International Journal of Modern Engineering Research (IJMER)
www.ijmer.com Vol. 3, Issue. 3, May - June 2013 pp-1623-1629 ISSN: 2249-6645

Performance Comparison of H-infinity and LQR Controllers for the Pressure Regulation of a Hypersonic Wind Tunnel

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Abstract : *Hypersonic wind tunnels are used to study the effect of air moving past the fighter planes, space vehicles and similar specimens under test. This paper aims to compare the performance of a h-infinity controller with that of a linear quadratic regulator (LQR) controller for regulating the pressure inside the settling chamber of a hypersonic wind tunnel. The linear model for both the controllers is one and the same and it is state controllable and observable. The h-infinity controller design is based on the selection of weighing function whereas the design of the LQR controller depends on the selection of optimal state feedback controller gain matrix. Performance comparison of both the controllers is carried out based on the settling time, peak overshoot and rise time. Simulation results show that h-infinity controller has better settling time compared to LQR controller.*

Keywords: *H-infinity controller, Hypersonic wind tunnel, Linear quadratic regulator, Settling chamber pressure, Weighting function.*

I. Introduction

Hypersonic wind tunnels are used in aircrafts and space vehicles to investigate the aerodynamic properties of the specimen in hypersonic flow regime. The speed of wind tunnel is indicated by Mach number which is defined as the ratio of speed of aircraft to speed of sound in gas. Hypersonic wind tunnel has a mach number greater than 5. The block diagram representation of the hypersonic wind tunnel is shown in Fig.1. The main parts of a hypersonic wind tunnel are high pressure system, pressure regulating valve, heater, settling chamber, nozzle and test section [1], [2]. Compressed air from the air storage tank is released through a pressure valve to the heater where it is heated to the required temperature and is straightened in the settling chamber and passed to the test section through the nozzle. Settling chamber pressure is controlled by designing a suitable controller for the effective operation of the pressure valve. Here the effectiveness of two controllers with different design strategy in controlling the settling chamber pressure of the hypersonic wind tunnel in terms of their settling time, peak overshoot and rise time is carried out.

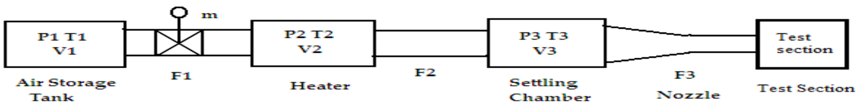

Fig.1: Block diagram of hypersonic wind tunnel

H-infinity methods are used to synthesis controllers that minimize the closed loop impact of a perturbations. H-infinity controllers are designed by properly selecting weighing function [3]-[5]. LQR controller is an optimization-based synthesis problem used to track the output and follow the changes in set point. Based on the performance requirements, the optimal state feedback controller gain matrix is designed for the controller [6]-[9].

II. Model and Analysis of The System

The system performance is decided by the speed of settling chamber pressure and it is accurately controlled by two controllers viz h- infinity and LQR. For modelling the wind tunnel system, the continuity equations and parameter values are selected for the pressure vessels [10] - [12]. The state space model of the system is given in (1) & (2).

$$\begin{bmatrix} \dot{P}_1 \\ \dot{P}_2 \\ \dot{P}_3 \end{bmatrix} = \begin{bmatrix} -K_1/C_1 & 0 & 0 \\ K_1/C_1 & -K_3/C_2 & -K_4/C_2 \\ 0 & K_3/C_3 & K_4 - K_n/C_3 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} + \begin{bmatrix} -K_2/C_1 \\ K_2/C_2 \\ 0 \end{bmatrix} m. \quad (1)$$

$$Y_1 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}. \quad (2)$$

where P_1, P_2 are the upstream and downstream pressures, P_3 is the settling chamber pressure, m is the stem movement of pressure valve, K_1, K_2, K_3 and K_4 are constants, K_n is the nozzle flow constant and, C_1, C_2, C_3 represents the capacitance of the three pressure vessels respectively.

The system is linearized and the transfer function [10] is given in (3).

$$G_p(s) = \frac{-2.369e006s^2 + 7.897e007s + 4.21e005}{0.015s^5 + 0.7802s^4 + 9.89s^3 + 18.46s^2 + 3.377s + 0.01937} . \qquad (3)$$

1.1. Stability Analysis

Stability analysis is carried out on the system model before considering the implementation of h-infinity and LQR controller. By substituting the values of the parameters K_1, K_2, K_3, K_4, Kn, C_1, C_2 and C_3 from [10] in (1) and (2), the state model is obtained with

$$[A] = \begin{bmatrix} -0.0045 & 0 & 0 \\ 0.0045 & 2.51 & 2.51 \\ 0 & 12.85 & -14.14 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 617679.68 \\ 6133259.91 \\ 0 \end{bmatrix}$$

$$[C] = [0 \;\; 0 \;\; 1]$$

Controllability and observability tests [13] are carried out on the model using Kalmans test with (4) and (5) respectively.

$$Q_C = [B \quad AB \quad A^2B] \neq 0 \qquad (4)$$

$$Q_O = \left[C^T \quad A^T C^T \quad {A^T}^2 C^T\right] \neq 0 \qquad (5)$$

It is found that $Q_C \neq 0$ and $Q_O \neq 0$ and rank of the matrix is 3, which is equal to the dimension of the system and hence the system is completely state controllable and observable[13].

1.2. Open Loop Response of the System

The system in (1)-(3) is simulated in Matlab and the open loop response for the settling chamber pressure is obtained in Fig.2. From the figure, it is observed that the peak value of settling chamber pressure is 130 * 10^5 Pa and settling time is 450 secs which is very high for the short duration test.

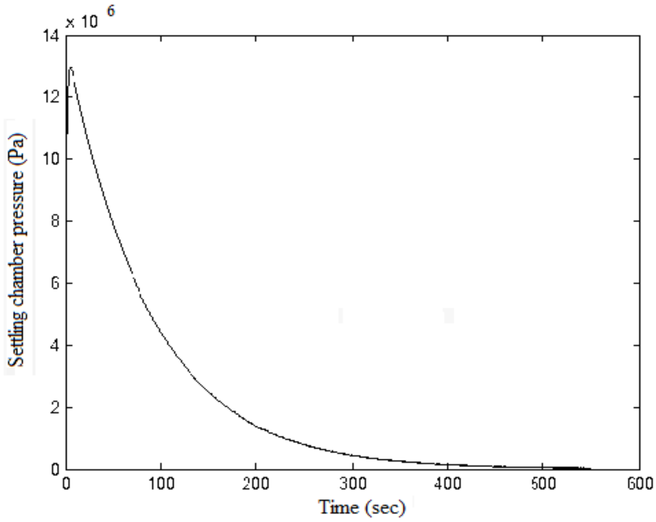

Fig.2. Open loop response of the settling chamber pressure

III. Performance of H-infinity Controller

The main objective of h-infinity controller is to minimize the h-infinity norm which is the energy gain of the system. Standard feedback configuration with weights [3], [14] is given in Fig.3. The controller is designed by properly selecting the weighing functions [14]. Here G is the plant transfer function, G_d the transfer function corresponding to input disturbance, r the set point, u the actuator, v the sensor measurement, K the controller, d the disturbance, n is measurement noise, Z_1 is the settling chamber pressure, Z_2 is control output, weight W_p is the second order transfer function and is selected such that $|S(j\omega)| < 1/(W_p(j\omega))$, where S is the sensitivity function. Weight W_u indicates control input weight and sensor noise effects are W_n [14].

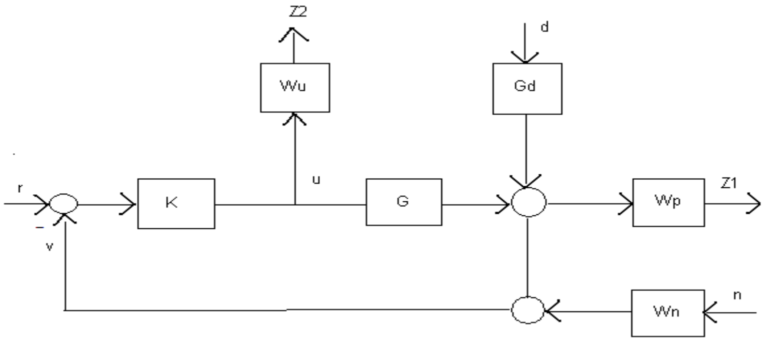

Fig.3. Standard feedback configuration with weights

The multiplicative uncertainty weight W_u is selected [4], [15], [16] by satisfying the stability conditions,

$$|W_u(j\omega)| \geq l_u(\omega), \quad \forall\omega \ . \tag{6}$$

where l_u is the relative error of the plant transfer function and W_u is selected as

$$W_u = \frac{600s + 210}{20s + 0.0001} \ . \tag{7}$$

The sensitivity function S(s) [4], [15], [16] is

$$S(s) = (1 + K(s)H(s))^{-1} \ . \tag{8}$$

The performance requirement is guaranteed if and only if the condition $|S(j\omega)| < \frac{1}{W_p(j\omega)}$, $\forall\omega$ is satisfied. The nominal performance criterion [17] is given in (9).

$$|W_p(j\omega)| < |1 + G_m(j\omega)|, \quad \forall\omega \ . \tag{9}$$

The robust performance [5] is defined by criterion

$$|W_p(j\omega)S_p(j\omega)| < 1, \quad \forall \ S_p, \ \omega \tag{10}$$

Using the performance criterion in (9) and (10), the weighing function W_p is selected [5] as

$$w_p = \frac{30S+20}{20S+1} \tag{11}$$

The weighing function W_n is chosen by trial and error method [5] as

$$W_n = \frac{1}{10} \ . \tag{12}$$

After selecting the three weights W_u, W_p, W_n, the h-infinity controller is simulated in Matlab, with the input disturbance transfer function G_d = 1 (with minimum disturbance) and the set points equal to $100*10^5$, $80*10^5$, and $50*10^5$ Pa is shown in Fig. 4, 5, and 6 respectively.

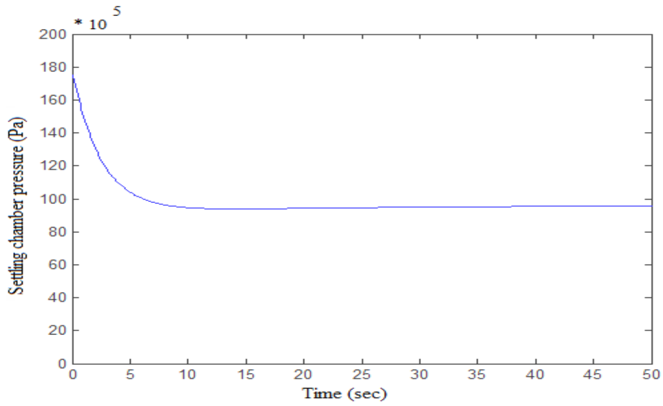

Fig.4. Settling Chamber Pressure with H-infinity Controller for Set point $100 * 10^5$ Pa

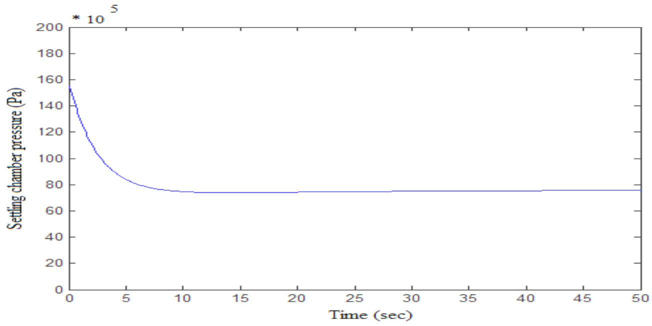

Fig.5. Settling Chamber Pressure with H-infinity Controller for Set point 80 * 10^5 Pa

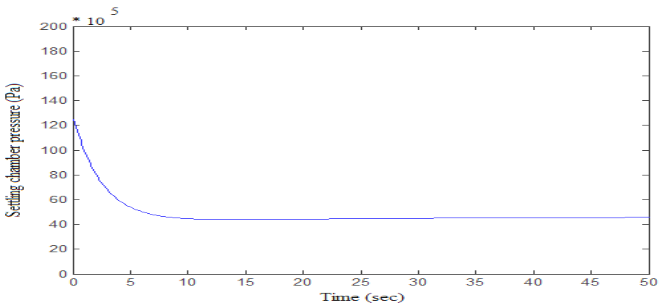

Fig.6. Settling Chamber Pressure with H-infinity Controller for Set point 50 * 10^5 Pa

From the simulation, the h-infinity controller matrix, K is obtained as

$$K = (1.0e+005)*$$

$$\begin{bmatrix} -2.0733 & 1.6054 & 1.1876 & 0.2099 & -0.168 & -0.0007 & 0 & 0 & 0.0001 \\ -1.8229 & 1.4115 & 1.0442 & 0.1845 & -0.1291 & -0.0006 & 0 & 0 & 0 \\ -1.3565 & 1.0503 & 0.7770 & 0.1373 & -0.0960 & -0.0004 & 0 & 0 & 0 \\ 0.9738 & -0.7540 & -0.5578 & -0.0987 & 0.0689 & 0.0003 & -0 & 0 & 0 \\ -0.2895 & 0.2241 & 0.1658 & 0.0294 & -0.0209 & 0.0003 & 0 & 0 & 0 \\ 0.0008 & -0.0007 & -0.0005 & -0.0004 & -0.0001 & -0.000 & 0 & -0.0001 & 0 \\ -0.0035 & 0.0027 & 0.0020 & 0.0004 & -0.0003 & -0.000 & 0 & 0 & 0 \\ -0.0001 & 0.0001 & 0.0000 & 0.0000 & -0.0000 & -0.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\infty \end{bmatrix}$$

With set point 100*10^5Pa, the settling time of settling chamber pressure is 12 sec and peak overshoot is 70%. When the set point is changed to 80*10^5Pa, the settling time remains 12 sec whereas peak overshoot is increased to 90% and when the set point is further reduced to 50*10^5Pa, the settling time is 11 sec and the peak overshoot is drastically increased and is >90%. The rise time for the three set points is 1sec. From this it is clear that a change in set point does not effect the settling time whereas there is a drastic increase in peak overshoot with decrease in set point and the rise time remains constant.

IV. Performance of LQR Controller

LQR controller design problem deals with optimizing an energy function, J by designing the state feedback controller, K.
A system in state variable form is

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

with x(t) ε R^n and u(t) ε R^m . x is the state of the system and u is the control input. The initial condition is x(0) and states are measurable. The state-variable feedback (SVFB) control law is

u = -Kx

where K is the linear optimal feedback control gain matrix [9], [18]. The closed-loop system using this control becomes

$$\dot{x} = (A - BK)x$$

where v is the new command input. The objective of the controller design is to find the optimal control law that minimizes the following performance index. The performance index (PI) [8], [9], [18] is

$$J = \frac{1}{2}\int_0^\infty (x^T QX + u^T Ru)\, dt \qquad (13)$$

where J is the energy function which keeps the total energy of the closed-loop system small. The two matrices Q and R are selected such that Q is positive semi-definite and R is positive definite [6], [9], [18]. The control value u is called optimal control [18] which is given by,

$$u(t) = -R^{-1}B^T Px = -Kx \qquad (14)$$

where P(t) is the solution of Riccati equation and is a real symmetric matrix. Solving the above (14),

$$PA + A^T P - PBR^{-1}BP + Q = 0 \qquad (15)$$

where Q and R are the optimal controller weight matrices and K is obtained as,

$$K = R^{-1}B^T P$$

The plant with LQR controller is shown in Fig.7.

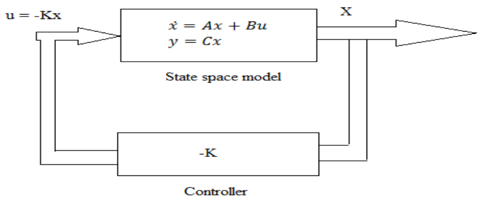

Fig.7. State Feedback Representation of the System with Feedback Gain

Matrices Q and R are selected by trial and error method to find optimal gain matrix, K. Q and R are given by

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

R = 1,

The optimal gain matrix K is obtained.
K = [-0.9839 1.1041 0.3943].

With these values of gain, the system is simulated to get the response of settling chamber pressure with three different values of set points. Fig. 8, 9, and 10 shows the variation of settling chamber pressure with LQR controller with set points $50 * 10^5$ Pa, $80 * 10^5$ Pa, $100 * 10^5$ Pa respectively.

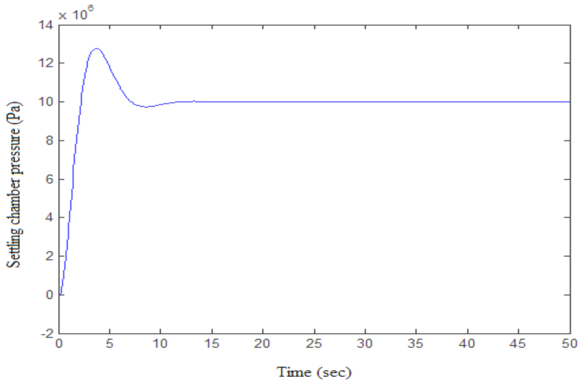

Fig.8. Settling chamber pressure with LQR controller with set point $100*10^5$ Pa .

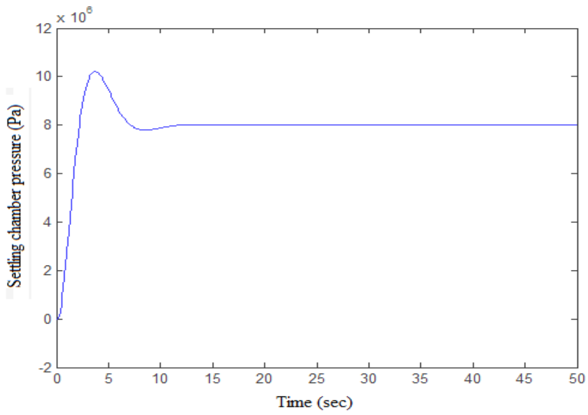

Fig.9. Settling chamber pressure with LQR controller with set point $80*10^5$ Pa

From the responses, it is observed that with set point $100*10^5$Pa, the settling time of settling chamber pressure is 18 sec, peak overshoot is 30% and rise time is 2 sec. When the set point is changed to $80*10^5$Pa, the settling time is 19 sec, peak overshoot is increased to 31.25% and rise time is 2 sec and when the set point is further reduced to 50*10^5Pa, the settling time is 18 sec, the peak overshoot is 30% and the rise time remains 2 sec. From this it is clear that a change in set point does not have much effect on the settling time, peak overshoot and the rise time in the case of a LQR controller .

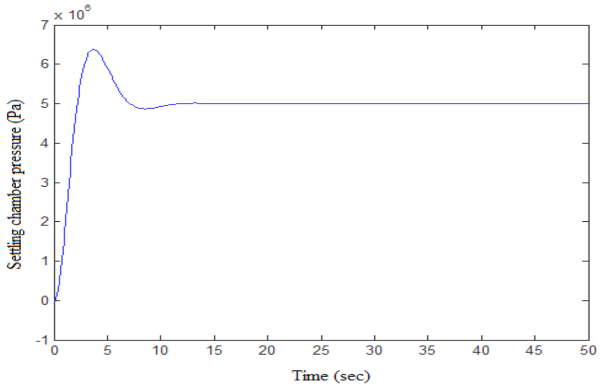

Fig.10. Settling chamber pressure with LQR controller with set point 50*10^5 Pa.

V. Result and Discussion

Stability analysis is carried out on the hypersonic wind tunnel system model and its open loop response is plotted. From the response, it is clear that the settling time is 450 secs which is further to be improved using a suitable controller. Here the effectiveness of an h-infinity controller and an LQR controller in regulating the settling chamber pressure is studied for different set points. The performance comparison of these two controllers in terms of settling time, peak overshoot and rise time is evaluated and is tabulated in table.1.

Table 1: Performance Comparison Table

	Settling Time(sec)		Peak overshoot(%)		Rise Time(sec)	
Set point	H-infinity	LQR	H-infinity	LQR	H-infinity	LQR
100*10^5Pa	12	18	70	30	1	2
80*10^5Pa	12	19	90	31.25	1	2
50*10^5Pa	11	18	> 90	30	1	2

From the table, it is observed that for an h-infinity controller, the settling time is much lesser than that of an LQR controller. In both the cases settling time does not vary much with change in set points, however the peak overshoot is much higher in the case of an h-infinity controller and the variation with set point is also drastic. The peak overshoot with smaller set point is very high in case of h-infinity controller whereas there is no drastic variation in the case of an LQR controller. The values of peak overshoot with set point 100*10^5Pa are tolerable for both the controllers whereas with reduction of set point, the peak overshoot is very high for h-infinity controller. The rise time in the case of h-infinity controller is lesser than that of LQR controller. With change in set point, there is no variation in rise time for both the controllers. In case of a hypersonic wind tunnel system, the settling time is more important than that of peak overshoot as the test duration is very short. Hence from these results, it is clear that an h-infinity controller with a set point of 100*10^5Pa would be a better choice for regulating the settling chamber pressure of a hypersonic wind tunnel.

VI. Conclusion

The performance of two controllers viz h-infinity and LQR for regulating the settling chamber pressure of a hypersonic wind tunnel is compared. The settling time of h-infinity controller is lesser than that of LQR controller and hence it is found to be more applicable in the present study. However the peak overshoot is slightly higher for the h-infinity controller with the same set point as that of LQR controller. As the test duration is very short, lower value of settling time makes h-infinity controller more suitable when compared to LQR controller. The set point of settling chamber pressure while using h-infinity controller cannot be decreased to a very low value as it effects the peak overshoot, however the results can be improved by considering higher values of set points and its effects on the performance characteristics. The results can further be improved by considering nonlinear models of the same system.

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