Response Spectrum Modal Analysis of Buildings using Spreadsheets

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ABSTRACT: *Seismic analysis of any structure is now mandatory requirement for design of structures. These analyses are very time consuming and tedious. Problem is to be analyzed as per country code only (e.g. IS 1893-2002 for India) in which building is situated. An attempt has been made to make these analyses simple using spread sheets. Spread sheet has been prepared for analysis of structure by response spectrum analysis using SRSS method and CQC method.*

Keywords: *Earthquake analysis, Response spectrum analysis, Modal combination, SRSS, CQC.*

I. INTRODUCTION

Understanding and mastering the seismic analysis of structures is a challenging yet essential element in the field of civil engineering [1-2]. Use of spreadsheets in Civil engineering is reported by various authors [3-7]. In this paper, an Excel-based analysis tool is presented, which attempts to address some of the challenges associated with the problem. The basic principle of the approach is anchored in the need and desire to empower the designer to explore effects of the various parameters on the response of the structure in an environment designed to generate rigorous solutions. The paper also explores the benefits of coupling programming and spreadsheet calculations.

II. RESPONSE SPECTRUM MODAL ANALYSIS OF BUILDINGS USING IS 1893(PART1)-2002

As per IS 1893 (part1)-2002 [8], dynamic analysis shall be performed to obtain the design seismic force, and its distribution to different levels along the height of the building and to the various lateral load resisting elements, for the following buildings:
a) Regular buildings — Those greater than 40 m in height in Zones IV and V, and those greater than 90 m in height in Zones II and III.
b) Irregular buildings - All framed buildings higher than 12 m in Zones IV and V, and those greater than 40 m in height in Zones II and III. Dynamic analysis may be performed by The Response Spectrum Method. Procedure is summarized in following steps.

(a) Modal mass (M_k) – Modal mass of the structure subjected to horizontal or vertical as the case may be, ground motion is a part of the total seismic mass of the Structure that is effective in mode k of vibration. The modal mass for a given mode has a unique value, irrespective of scaling of the mode shape.

$$M_k = \frac{\left[\sum w_i \, \phi_{ik}\right]^2}{g \sum w_i \, \phi_{ik}^2}$$

Where
g = acceleration due to gravity,

ϕ_{ik}= mode shape coefficient at floor i in mode k

W_i = Seismic weight of floor i.

(b) Modal Participation factor (P_k) – Modal participation factor of mode k of vibration is the amount by which mode k contributes to the overall vibration of the structure under horizontal or vertical earthquake ground motions. Since the amplitudes of 95 percent mode shape can be scaled arbitrarily, the value of this factor depends on the scaling used for the mode shape.

$$P_k = \frac{\sum w_i \, \phi_{ik}}{\sum w_i \, \phi_{ik}^2}$$

(c) Design lateral force at each floor in each mode – The peak lateral force (Q_{ik}) at floor i in

Mode k is given by

$$Q_{ik} = A_{hk} \phi_{ik} \, P_k \, W_i$$

Where,

A_{hk} = Design horizontal spectrum value using natural period of vibration (T_k) of mode k.

= $(Z/2)(I/R)(S_a/g)$
Z= zone factor for the maximum considered earthquake (MCE), Z/2 stands for DBE
I= Importance factor depending upon the functional use of the structures
R= Response Reduction factor

S_a/g= Average response acceleration coefficient for rock or soil sites as given by response spectra and based on appropriate natural periods and damping of the structure

(d) Storey shear forces in each mode – The peak shear force (V_{ik}) acting in storey i in mode k is given by

V_{ik} = □ □ Q_{ik}

(e) Storey shear force due to all modes considered – The peak storey shear force (V_i) in storey i due to all modes considered is obtained by combining those due to each mode as per SRSS or CQC methods.

SRSS method

If the building does not have closely spaced modes, than the peak response quantity (□) due to all modes considered shall be obtained as per Square Root of Sum of Square method

$$\lambda = \sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2 + \lambda_4^2 + \dots}$$

Response Quantity λ could be any response quantity of interest such as base shear, force in a member etc.

CQC method

$$\lambda = \sqrt{\sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i \rho_{ij} \lambda_j}$$

Where,

r = Number of modes being considered,
□$_{ij}$ = Cross-modal coefficient
□ i = Response quantity in mode i (including sign)
□ j = Response quantity in mode j (including sign)

$$\rho_{ij} = \frac{8\zeta^2(1+\beta)\beta^{1.5}}{(1-\beta^2)^2 + 4\zeta^2\beta(1+\beta)^2}$$

β_{ij} = frequency ratio = ω_j/ω_i

ω_i = circular frequency in *i*th mode

ω_j = circular frequency in *j*th mode

ζ = modal damping ratio (2% and 5% respectively for steel and RC buildings)

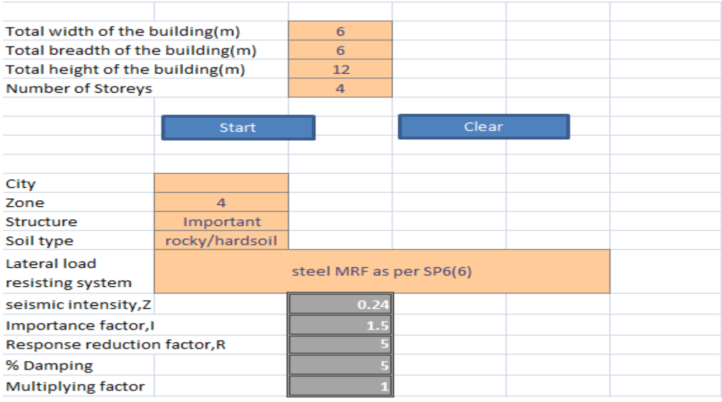
III. BUILDING THE SPREADSHEET

A large set of parameters, some of which are interrelated, is involved in the analysis. In order to facilitate the interaction between the designer and the spreadsheets, which may seem intimidating at first glance, the cell values that require an input by the user are highlighted in light pink and the ones that will be automatically generated are highlighted in dark grey. Calculations are highlighted using orange colour and outputs of those calculations are shown in light grey.

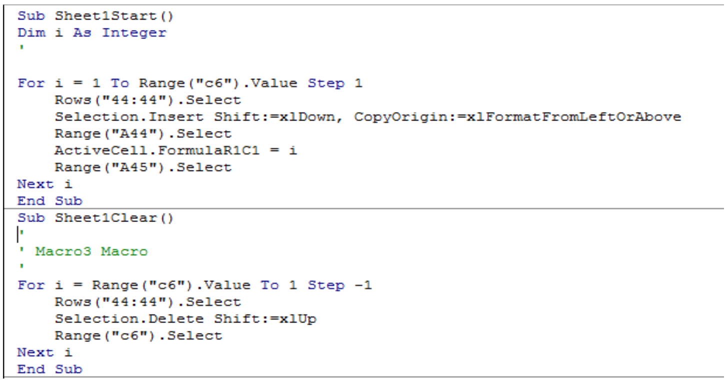
Some of the calculations involve trial and error procedures and loops. In such instances, Visual BASIC macros were assigned to specifically created buttons for the purpose of running these loops and obtaining the required parameter value with minimal effort. The idea is to lessen time spent on performing tedious and repetitive tasks which encroach on the "quality-learning time" and reduce the tedium to a few mouse clicks while maintaining the advantages of spreadsheet interfaces. It is important to add here that the automated processes do not create "black boxes" invisible to the student, but merely perform those tasks that are fully understood and clearly explained.

After entering all the required parameters the response of the structures, expressed in terms of forces can be calculated using the formulae given in the spread sheet. Macros are used for the generation and clearing of rows corresponding to each storey. Logic used for macros is shown in figure. Macros are displayed in blue in colour. Necessary parameters are given as input in specified cells the values that are shown in dark grey cells are used for calculation of base shear and lateral storey forces. In this method weights, fundamental time period of modes and mode shapes of structure are to be inputted in their specified location. No. of stories is also to be given as input. Macros are used here also for adding and clearing of rows corresponding to floors for the convenience of calculations.

Calculations for A_h, Sa/g and frequencies are also shown. Using these inputs and with formulae given in [8], base shear (V_B) has been calculated using excel. The design base shear (V_B) obtained by response spectrum modal analysis shall be compared with a base shear (V'_B) calculated using a fundamental period *Ta*, Where V_B is less than V'_B, all the response quantities (for example member forces, displacements, storey forces, storey shears and base reactions) shall be multiplied by (V_B/V'_B). Calculations and results of this method are shown in the following figures



Total width of the building(m)	6		
Total breadth of the building(m)	6		
Total height of the building(m)	12		
Number of Storeys	4		
	Start		Clear
City			
Zone	4		
Structure	Important		
Soil type	rocky/hardsoil		
Lateral load resisting system	steel MRF as per SP6(6)		
seismic intensity,Z		0.24	
Importance factor,I		1.5	
Response reduction factor,R		5	
% Damping		5	
Multiplying factor		1	



```
Sub Sheet1Start()
Dim i As Integer
'

For i = 1 To Range("c6").Value Step 1
    Rows("44:44").Select
    Selection.Insert Shift:=xlDown, CopyOrigin:=xlFormatFromLeftOrAbove
    Range("A44").Select
    ActiveCell.FormulaR1C1 = i
    Range("A45").Select
Next i
End Sub
Sub Sheet1Clear()
'
' Macro3 Macro
'
For i = Range("c6").Value To 1 Step -1
    Rows("44:44").Select
    Selection.Delete Shift:=xlUp
    Range("c6").Select
Next i
End Sub
```

An example of complexity of formula for selecting Sa/g value is shown in below figure

Total width of the building(m)	6
Total breadth of the building(m)	6
Total height of the building(m)	12
Number of Storeys	4

Start Clear

City	
Zone	4
Structure	Important
Soil type	rocky/hardsoil
Lateral load resisting system	steel MRF as per SP6(6)
seismic intensity,Z	0.24
Importance factor,I	1.5
Response reduction factor,R	5
% Damping	5
Multiplying factor	1

CALCULATIONS

Fundamental natural period of vibration of the strcture with brick infill,

in the direction of		Ta=0.09h/√d	Ta=0.075h^0.75
	Width	0.484	
	Breadth	0.441	
S_a/g in direction of			
	Width	2.068010798	
	Breadth	2.068010798	
A_h in direction of			
	Width	0.074448389	
	Breadth	0.074448389	

seismic weight= dead weight of the structure+part of imposed load

Q_{i1}	Q_{i2}	Q_{i3}	Q_{i4}
21.33272	2.51865	54.35947	7.344951
43.69239	2.826667	-31.9922	-15.2732
61.68749	-1.014	-26.0497	42.76274
71.32689	-4.09144	29.18655	-22.6613
0	0	0	0

V_{i1}	V_{i2}	V_{i3}	V_{i4}
198.0395	0.239871	25.50414	12.17318
176.7068	-2.27878	-28.8553	4.828225
133.0144	-5.10545	3.136873	20.1014
71.32689	-4.09144	29.18655	-22.6613

		correction factor	4.279801
V_1	200.0458	V_{1c}	856.1565
V_2	179.1268	V_{2c}	766.6272
V_3	134.6581	V_{3c}	576.3098
V_4	80.43417	V_{4c}	344.2423

For calculation of base shear using this CQC method, user has to input frequency ratio matrix in the specified matrix. Using the formulae given in IS 1893, cross modal coefficients are calculated and shown in the form of a matrix shown in orange colour. λ matrix is the base shears calculated for each mode. This λ and λ^T matrices are used for calculating the base shears of the structure. Correction factor is again to be calculated and multiplied to the outputs. Inputs, calculations and results for some input values are shown in the following figures.

Input frequency ratio matrix

	β_1	β_2	β_3	β_4
β_1	1	2.919	0	0
β_2	0.343	1	0	0
β_3	0	0	0	0
β_4	0	0	0	0

Cross modal coefficient

	ρ_1	ρ_2	ρ_3	ρ_4
ρ_1	1	0.006857	0	0
ρ_2	0.006876	1	0	0
ρ_3	0	0	0	0
ρ_4	0	0	0	0

V_{i1}	V_{i2}	V_{i3}	V_{i4}
198.0395	0.239871	25.50414	12.17318
176.7068	-2.27878	-28.8553	4.828225
133.0144	-5.10545	3.136873	20.1014
71.32689	-4.09144	29.18655	-22.6613

V_{i1}^T	198.0395	176.7068	133.0144	71.32689
V_{i2}^T	0.239871	-2.27878	-5.10545	-4.09144
V_{i3}^T	25.50414	-28.8553	3.136873	29.18655
V_{i4}^T	12.17318	4.828225	20.1014	-22.6613

Base shear (CQC Method)

V_1	266.3184	V_{1c}	856.1565
V_2	2.28973	V_{2c}	7.36099
V_3	38.37948	V_{3c}	123.3818
V_4	13.1265	V_{4c}	42.19889
Correction factor		3.214785	

IV. CONCLUSION

In this paper, the process of development and use of a spreadsheet-based tool for the analysis of multi storey frame under seismic loading is documented. In summary, making of excel spreadsheet for seismic analysis of a multi storey frame is found to be fruitful since the calculation time and energy consumed for these analyses using this spreadsheet is negligible when compared to manual calculations.

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