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Dynamic Analysis of Delaminated Sandwich Composites

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ABSTRACT: *The analytical formulation for free vibration of sandwich beams consisting of two different layers and is split along the interface with single and multiple delaminations is presented here. The influence of the delamination size and location on the natural frequency is investigated experimentally for beams with single delamination and the results agree well with present analytical formulation. Modal analysis has also been conducted using finite element package ANSYS and has been compared with the experimental results. It is observed that the delamination reduces the natural frequency and change the mode shape. To identify the exact location and extent of delamination, strain plots are used.*

Keywords: *Debond, Delamination, Free Vibration, Mode Shapes, Natural Frequency*

I. Introduction

Composite materials have fully established themselves as workable engineering materials and are now quite extensively used in various engineering applications wherein weight saving is of paramount importance. However composites are very sensitive to the anomalies induced during their fabrication or service life. These materials are prone to damages such as fibre breakage, matrix cracking and delamination. Delamination is the critical parameter for laminates under compression and one of the most common failure modes in composite laminates. Delamination reduces the load-carrying capability of the structure and can thus lead to an early failure. Delamination can promote early failure by interacting with other failure modes. The presence of delamination in a composite structure affects its integrity as well as its mechanical properties such as stiffness and compressive strength. Vibration based methods are a new approach for damage detection and are more globally sensitive to damage than localized methods such as ultrasonic and thermography methods.

Ahmed [1] presented a finite element analysis technique which includes the effect of transverse shear deformation for sandwich beams without any debond Wang et al [2] examined the free vibration of an isotropic beam split by a through-width delamination. They analyzed the beam as four joined Euler- Bernoulli beams, which were assumed to vibrate 'freely' without touching each other. This was later improved by Mujumdar and Suryanarayanan [3] who then proposed a solution based on the assumption that the delaminated beams were constrained to have identical transverse displacement. Their solution compared favorably with their experiments on homogeneous, isotropic beams with single delaminations. But multiple delaminations further weaken a sandwich beam and cause a shift in natural frequencies. An increased number of delaminations also considerably complicate the problem and hence no analytical solution has been studied in detail so far. In the present study, the formulation of Mujumdar and Suryanarayanan [3] has been extended for beams with multiple delaminations. Pandey and Biswas [4] used mode shape curvature method for damage assessment in composites. This methodology is modified using strain plots for identifying the location and extent of delamination in the present study.

II. Analytical formulation

Fig 1 shows a beam with two arbitrarily located through-width delaminations. The beam is assumed to be homogeneous and isotropic. For the analysis, beam has been subdivided into seven regions, namely two delaminated regions and three integral regions. The delamination region itself is made up of two separate component segments above and below the plane of delamination, joined at their ends to the integral segments. Each segment is modeled as an Euler beam. It is assumed that transverse displacement of the delaminated layers is identical and there is no natural gap between the delaminated layers.

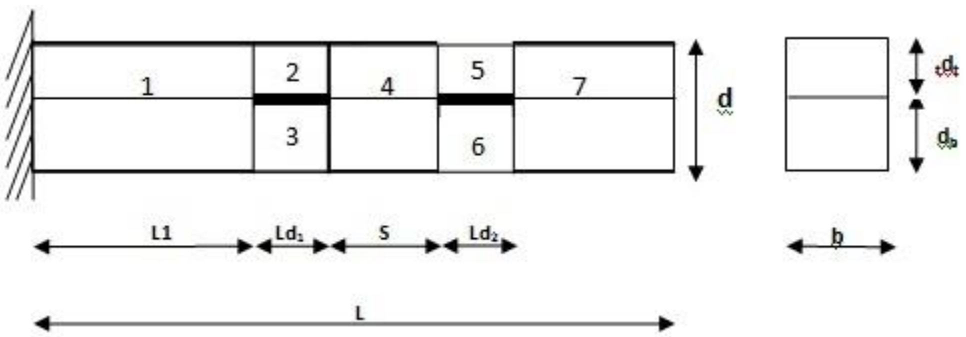

Fig .1 Model of multiple debonded two layered sandwich beam

The governing equations of transverse equilibrium for the intact regions are,

$$\frac{\partial^4 w_i}{\partial x^4} + \frac{\rho A}{EI}\frac{\partial^2 w_i}{\partial t^2} = 0 \text{ , i=1,4,7…………………} \tag{1}$$

In the case of delaminated segments 2 and 3, the governing equation can be written as

$$\frac{\partial^4 w_2}{\partial x^4} + \frac{\rho(A_2 + A_3)}{E(I_2 + I_3)}\frac{\partial^2 w_2}{\partial t^2} = 0 \quad \text{, since } w_2 = w_3. \tag{2}$$

Similarly for delaminated segments 5 and 6, the governing equation can be written as

$$\frac{\partial^4 w_5}{\partial x^4} + \frac{\rho(A_5 + A_6)}{E(I_5 + I_6)}\frac{\partial^2 w_5}{\partial t^2} = 0 \text{, since } w_5 = w_6. \tag{3}$$

Assume harmonic function $w_i(x,t) = W_i(x)e^{i\omega t}$ (4)

where ω is the natural frequency and W_i is the mode shape.

Substituting Eq.(4) in Eqs.(1), (2), (3) and introducing non-dimensional variable $\xi = \frac{x}{L}$, we can obtain the generalized solutions of the Eqs.(1), (2) and (3) as

$$W_1(\psi) = A_1\cosh(\lambda\psi) + A_2\sinh(\lambda\psi) + A_3\cos(\lambda\psi) + A_4\sin(\lambda\psi) \tag{5}$$

$$W_2(\psi) = A_5\cosh(\lambda_{d_1}\psi) + A_6\sinh(\lambda_{d_1}\psi) + A_7\cos(\lambda_{d_1}\psi) + A_8\sin(\lambda_{d_1}\psi) \tag{6}$$

$$W_4(\psi) = A_9\cosh(\lambda\psi) + A_{10}\sinh(\lambda\psi) + A_{11}\cos(\lambda\psi) + A_{12}\sin(\lambda\psi) \tag{7}$$

$$W_5(\psi) = A_{13}\cosh(\lambda_{d_2}\psi) + A_{14}\sinh(\lambda_{d_2}\psi) + A_{15}\cos(\lambda_{d_2}\psi) + A_{16}\sin(\lambda_{d_2}\psi) \tag{8}$$

$$W_7(\psi) = A_{17}\cosh(\lambda\psi) + A_{18}\sinh(\lambda\psi) + A_{19}\cos(\lambda\psi) + A_{20}\sin(\lambda\psi) \tag{9}$$

$$\text{where } \lambda = \left[\frac{\rho A\omega^2 L^4}{EI}\right]^{1/4}, \ \lambda_{d_1} = \lambda\left[\frac{I}{I_2 + I_3}\right]^{1/4} \text{ and } \lambda_{d_2} = \lambda\left[\frac{I}{I_5 + I_6}\right]^{1/4} \tag{10}$$

The 20 unknown coefficients A_1 to A_{20} can be determined using appropriate boundary conditions of intact regions and continuity/equilibrium conditions at delamination boundaries.

The boundary conditions for a cantilever beam are as given below:

1. The deflection at the fixed end is zero, ie. at ψ =0, $W_1=0$ (11)

2. The slope at the fixed end is zero, ie. at ψ =0, $\frac{dW_1}{dx_1}$ =0 (12)

3. The bending moment at the free end is zero, ie. at $x=L$; $\psi = 1$, $\frac{d^2W_7}{{dx_7}^4} = 0$ (13)

4. The shear force at the free end is zero, ie. at $x=L$; $\psi = 1$, $\frac{d^3W_7}{{dx_7}^3} = 0$ (14)

The Continuity/Equilibrium conditions are as given below:

At the delamination boundary, $x=L_1$, $\psi = \frac{L_1}{L} = \psi_1$. Applying the compatibility conditions, the following equations are derived.

1. Deflection compatibility states that $W_1(\psi_1) = W_2(\psi_1)$ (15)

2. Slope compatibility states that $W_1'(\psi_1) = W_2'(\psi_1)$ (16)

3. Bending moment compatibility condition states that

$$EIW_1''(\psi_1) = E(I_2 + I_3)W_2''(\psi_1) \quad (17)$$

4. Shear force compatibility condition states that

$$EIW_1'''(\psi_1) = E(I_2 + I_3)W_2'''(\psi_1) \quad (18)$$

At the delamination boundary, $\psi = \psi_2 = \dfrac{L_1 + L_{d_1}}{L}$, applying compatibility conditions, the following equations are derived.

5. Deflection compatibility states that $W_2(\psi_2) = W_4(\psi_2)$ (19)

6. Slope compatibility states that $W_2'(\psi_2) = W_4'(\psi_2)$ (20)

7. Bending moment compatibility condition states that

$$EIW_4''(\psi_2) = E(I_2 + I_3)W_2''(\psi_2) \quad (21)$$

8. Shear force compatibility condition states that

$$EIW_4'''(\psi_2) = E(I_2 + I_3)W_2'''(\psi_2) \quad (22)$$

Similarly at the second delamination boundaries, we can get 8 more equations. This forms a set of 20 homogeneous equations for 20 unknown coefficients. Solving the eigen value problem, we obtain the natural frequencies and mode shapes.

The above formulation has been used for finding the natural frequencies of beams with different end conditions, viz. both ends fixed and both ends free.

III. Finite Element Analysis

The finite element models used for the present study are generated using a meshing application developed in Visual Basic. A single keypoint is created at the ends of the delamination and two keypoints are created anywhere along the delamination using this program as in Fig 2. The volumes above the delaminated region are defined using one of the two keypoints and the volumes below are defined using the other keypoint. This ensures no connectivity between the volumes above and below the delaminated region and thus it behaves as a delaminated beam. Layered brick element of SOLID45 is used for the modeling of sandwich composite beams. It is an 8-noded layered structural solid element. The element has three translational degrees of freedom per node in the nodal x, y, and z directions [5].

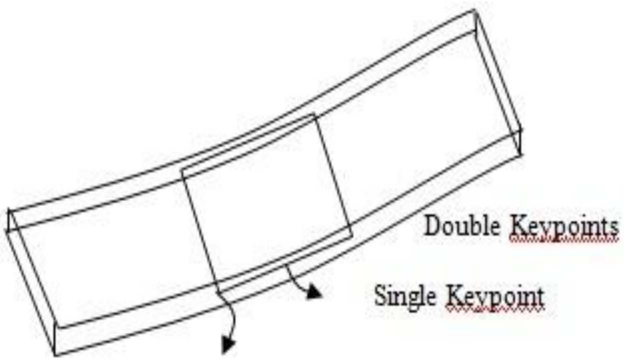

Fig 2. Delamination Modeling in FEM

IV. Experimental Study

An experimental study is carried out to show the validity of the analytical results obtained. The beam specimens are made by bonding thin aluminum strips using a very thin epoxy film adhesive. The delamination is simulated by bonding only a part of the beam surfaces to create unbonded regions at various locations. Delaminations are created by inserting celephone sheet of 50 microns along the width for the required length (50 mm) of delamination.

In order to incorporate the effect of central layer delaminations, the thickness for each aluminium strip is kept as 2mm. The beams are of sizes 300mm×25mm×2mm each. The delamination length of the manufactured specimens is kept as 50 mm.

Fig 3. Two layered metallic beam specimens with 50 mm delaminations

A piezoelectric accelerometer (Model 8730AE500, SNC190699 manufactured by Kistler) with a measuring range of ± 500 g, and sensitivity 9.52mV/g is used for testing. The instrumented hammer is Bruel & Kjaer (Model No. 8202) equipment along with a force gauge (Model No 8200) with an output sensitivity of 9.52mV/g. Weight of the hammer is 280 grams. A PC based 2 channel FFT analyzer is connected to the PC through data acquisition card (PC I MCA card. manufactured by OROS).

Transfer functions only are measured. Frequency versus Phase angle and Frequency versus Amplitude graphs are obtained. The beams are tested for two different boundary conditions, viz. clamped-free and free-free.

V. Results and Discussions

The experimental results obtained for the composite beam specimens with single delamination are found in good agreement with the present analysis results as given in Table 1. Hence the present model can be used for studying the dynamics of debonded sandwich beams. The finite element analysis results are also found to be comparable. The length of delamination is kept constant and the location is varied, near to the fixed end (0.25l-0.4L), middle (0.5L-0.7L) and towards the free end (0.75L-0.9L).

Table 1. Comparison of natural frequency of beams with single debond (50mm)

Specimen details	First mode frequency (Hz)					
	Cantilever			Free- Free		
	Experimental	Analytical	FEM	Experimental	Analytical	FEM
Intact	35.25	35.95	35.39	198.75	215.60	225.00
Debond 0.25L-0.4L	31.25	30.25	32.25	196.25	213.80	223.00
0.50L-0.7L	33.75	33.73	35.30	198.75	215.36	224.00
0.75L-0.9L	34.75	35.80	35.30	195.00	207.75	222.35

The variation in natural frequency for various percentages of debonding lengths for a cantilever beam is given in Table 2 from which it is clear that the presence of debonding reduces the natural frequencies of the composite beam.

Table 2 Natural frequencies for various debonding lengths for a cantilever beam

Size of delamination	Frequency		
	First mode	Second mode	Third mode
Nil	35.393	221.62	619.74
10%	35.358	219.04	601.9
20%	35.122	204.13	540.13
30%	34.515	180.19	513.57
40%	33.447	160.62	508.83
50%	31.967	149.92	435.75
60%	30.25	146.33	304.27
70%	28.506	146.03	224.42
80%	26.908	144.48	172.32
90%	25.559	136.47	139.95

To study the effect of location of debonding on the modal parameters, various parametric studies are conducted by providing a constant delamination length, 50mm at three different locations along the longitudinal directions as shown in

Table 3. The variation in natural frequency with the debond location is as shown in Fig 4. It can be concluded that the debond has a minimum effect on modal frequency if it is towards the free end of the cantilever and has the maximum decrease when it is near to the fixity. Thus, the same bonding length has different effects, with respect to the change in location.

Table 3 Effect of the location of debonding on natural frequency for a cantilever beam

Debond location	First mode frequency (Hz)
Intact beam	35.393
(1) Near to the fixity (debond 25-75mm)	35.236
(2) Left of the centre (debond 100-150mm)	35.263
(3) Right of the centre (debond 150-200mm)	35.301
(4) Near the free end (debond 225-275mm)	35.370

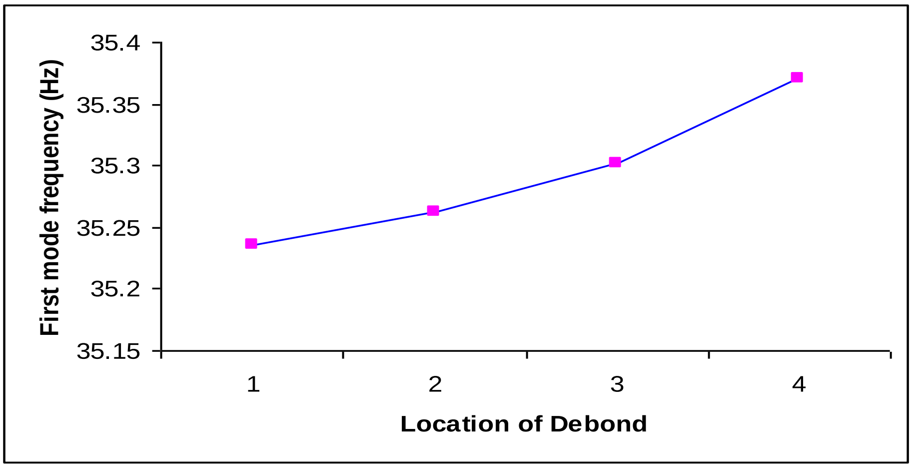

Fig 4. First mode frequency Vs Location of debonds

The mode shapes for the first mode of the cantilever beam, for various debonding lengths (60 mm, 120 mm and 180 mm) are shown along with that for an intact beam in Fig 5.

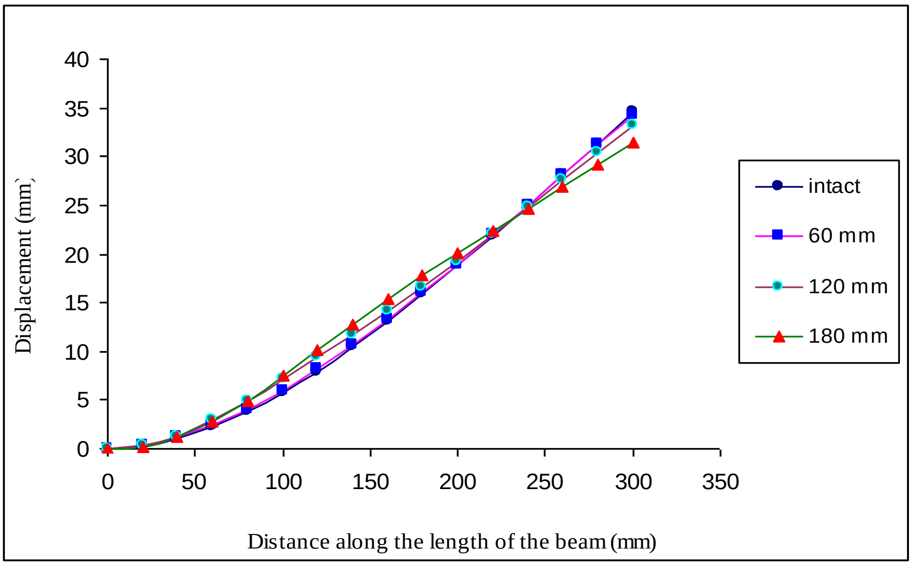

Fig 5. Mode shapes of intact and debonded beam

It can be seen that when the debonding length is very small, the mode shape is identical for both intact and debonded beams A significant change in mode shape is observed only for larger delaminations. Hence to identify the exact location and extent of damage in sandwich composite beams, the strain plots can be used. Fig 6 represents the variation of longitudinal strain for a debond of 50% of total beam length starting extending from 10 mm to 140 mm.

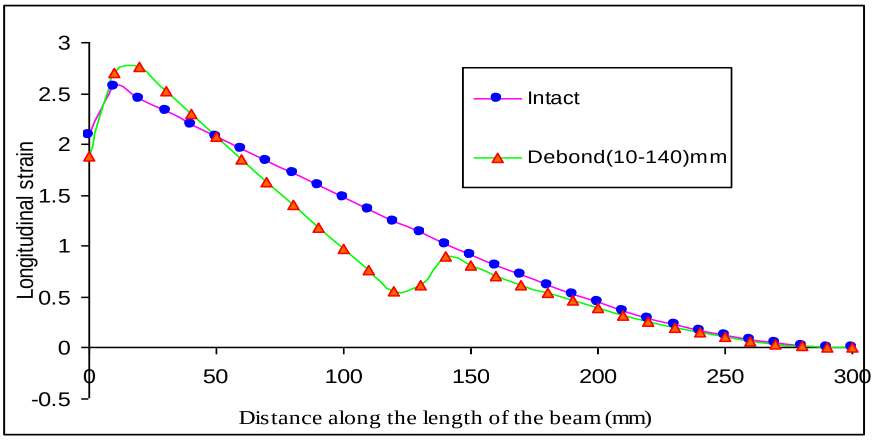

Fig 6 Strain plot of an intact beam and a debonded beam

Table 4 shows the natural frequencies of a double delaminated cantilever beam obtained using the present analytical formulations. The length of first debond (Ld_1) and it's location (L_1) is kept same in all the cases. The length of the second delamination (Ld_2) and the distance between the two debonds (s) is varied.

Table 4 natural frequencies of beam with multiple (two) delaminations

L_1=10 mm, Ld_1=50 mm						
S / Ld_2	5 mm	10 mm	15 mm	20 mm	50 mm	100 mm
	Frequency (Hz)					
10 mm	22.27	22.27	22.27	22.43	22.59	22.91
20 mm	21.48	21.64	21.79	21.80	22.27	22.75
50 mm	20.20	20.52	20.68	20.84	21.63	22.59
100 mm	19.41	19.57	19.88	20.04	21.32	22.60

It is observed that the presence of a second debond does not produce a considerable reduction in natural frequencies compared to that of a beam with single debond. It is clear that as the spacing between two similar debonds is greater than the length of the debond, there is no reasonable change in frequency values and mode shapes. Hence for identifying the exact location of multiple debonds also, the strain plots can be effectively used.

VI. Conclusions

An analytical solution for the free vibration of two layered sandwich beam with multiple delaminations is presented. The influence of the delamination sizes and locations on the natural frequency as well as the mode shapes is investigated experimentally for beams with single delamination and the results agree well with present analytical formulation. In multiple delaminated beams, the effect of a second debond is felt only when it is very close to the first one It is found that for small debonding lengths (less than 30% of the length of the beam), the changes in mode shapes and reduction in natural frequencies compared to that of an intact beam are not significant. But the variations of longitudinal strains for a delaminated beam suffers more irregularities at the delamination location and hence this feature can be effectively used for identifying the extent and location of debond.

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