Area Model Mediating Learning of Area Measurement: A Case Study of African American Students

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Abstract: *The over-arching gap in research is regarding the instructional needs and the cultural capital that African American students bring to the mathematics classroom. Given the reality that most African American students in the United States will learn mathematics from a white, middle-class teacher. Using models of learning in teaching of mathematics for diverse students might bridge the cultural differences between teachers and students. The case study of 5 fifth grade African American students illustrate that using gradual release model with area model in mediating learning of area measurement, allowed these students to achieve the higher levels of abstraction in their thinking.*

Keywords: *area model, cognitive variability, learning trajectories, internalization, context-based problems*

I. Introduction

Measurement is an indispensable everyday mathematical activity worldwide. The idea of measurement is too broad and attempting to define it will do disservice to the topic itself [1]. However, Russell (1900) cited by [1] give a broader description that encompasses aspects of measurements related to this study. They describe measurement of magnitude as a method used to define some properties of a particular object by using mathematical concepts like number, relations and functions. Therefore, measurement is the activity of determining the magnitude of a quantity by comparison with a standard for that quantity. In this paper measurement of magnitude refers to measuring the surface area that uses the properties of the shape measured, using number, rows and columns as vehicle to expose students to the relations between length and area and conceptualize the formula of $l \ X \ b = area$. Area measurement integrates mathematical concepts as [2] state, "it can develop other areas of mathematics including reasoning and logic" (p.163). These other areas are patterns leading to algebraic thought, geometric reasoning leading to logic, and number operations as tools for generalizing etc.

Measurement concept performance of fourth grade US students is lower than number and probability [3], [4], [5]. Black students' average performance on math content in [5] was 217 for measurement, 220 for number, 227 for probability, 229 for algebra and 226 for geometry. According to these results measurement has the lowest average performance for Black students. Few studies [6]; [7] and [8] have investigated fifth grade students' processes in understanding 3D arrays and 2D arrays. Their results showed that students demonstrated complicated structures in working with these arrays. [9] suggested an inquiry-based approach in studying these complicated mental structures. [10] used a cognition-based assessment in understanding these structures. This paper is contributing towards extending their work focusing on area measurement and using area model as a mediation tool. This paper aims to use an area model as a tool in mediating and eliciting processes of conceptual development of area to 5 fifth grade African American students. To describe these thought processes the following question will be addressed:

What role can area model play in mediating learning of area to African American fifth grade students?

This paper is structured into four parts: the *first part* present and describe the theoretical framework employed as lenses for discussing the findings. This is followed by discussion of the area concept this study employs in eliciting African American students' thought processes. The *second part* focuses on the methodology of the reported study. The *third part* presents findings. These findings are presented using themes with measurement learning trajectories hypothesized by [2]. The *fourth part* engages with the current literature in discussing the findings of the study and then draws and presents the final conclusions on the reported study.

II. Theoretical Framework

The theoretical framework used in this paper is influenced by Vygotskian theory of internalization of ideas, and Ernest's philosophy of connectionist approach to teaching of mathematics.

II.1 Vygotskian theory of internalization

Internalization refers to the psychological process of transforming intermental to intramental through mediation of student's cultural tools [11]. The mental structures that form when conceptualization of a concept takes place [2]. [12], [13] asserts that ideas/concepts are tools. When these tools are not personalized they are external however, when they become personalized meaning they become internal tools [14]. This psychological process of personalizing meaning is internalization/interiorization in Vygotskian language.

[15] defines internalization as a five phases process. The phases are "(1) the phase in which the given phenomenon does not manifest itself yet; (2) the phase in which its initial traces seem to appear for the first time, always with corresponding analysis of the psychological tools and social forces that bring this phenomenon to life; (3) the phase in which the phenomenon reaches its climax, always linked to social interaction and usage of tools; (4) the phase of its gradual interiorization; (5) and finally the phase in which it appears that the phenomenon has been there, quite naturally in our heads, resembling inherited individual property that was just waiting its time to be actualized". Ageyev's phases give a clear

developmental path for internalization of ideas using cultural mediation tools. Activities with peers or with an educator encourage social engagement that brings the phenomenon to reach the climax. Clearly for this internalization to occur educator's role is to create activities that connects with students' mediation tools pushing for intermental ideas to be intramental [14]. Social interaction with peers and educator plays an integral part for moving a student towards mental climax. The educator fulfills the purpose of philosophy of mathematics education in assisting students' mental structures to reach actualization of mathematical concepts [15].

II.2 Ernest's philosophy of connectionism

[15] claims that educators have distinct philosophies of mathematics that influence their perspective of philosophy of mathematics education. Absolutist philosophy of mathematics is demonstrated in practice by instruction that is transmissionist. This instruction is characterized by memorization of rules and formulas, [17] while fallibilist philosophy of mathematics is transferred towards creation of ideas that acquire broader instruction [18]. Fallibilist philosopher believes that mathematical knowledge is created instead of being discovered or a set of rules that already exist. This study is influenced by the fallibilist philosophy that if mathematics is created, the creator has to connect new ideas with existing mental structures of students recognizing the influence of their environment as well as their culture that determines their way of knowing. This instructional approach in Ernest' term is a connectionist instructional approach. The teaching experiments employed by this reported study are influenced by connectionism and internalization of concepts or ideas in teaching area measurement.

II.3 Area measurement conceptualization

Research that demonstrates how young children learn reveals that mediation and use of technology lead to internalized measurement concepts [19]; [20]; [21]; [22]; [23]. Nonetheless, research on older children's processes of conceptualizing measurement concepts is still limited.

[24] Described area as a concept that connects a number of mathematical concepts. Understanding of area measurement demands coordination of all the concepts involved, i.e., structuring, adding, and multiplication that leads to functions. [2] Divide the area foundational concepts into six sub-concepts of area that follows:

Attribute of area: Quantitative meaning of space
Equal partitioning: Using congruency is dividing two-dimensional space into equal parts.
Units and unit iteration: Covering space with the same units without gaps or overlaps.
Accumulation and additivity: Adding rows and column repeatedly.
Structuring space*: The concept is* realizing multiplicative structures of the rows and columns.
Conservation: Composing and decomposing of shapes exposing students to the idea of area staying the same even if it is rearranged. [2] argue that these foundational concepts form basis of building students' conceptual understanding of area measurement.

III. Methodology

III.1 Context

The study was conducted in a public school in Western New York. The sampled school comprised of 30 % black population, 10 % of Hispanic students, and the majority (60%) was white students. About 31% of the students qualified for free lunch.

III.2 Participants

There were fourteen 5^{th} grade African American students in the whole school. Seven of them were in a math special class and the other seven in a mainstream math class. Only students in the mainstream math class participated in this study. Out of the seven students, six brought back their parents' consent forms. The math teacher was requested to share students' performance results for the previous year. From the six students with consent, a selection of five who represented varying performance was conducted. Two of these students attended both mainstream and special math classes, two others were high performers while the other one was on average.

III.3 Data Collection

The reported study employed clinical individual pre-interviews and post-interviews with students and teaching experiments. The pre-interviews were conducted to inform the designing of teaching experiments and to instruct at the thinking level of students. Whereas the post-interviews were used as a measuring stick of the area concept gains student made during teaching experiments and to study the role of the area model.

III.3.1 Interviews

The pre and post clinical interviews were conducted prior and after teaching experiments. Each interview was between 10 to 15 minutes. Video and audio recorders were used to collect data. Field notes were taken during interviews for the purpose of triangulating data.

II.3.2 teaching experiments

The teaching experiments focused on mediation of new area measurement concepts, after establishing students' zone of proximal development. [12] zone of proximal development has two linked perspectives that are psychological perspective on student's development and pedagogical development on instruction. He argues that these two perspectives are connected. The student's zone of proximal development (ZPD) is the level the student demonstrates when solving a task independently. Then mediation of new ideas is connected to the student's ZPD for student intellectual development to occur.

The individual teaching experiments were used in order to describe and engage with students personalized ideas that they use in solving problems that involve measurement concepts. One main teaching experiment was designed with follow up episodes intended to address students' needs in understanding the area concept. Each lesson lasted 20 minutes with each student. The follow up episodes varied in time depending on students' level and pace. The teaching experiments were both audio and video taped for audibility and capturing of all different kinds of communication the student undertook during a lesson. The field notes were taken with students' written work to triangulate for credibility in describing students' thinking.

Figure 1 represents the triangulation of the data sources, techniques and tools used in this study.

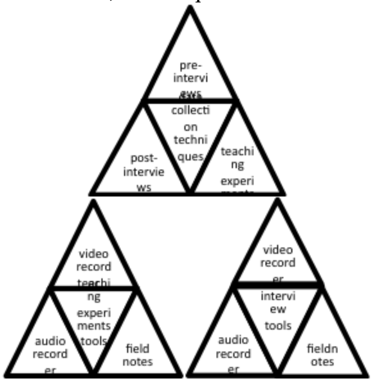


Figure 1: Triangulation of data sources, techniques and tools

II.4 Data analysis

In this study data collection and analysis were inseparable. The analysis continuously informed data collection and also assisted in engaging with the data [25]. Analysis of the pre-interviews of area concepts was conducted using a table. Each student's responses were put in the table and grouped by color-coding similar levels of thinking using [2] learning trajectories for area.

Each color represented a specific level for the area attribute. Below is a summarized description of the learning trajectories used to analyze the interviews reported in this study [14].

II.4.1 Hierarchical Area Measurement learning trajectories

Pre-Area Quantity Recognizer: Students show little specific concepts of area. For example, when students have a task to cover any surface they cover by packing tiles on top of each other without a plan of covering the space at all.

Area Simple Comparer: Students may compare areas using one side. In this case it is possible for students to compare only lengths of the areas without considering the width of the area.

Side-to-Side Area Measure: Students cover a rectangular space with physical tiles but cannot organize, co-ordinate the 2D space. Mostly they cover sides and fiddle with the space in the middle.

Primitive coverer: When counting squares covering the area students skip count, loose track, sometimes count some squares repeatedly. Covering of spacing might have gaps or overlapping.

Area Unit Relater and Repeater: Students count one row at a time. Student has not developed the structure of rows and columns. S/he counts one unit at a time keeping track.

Partial Row Structurer: Counts rows but sometimes does not count all rows. The column existence is not yet realized. The relationship between the rows and the columns has not been recognized yet.

Row and Column Structurer: Students count rows and draw rows of squares to determine area.

Area Conserver: Conserve area by determining different looking surfaces as equal.

Array Structurer: Use multiplicative structures in filling up and counting area [10].

Conceptualization of area measurement is an integral part towards mastering calculating skills of area. The area formula is not the same for different shapes, therefore knowing formulas without understanding creates barriers for further development in understanding the Pick's theorem that allows students to calculate area of any polygon. This might sound abstract but with the understanding or the six sub-concepts of area suggested by [2] give this fundamental understanding. Internalization of these concepts has its own developmental path for each student. The learning trajectories hypothesized by [2] became the relevant tool for analyzing the developmental path of these five African American students.

The analysis of the post-interviews was simply analytical using the learning trajectories. The results of this analysis supported the teaching experiment themes that respond to the question of the study reported. The teaching experiments data was transcribed from the video tapes, and audio tapes. Each transcription was typed with numbered rows to assist analysis. The field notes taken from the interviews were also typed and numbered in rows too. Data from each of the three data sources were analyzed separately. Each typed data was annotated with low inference phrases. Those annotations were put into tables. Each table was color coded to group similar phrases and single out odd ones. Once those annotations were in colors analytical memos were written for each annotated data to make sense of the patterns and odd annotations. Descriptive codes emerged from the three groups of data. The codes were triangulated and area measurement themes emerged. Those themes were context-based instruction, area model, and responsibility.

The three themes responded on "the role of area model in mediating leaning of area and the processes these students went through in conceptualizing area" Each student is reported under the themes that emerged. Figure 2 illustrates how themes were divided in response to the study's question.

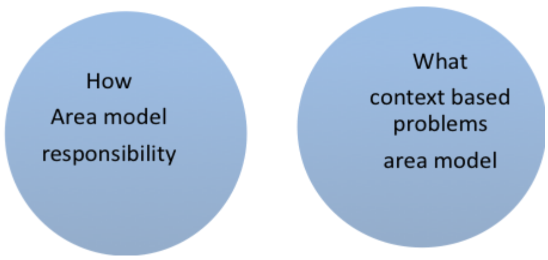


Figure 2: Division of area measurement themes

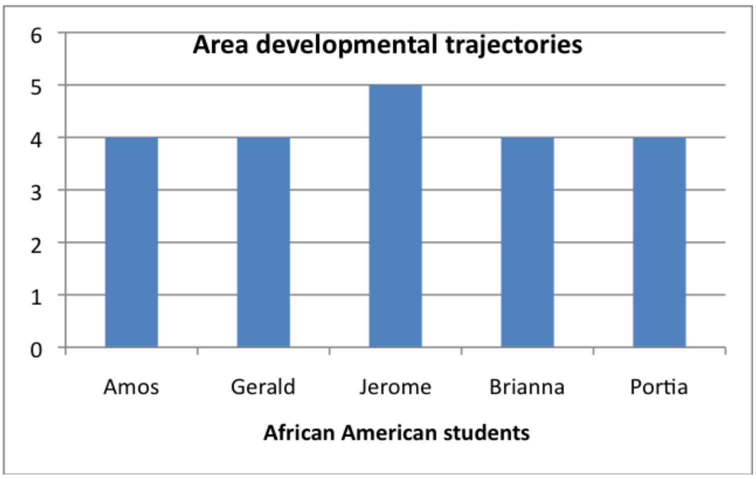
IV. Findings

The findings are presented firstly by describing the area measurement understanding of the fifth grade African American students' prior teaching experiments. Then, emerged themes from the area teaching experiments will be presented and discussed. The findings from the pre and post interviews are presented graphically in order to demonstrate the trajectory levels of the African American students' prior and post teaching experiments.

IV.1 Students' Developmental Progression Levels

The learning trajectories used in this study were designed for young children's developmental progression in measurement concepts [2]. The research conducted to develop these learning trajectories was undertaken from pre-school to 8 yr olds. However, the pre-interviews of this study revealed that these fifth grade students' developmental levels were still missing some developmental progression levels in the early childhood standards. The pre-interview data present the levels of development of the five African American students' prior teaching experiments.

Out of the five students, Portia and Jerome (pseudonyms) were able to cover the shape with the correct number of squares without touching them. They were both able to see the relationship between the square and the triangle, that two triangles cover the square and use their square covering response to get the correct answer for covering surface area with triangles. The other three students could not cover the shape correctly physically, they had overlaps and gaps. They were also unable to neither cover the shape with the triangle nor see any of the relationships between the two shapes. Four of them were primitive coverers only one of them was an Area Unit Relater and Repeater. Below is the graphical presentation of their developmental trajectory levels prior teaching experiments.

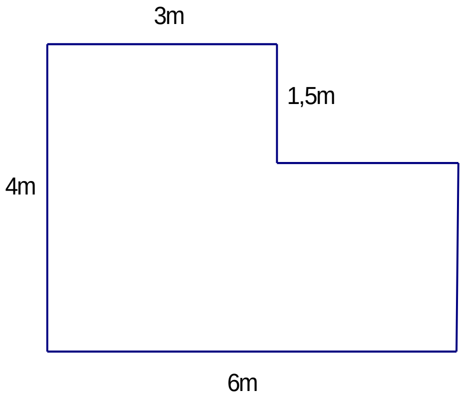


V1.2 teaching experiments themes

The first teaching experiments theme, context based instruction addresses the instructional support that meets African American needs regardless of their challenges. Then two themes, responsibility and area model responded to the African American students' ways of learning area measurement.

IV.2.1 Context based instruction

The shape used to find area had two challenges for the students, first the shape had missing lengths, and secondly it was an L shape. The rationale behind these challenges is to diagnose student's ZPD so as to give the instruction at their actual level and establish their potential. Students' knowledge of a rectangle was the connecting concept as all students knew the properties of a rectangle and could easily use them to learn the area model. For example, Jerome and Amos had a similar approach in attempting to find the missing lengths of the shape of the land. The following was my dialogue with Amos:



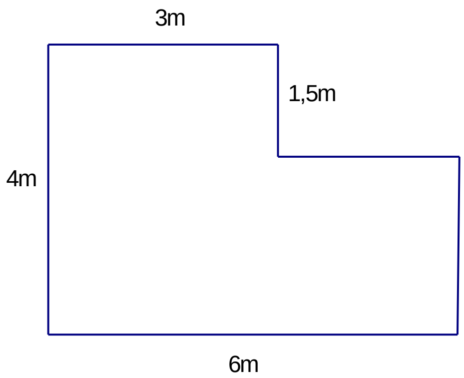
R: This diagram represents the gardening land my uncle has. He wants to calculate the amount of space he has but need fence to protect the land from animals first. He has some missing lengths on it. Help him calculate the missing lengths of his land?

Amos: His length is 14.5 meters.

This step showed that the context of the story made it simple to do perimeter informally. An opportunity presented itself for learning and teaching and the following was my approach in connecting all that Amos brought with him, completing the perimeter problem that was emerging was vital for in-depth understanding of area.

R: Does that mean this side is not going to be fenced and this other side. He does not want any animals in his land. How are we going to get the missing lengths?

Amos: This side is



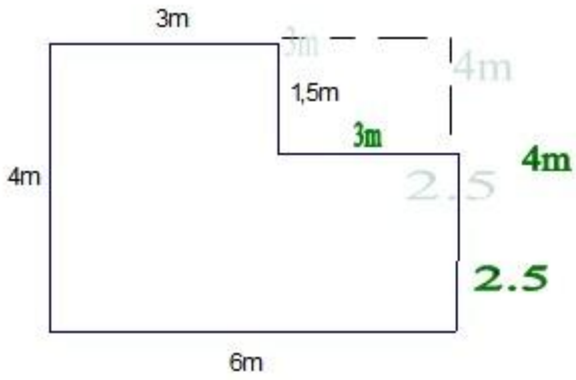
R: Oh you think this side (the 3m side) and this side are the same? (The side he labeled 3m)

Amos: Yes

R: Ok what if we do this. We try and make this shape a familiar one by using some dotted lines as follows:

R: What will be this side if you have to give us the whole side (side marked with arrows)?

Amos: 4m



R: Why is it 4 now?
Amos: Because it's the same as the other one on the other side (pointing opposite side). The side on that side of 6m will also be 6 because they are the same.
R: So what will be the length from this corner to here? (The one below 1.5m)
Amos: 3m
R: Oh, why?
Amos: Because this one is 6m and that one has 3m and the missing one is 3m.
R: Write it then where it belongs. What about the other missing length on that side opposite to 4?
Amos: wrote the following on the paper

1.5
+3.5
5.0

He then wrote again the following:

1.5
+2.5
4.0

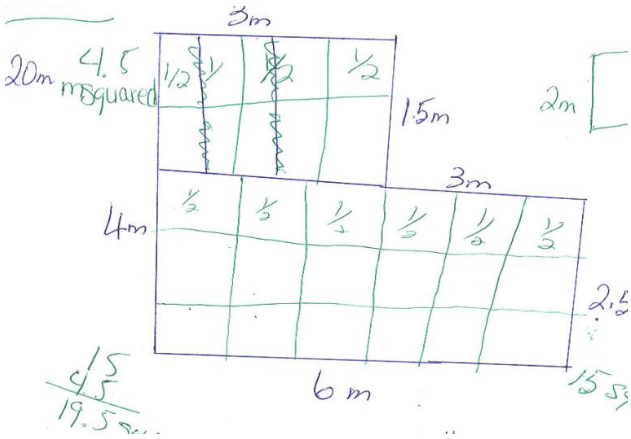
The missing length is 2.5m.

Finding these missing lengths with these five students informed the researcher of their internalized ideas and ideas that have not been internalized yet. The two boys Amos and Jerome were able to transfer their understanding of properties of a rectangle to the L shape in finding lengths. While the other three students, Gerald, Brianna and Portia could not find the missing lengths on their own. They demonstrated that they have not yet internalized the knowledge of properties of a rectangle. It was still external knowledge to them, as they could not transfer it to another context. The researcher had to work with them using rectangles. These teaching episodes are not reported, as they are not the focus of this paper. Area measurement mediation is the focus of this paper.

Area model

The actual problem students have to solve was to calculate the area of the land. Each student experience and approach is reported in learning area using area model. Each student was given the L shape to start with. The Area model pushed individual students differently. The results show that it assisted others to connect ideas that were already there easily.

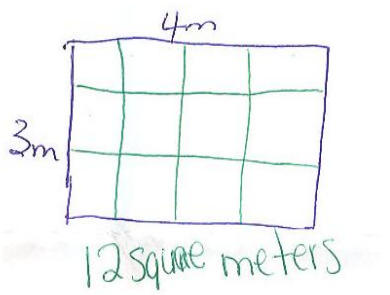
Jerome's journey
R: Divide the garden and calculate the area of each portion.
Jerome: (divides the land this way)



R: Now that you have divided the land, tell me which portion of it you want me to work with you on? Choose the one you want me to help you with? We have a 3 by 1.5 land and a 6 by 2.5 land.
Jerome: This one (pointing a 6 by 2.5 land)
R: If I want to get squares that cover this land. I am going to make columns that are 6 because of the 6m. (Making the 6 columns). Does that make sense? Still I do not have squares I have only columns.
Jerome: I know.
R: What do you know?
Jerome: You need to make rows now that will help you make the squares.
R: Yes. But how many?
Jerome: You need 2
R: Only 2
Jerome: And then you need half way.
R: This is half. So you think we have squares now?
Jerome: Mh mh
R: So how many squares is this portion? Can you count them?
Jerome: 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18. 18 squares.
R: I do not agree with you. These (pointing the row of halves) are not squares they are half squares.
Jerome: Oh---.1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15. 15 squares.
R: You have 15 square meters. Remember Uncle wants to know the Area of the whole land. So you need to do the other portion too.
Jerome: I need 3 columns. I need one and a half across. I have three.
R: What about these? (The row of halves)
Jerome: 4 ½ square meters?
R: You can even write square meters as m^2. What is the area of the whole land then?
Jerome: writes

15
+4.5
19.5 square meters.

R: Can you try and find Area of this one?



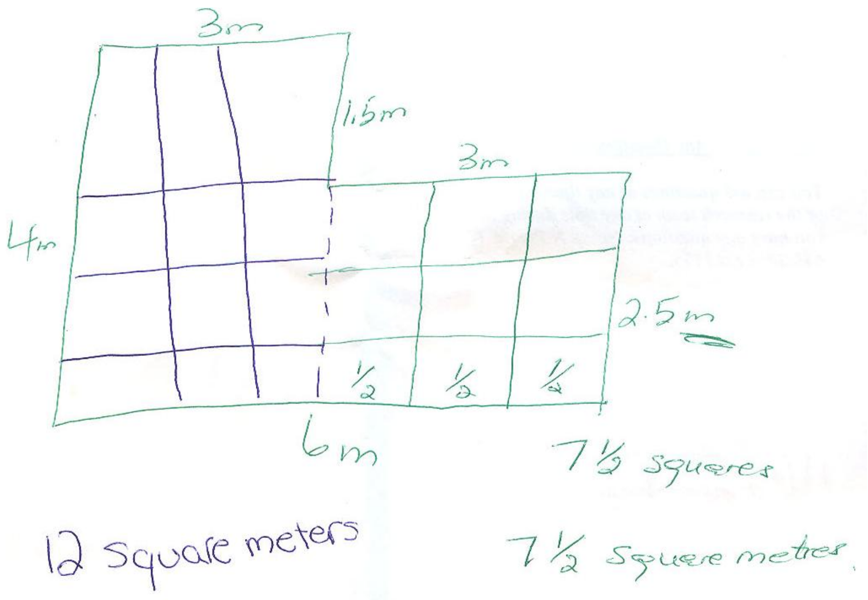
Jerome: (divide the land independently as shown in the diagram above and wrote.) 12 square meters (without counting them).

Jerome started at the second phase of internalization the phenomenon of columns and rows. Before this teaching experiment he could not count all the rows but now a connection took place in his mind that made him see that he needs to draw rows on those columns to get the squares he needed. Seeing the columns the researcher drew caused connection of some mental structures. His pace of internalizing the area model required only 25% of the researcher's demonstration and he took over and ran with it. His challenge was dealing with halves and conceptualizing square as units. This caused climax as he kept on counting half squares as full squares and could not see the relationship between rows and columns before. His area measurement learning path started from partial row structurer straight to an Array Structurer jumping three learning trajectories.

Gerald's journey

This episode demonstrates Gerald's pace and approach in internalizing the area model.
R: Now that the garden is fenced uncle wants to divide it into two portions. Divide the garden and calculate the area of each portion. Do you know the area?

Gerald: Yes
R: Divide the land into two portions.



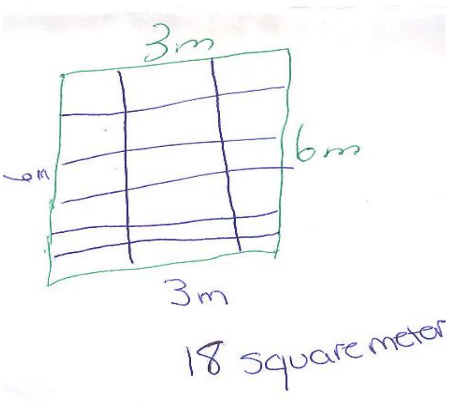
Gerald: (divide the land as above.)
R: By Area he wants to know how many square meters cover his land. Do you know how many squares?
Gerald: No

R: I will make squares with you. For the side with 3m I will make 3 columns. (working on the small portion). Then on the 2.5m I will make two and a half rows. (I wrote a ½ inside each half square). Now see I have squares can you count them for me.

Gerald: 9
R: Wow I don't see 9 show me.
Gerald: Counts them one by one. 7 ½ squares.
R: These squares have names. What is the name?
Gerald: Square meters.
R: Now I want you to do the big one. How many rows are you going to make?
Gerald: Four rows.
R: Ok make them.
Gerald: Draw columns and rows in the 3 by 4 side as shown in the diagram above. (4 rows and 3 columns)
R: How many squares is that portion?
Gerald: It's 12 squares.
R: Is it only 12 squares?
Gerald: It's 12 square meters.
R: What is the total area now? Can you add it up for him?
Gerald: writes

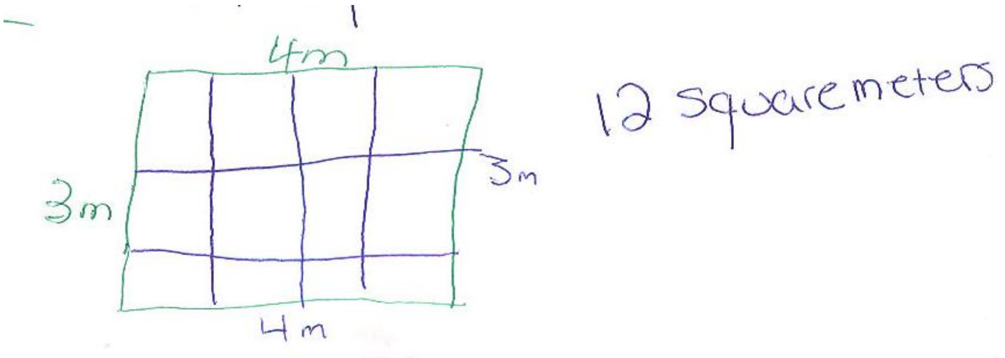
12
+ 7 ½
19 ½ square meters

R: Do you know that you have just done Area? Let's see if we can find area of simple shapes now that one was complicated. Work out this one. Find the sides that are not labeled first.



Gerald: (first labeled the missing length).
That is good. Calculate the area now.
Jerome: (draw rows and columns as shown in the drawing and writes) 18 square meters.

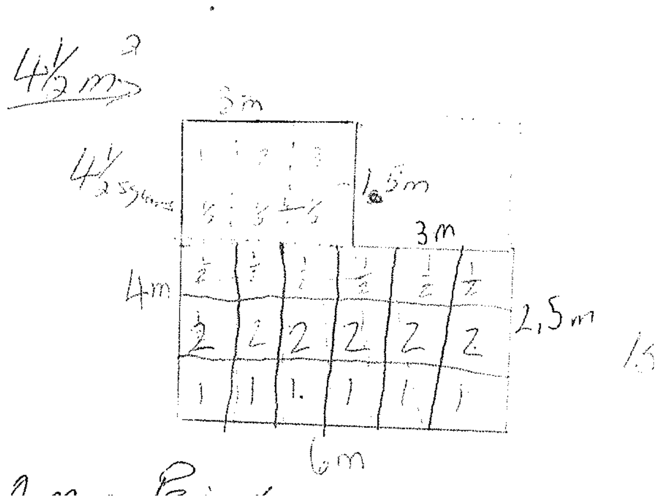
R: Let's do the last one:



Gerald: (label missing lengths and writes) 12 square meters.

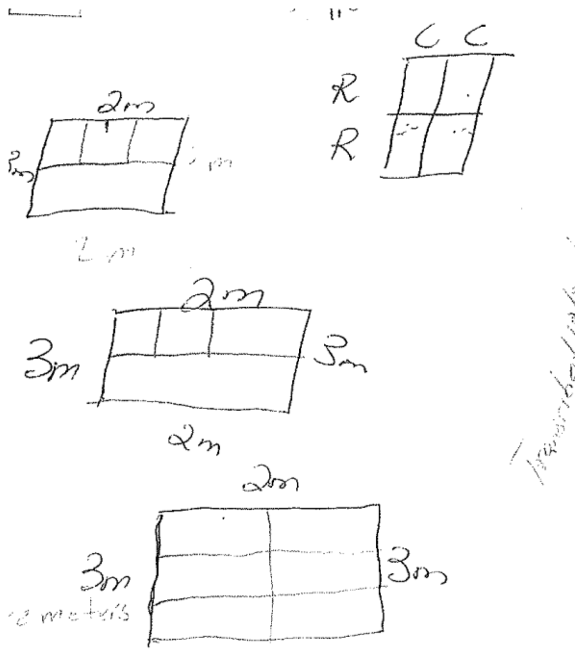
Gerald showed that he needed the area model to make sense of area. He needed full demonstration before he used it. He was challenged by fractional parts but through mediation he progressed. The model assisted his actualization of area and pushed him to an Array structurer. He had to master the area unit relater and repeater first and jumped to Array structurer from there.

Amos's journey
R: Now that the garden is fenced let us divide it into two portions. Divide the garden and calculate the area of each portion.
R: Do you know how we can find square meters?
Amos: I will try and draw squares.
R: Great. To make it easy for us divide this into two lands. So that it can be easy to work with. Where would you divide it?
Amos divided the diagram as follows:

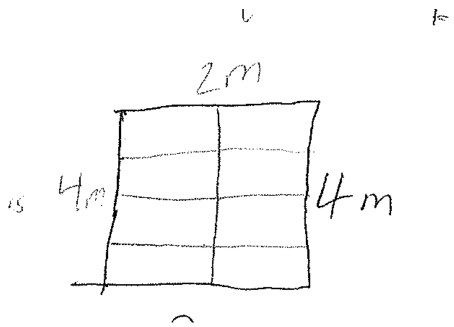


R: So you have the land cut into two portions one with 3m by 1.5m and 6m by 2.5 m. Let's work with one portion at a time. Do you know how we are going to make these squares? We have three meters and then I think we can divide it into three equal portions. Then we divide it again into 1 1/2. How many square meters is that portion?
Amos: 5 ½
R: How come?
Amos: I counted them.
R: Remember we have half squares there. Can you show me?
Adam: Yes
R: How many are they?
Amos: 3
R: So how many squares do you get then?
Amos: I get 4 ½ squares
R: How did you get that?
Amos: I counted the three full squares then combine two half squares to one and have four then added the other half.
R: What kind of squares are they? Let's look back on our division what were we dividing?

Amos: Meters
R: So our squares are square meters. Now I want you to do the other portion by your self.
Amos: (quietly divide the bottom part as follows: Check the diagram above to see his work. He labeled his first row of squares 1 each square then the second row he labeled 2 each square then the third row he labels a ½ each square then he started counting them.) 1, 2, 3,4,5,6 (then when he gets to the second row he counts) 8, 10, 12, 14
R: Ok tell me what is it that you are counting why you are adding two's now, show me.
Amos: I am adding what is in the squares.
R: Oh why do you have two in those squares?
Amos: They are in the second row.
R: Why do you have halves in the third row then?
Amos: Because I did not want to forget that they were halves.
R: What makes the second row two's then?
Amos: I don't know.
R: What must you do?
Amos: Count the squares as one each.
R: How many square meters is this portion?
Amos: 15 square meters.
R: Ok I think we need to do more. You said the first portion is 4 ½ square meters and this one is 15 meters. What is the total area of uncle's land?
Amos: 19 ½ square meters.
R: I would like you to try working the area of this one.
Amos first labeled the sides that were not labeled. He drew the columns like this twice. Check below. I had to draw a diagram teaching him columns and rows.



R: How are you dividing this land?
Amos: Three rows and two columns.
R: Can you show me columns first?
Amos: Point the rows.
R: Ok me too I used to call those columns. But the line you made across forms rows and then the lines that go down for columns. Ok how many meters are on the side on top?
Amos: Two
R: So how many columns are you suppose to have then?
Amos: Two
R: What about the other side?
Amos: I must make 3 rows
Can you draw then now? Amos: draws the fourth diagram above.
This is six square meters.
R: That is good Amos can you try and find Area for this one too.

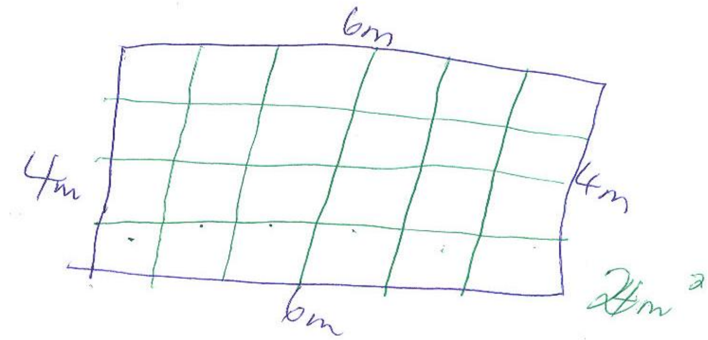


Amos: (label it first) then draw columns and rows as shown above. The Area is 8 square meters.

Amos' journey was unique compared to his peers. He did not grasp the area model after the researcher demonstration. When he started he was challenged by the rows and columns he had to master them first then move to the area unit relater and repeater before he moved to array structurer.

Portia's journey

R: Now that the garden is fenced. How many square meters cover his land?

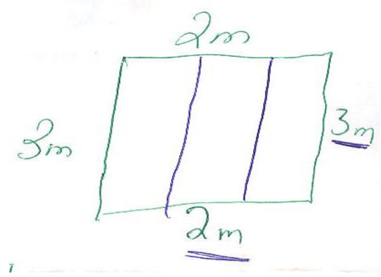


Portia: It has two square meters.

R: Let's see it has 6m long and then I will make 6 columns. (Making 6 columns). Then it is 4m wide then I make 4 rows. Whu, I have squares. How many squares is this land?

Portia: (count squares one by one) 24 squares.

R: Can you try and find area with this little land I give you now.

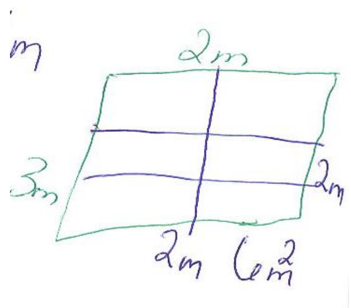


Portia: (label the sides first with bold labeling) She then makes three columns.

R: Why are you making 3 columns?

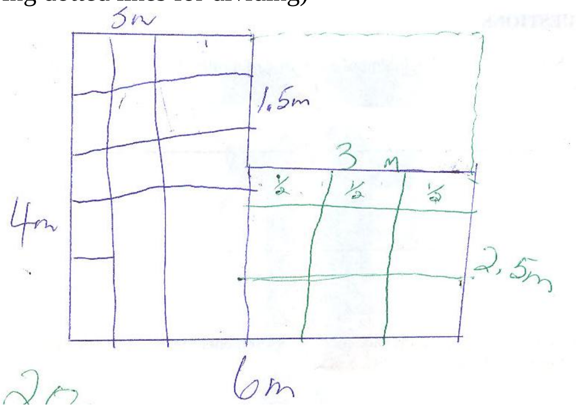
Portia: I am making the columns that are oh, I am suppose to make two.

R: I like it when you say oh, ok let's work with another diagram.

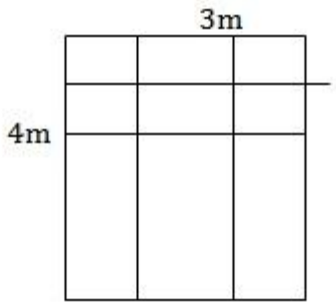


Portia: makes two columns as above and three rows.
R: How many square meters do you have?
Portia: 6 squares
R: Write $6m^2$ this is how we write them in mathematics.
Portia: writes $6m^2$

Briana's journery
R: Now that the garden is fenced uncle wants to divide it into two portions. Divide the garden and calculate the area of each portion.
Briana: (divides the land like this using dotted lines for dividing)



R: Now we have two lands. Which one do you want me to help you with?
Briana: The small one.
R: This is 3m long and 2.5m wide. Let's make squares. Because this is 3 I will make 3 columns (refer to figure). And because this is 2 ½ meters I will make 2 ½ rows.(refer to figure) Now I have squares. How many squares are they? Can you tell me?
Briana: 9 squares
R: I can't see 9. Show me how you got 9.
Briana: 1, 2, 3,4,5,6, ½ and ½ is 1 and there is a 1/2 and it's 7 ½ squares.
R: Now make your own squares in the other portion it is 4m long and 3m wide.
Briana: (makes three columns then struggle to make the rows)
R: Let me make another drawing for you.



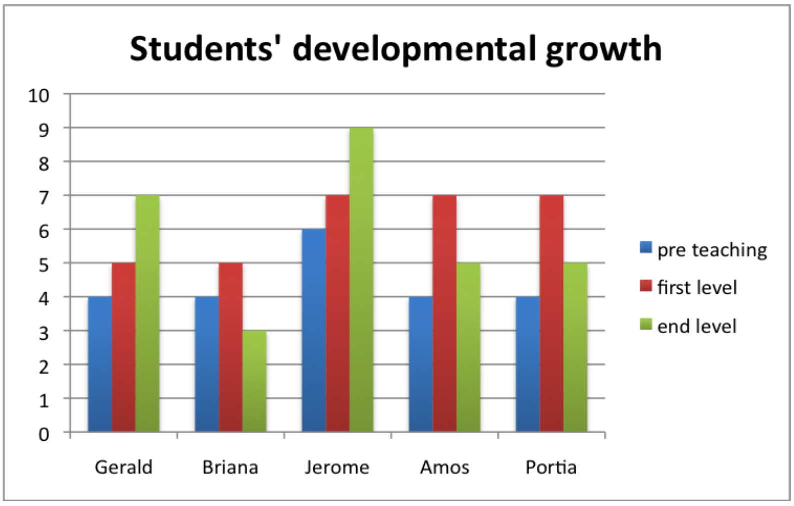
Briana: (continues to struggle in drawing)

Briana did not move further that row structurer during teaching experiments. Her development was not evident except that moved back to Side-to- Side Area measure showing regression at the end of her teaching experiments.

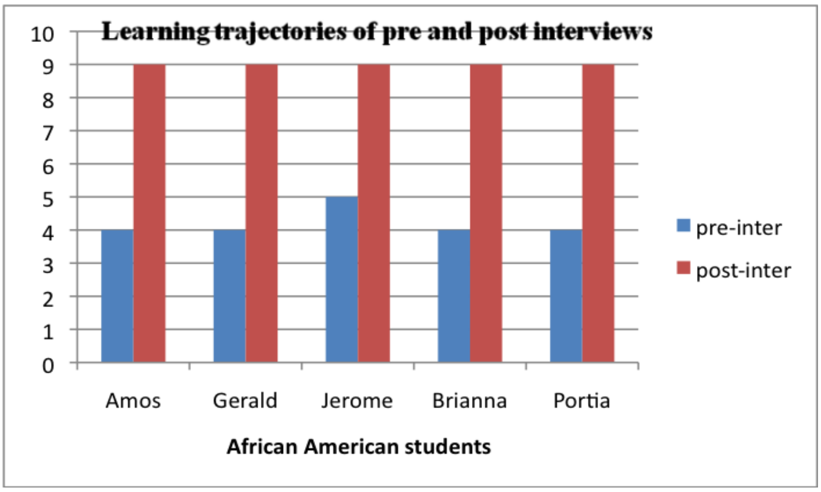
IV.2.2 Release of Responsibility

The area model was an attempt used in addressing the abstract conception of area. With the language challenge the study employed a gradual release of responsibility approach in mediating use of area model. A model in mathematics teaching and learning is used to introduce an abstract idea. Mathematical ideas are not tangible or visible, and because of that nature cannot be easily mediated. It is important to note that a model is not a concept but a model used to make the concept accessible. Thus, a model represents the concept. In this case releasing responsibility of doing area model gradually assisted instruction and comprehension of the problem. The researcher took 100% responsibility introducing the model except with Jerome who grasped it immediately. However, other students needed more time to grasp with Portia regressing. Below is a line graph presenting each student developmental path using the learning trajectories.

Each student's path is unique. Jerome and Gerald's path during teaching experiments was linear, while Amos and Portia needed to go back to area unit relater and repeater to have a solid understanding. Briana moved one level up and moved back two levels showing regression at the end of the teaching experiments.



The following graphical representations below present the difference between pre-interviews and post-interviews these students took before and after teaching experiments for the area measurement.



From this graph it is clear that Briana was not regressing but touch basing internalizing the concepts. Her route support the statement made by [2] about these learning trajectories that they are hierarchical but students develop through them differently. Some students develop linear but some mental structures are formed through intertwined progress.

V. Discussion and Conclusion

This study's results reveal four important components of learning area measurement concept by 5 fifth grade African American students of different performance levels. (1) Integrating a language model and a mathematics model gave access to abstract ideas of area measurement. (2) The learning trajectories can be used as assessment tools to inform practice. (3) Students cognitive structures are complicated and varied [26]. (4) African American students have potential for learning and are diverse in their learning. Area model became a powerful teaching tool that students were able to internalize and transfer to other area problems in unique ways [27]; [28]. However, on its own it could not assist students in conceptualizing area without the language model [29]. This paper supports the Vygotskian perspective of directing instruction to the student level of development. It also supports [26] argument of prevalence of variability in students' cognition at all levels. In this paper Briana demonstrated a unique way of learning by moving from a lower level of development to a higher level and then goes back two levels down. During assessment interviews Briana demonstrated an enormous growth that moved her from a third level to the highest level of abstraction, level 9. Briana's case demonstrated variability within [26]. Reflecting on the other four students in the study two students, Gerald and Jerome development was linear following the learning trajectories hierarchically. However, Amos and Portia showed some similarities in their development that moved to higher development first then regressed two levels down. Looking closely at their development they both developed uniquely up. Amos moved from Primitive cover to partial row structurer and took time to see relationships between rows and columns. When he did, understanding unit relater and repeater was easy for him. On the other hand Portia was still thinking in terms of length and focusing on length before she noticed rows, then columns. These two cases demonstrate variability between that could be explored further in research.

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