distributions along the radial section of the discs.

3.3 Tangential Strain Plot



3.4 Radial Displacement Plot



IV. CONCLUSION

The following conclusions are found from the thermal

Table1. Comparison of analytical and numerical results in rotating isotropic homogeny disc.

procedure	surface	$\sigma_{\theta(MPa)}$	U(mm)	ε_θ	$(\sigma_r)_{max}$ (MPa)
Numeric	Inner	524.996	1.051	.00350	39.0360
	Outer	279.605	0.93062	.00186	
Analytic	Inner	525.252	1.0505	.00350	39.0371
	outer	279.188	0.93062	.00186	

Maximum radial stresses, tangential stresses, radial displacements and tangential strains values obtained from ANSYS commercial finite element analysis program and the present analytical study at the inner and outer surfaces of the only rotating isotropic homogeny disc are given in Table 1 and their numerical results are depicted in Figure 9. As seen in the table, the analytical results are compared with the numerical results which are obtained from ANSYS and they are found to be consistent with each other. stress analysis of the functionally graded discs:

1) Thermal expansion coefficient decreases when elasticity modulus and density increase, or this situation is diametrically opposite for functionally graded materials.

2) The tangential stress components are found to be highest at the inner surface but lowest at the outer surface for both Discs. They decrease at the inner surface whereas increase at the outer surface by increasing temperature.

3) The radial stress components decrease gradually along the radial section when the temperature is increased.

4) The magnitudes of the tangential stresses are higher than those of the radial stresses.

5) The analytical solution gives the radial displacement component at each point. The radial displacements increase more and more at the inner and outer surfaces by increasing temperature for both discs.

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