

## Cross Spectral Density Analysis for Various Codes Suitable for Spread Spectrum under AWGN conditions with Error Detecting Code

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**ABSTRACT:** CDMA is based around the use of direct sequence spread spectrum techniques. Essentially CDMA is a form of spread spectrum transmission which uses spreading codes to spread the signal out over a wider bandwidth than would normally be required. By using CDMA spread spectrum technology, many users are able to use the same channel and gain access to the system without causing undue interference to each other. Here this paper concentrates on the area where how to spread the signal with spread codes, so that more number of users can be accommodated in the spreaded bandwidth. Analyzing various correlation properties comparison between Pseudo Noise and Gold codes, to conclude which code best suits. Walsh Hadamard code has the capability of error detection and correction capability. The codes are simulated using MATLAB.

**KEYWORDS:** Correlation, gold codes, linear feedback shift register, Pseudo noise sequences, Spread Sequences, Walsh hadamard code.

### I. INTRODUCTION

Continuous growth in telecommunication industry in terms of users has led to utilize the available resources effectively and it provide maximum number of services. Spread spectrum (SS) is a technique where an already modulated signal or high frequency signal is modulated a second time in such a way as it produce interference in a barely noticeable way with some other signal operating in the same frequency band. So, in this interfering signals are transparent to spread spectrum signals and spread spectrum signals are transparent to interfering signals. Note that spread spectrum is not a modulation scheme. In this the main objective is preserved for generating PN sequence, like maintaining randomness, which is helpful to differentiate between users and also in generating noise like sequences, a very low correlation value, useful to avoid interference in the transmission channel and a high autocorrelation value to reject multipath fading. This paper illustrates the comparative capabilities of gold sequences and pseudo noise sequences for finding the better sequence by using their correlation properties and the simulation is done in Matlab environment for PN sequences, Gold Sequences, As well as Walsh hadamard codes.

The whole paper is sectionalized as follows. In section 2 spread spectrum techniques, pseudo noise sequences, gold sequences, Walsh hadamard codes are described. In section 3 generating PN and gold codes are described and generation of spread signal, how to generate despreaded signal along with cross correlation and auto correlation. In section 4 simulation comparisons of PN and Gold sequences in various scenarios how the Walsh hadamard code signal appears for a spread signal received without error in both cross correlation and autocorrelation cases. In section 5 results are described.

### II. SPREAD SPECTRUM SEQUENCE

**II.1 Introduction:** Spread spectrum is a means of transmission in which signal occupies a bandwidth in excess of the minimum necessary to send the information; the band spread is accomplished by a code which is independent of data and a synchronized reception with the code at the receiver is used for despreading and subsequent data recovery. Spreading the bandwidth has several advantages like the resistance to narrowband interference and it covers the narrowband signal into a broadband signal.[3].

Spread codes are mainly:

- Pseudo Noise (PN) sequences
- Gold sequence
- Walsh Hadamard Codes
- Kasami Sequence

**II.1 PN sequence:** Pseudo noise (PN) sequence is a periodic binary sequence with noise like waveform but have deterministic value. The codes are usually generated by means of feedback shift registers, which are generated by using an algorithm which is deterministic and consists of some initial value called seed & therefore it produces sequence of numbers that are not stastically random. A pseudo noise (PN) sequence is a binary sequence of 1's and 0's. The PN code Properties are

**Balance Property:** In each period of maximum-length sequence, the number of 1s is always one more than the number of 0s. So there must be  $2^m - 1$  ones and  $2^m - 1$  zero in a full period of the sequence.

**Run Length Property:** Here, the 'run' represents a subsequence of identical symbols (1's or 0's) within one period of the sequence. The length of this subsequence is the length of the run. Among the runs of 1's and 0's in each period of a

maximum-length sequence, one-half the run of each kind are of length one, one-fourth are length two, one-eighth are of length three etc. For a maximum-length sequence generated by a linear feedback shift register of length m, the total number of runs is (N+1)/2 where  $N=2^m - 1$

**Correlation Property:** The codes must have a low cross correlation value. Lower cross correlation allows more users in the system. This condition holds for both full-code correlation and partial-code correlation. Because in most situations there will not be a full period correlation of two codes, it is more likely that codes will only correlate partially due to random-access nature [2].

**II.2 Linear Feedback shift Register:** The PN sequences are generated by using linear feedback shift registers. LFSR is designed with Flip-flops and XOR gates with shifting operation. The number of states depends on number flip flops/registers involved in the LFSR. At each end of the clock edge the content in the registers are shifted to right by one position [1].to give feedback there is predefined taps in the registers. They will give feedback to the left most register through XNOR or XOR gate.

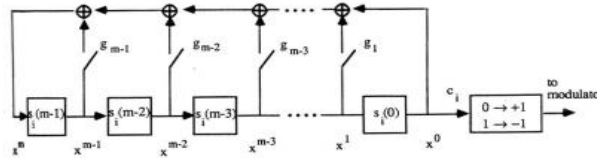


Figure 1:m-sequence Linear feedback shift Register

- Linear generator polynomial  $g(x)$  of degree  $m > 0$   $g(x) = g_m x^m + g_{m-1} x^{m-1} + \dots + g_1 x + g_0$
- Recurrence Equation ( $g_m = g_0 = 1$ )  $x^m = g_{m-1} x^{m-1} + g_{m-2} x^{m-2} + \dots + g_1 x + g_0$
- If  $g_i = 1$ , the corresponding circuit switch is closed, otherwise  $g_i \neq 1$ , it is open. Output of the shift-register circuit is transformed to 1 if it is 0 and -1 if it is 1.

One of the two main parts of LFSR is the shift register and the other one is feedback function. The main function of the shift register is to shift its contents into adjacent positions within the register or if it is the position on the end, out of the register. the position on the other end is left empty unless some new content is shifted into the register[1].

**II.3 Gold code:** The particular class of PN sequences called Gold sequences. They can be chosen such that, the cross-correlation values between the codes over a set of codes are uniform and bounded. Gold codes can be generated by modulo-2 addition of two maximum-length sequences with the same length. The code sequences are added chip by chip by synchronous clocking. The generated codes are of the same length as the two m-sequences which are added together. In addition to their advantages Gold codes generates large number of codes. To define a set of Gold codes, preferred pairs of m sequences are used. If a is an m-sequence of length N, the second sequence a' can be obtained by sampling every q<sup>th</sup> symbol of a. The second sequence is called decimation of the first sequence.

The figure2 illustrates about the gold sequence generator for the proffered pair gold sequences. Suppose we take an m sequence represented by a binary vector a of length N and generate a new sequence a' by sampling every q<sup>th</sup> symbol a. We use multiple copies of a until we have enough samples to produce a sequence of a' of length N. The sequence a' is decimation of sequence a and is written as  $a' = a[q]$ . for gold sequences we need to start with preferred m-sequences that meet following conditions.

- $n \neq 0$ ; that is, n is odd or  $n=2$
- $\lambda = a[q]$  where q is odd and either
- $q = 2^k + 1$  or  $q = 2^{2k} - 2^k + 1$
- $GCD(n, k) = \begin{cases} 1 & \text{for } n \text{ odd} \\ 2 & \text{for } n \text{ even} \end{cases}$

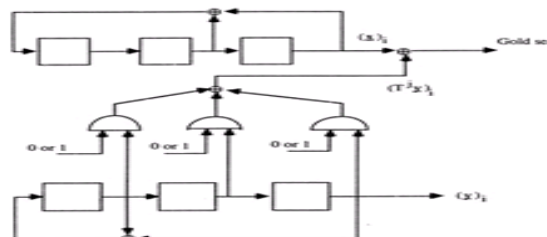


Figure2: Gold sequence generator for the preferred pair  $g_1(x) = x^3 + x + 1$  and  $g_2(x) = x^3 + x^2 + 1$

For shift register of length n, the cross correlation of the gold sequences produced by a preferred pair is bounded by  $|R| \leq 2^{(n+1)/2} + 1$  for n odd and  $|R| \leq 2^{(n+2)/2} + 1$  for n even. the two PN sequences are used as components in the program by XORing the two PN sequences we will get Gold-Code. Gold sequences help to generate more sequences out of a pair of m-sequences giving now many more different sequences to have multiple users. the m-sequences gave only one sequence of length  $(2^5 - 1)$ . by combining two of these sequences, we can obtain up to 31 plus the two m-sequences themselves, generate 33 sequences that can be used to spread different input messages.

**II.4 Walsh Hadamard code:** Walsh codes are orthogonal codes. These codes are used in DSSS systems and also in FHSS systems to select the target frequency for next hop. The Hadamard-Walsh codes are generated in a set of  $N=2^n$  codes with length  $N=2^n$ . The generating algorithm is as follows

$$H_{2N} = \begin{bmatrix} H_N & H_N \\ H_N & \overline{H_N} \end{bmatrix}$$

**Figure3: Walsh Hadamard Matrix**

Where  $N$  is a power of 2 and over score denotes the binary complement of the bits in the matrix. The smallest set of  $N=0$  is  $H_0 = [1]$  with the length 1. The rows or columns of matrix  $H_n$  are the Hadamard-Walsh codes since the matrix  $H_N$  is symmetric.

As shown above, in each set, the first row of the matrix consist all 1's and rest of the rows contains  $N/2$  0's and  $N/2$  1's. Also row  $N/2$  starts with  $N/2$  1's and ends with  $N/2$  0's. Orthogonality is the most important property of Hadamard-Walsh codes. Because of this orthogonality property, the cross-correlation between any two Hadamard-Walsh codes of the same set (matrix) is zero, when system is perfectly synchronized. Walsh codes are not maximal length or PN type codes for spread spectrum. Although the members of the set are orthogonal, they do not give any spreading. They are used in forward channel of IS-95 CDMA type system for their orthogonality. Walsh code spreading can be used if all users of the same channel are synchronized in time, because the cross-correlation between different shifts of Walsh codes is not zero. Orthogonal functions are employed to improve the bandwidth efficiency of spread spectrum systems. The Walsh and Hadamard sequences make useful sets for CDMA.

The orthogonal functions have the following characteristic:

$$\sum_{k=0}^{M-1} \varphi_i(k\tau) \varphi_j(k\tau) = 0, i \neq j$$

$\varphi_i(k\tau), \varphi_j(k\tau)$ :  $i^{\text{th}}$  and  $j^{\text{th}}$  orthogonal members of an orthogonal set.

### III SPREAD CODES GENERATION

**III.1 Correlation:** The correlation properties of PN codes play a major part in the code design for CDMA systems, since they determine not only the level of multiple access interference, i.e., the interference arising from other users of the channel and self interference due to multipath propagation, but also the code acquisition properties [6]. correlation is the concept of determining how much similarity one set of data has with another. It is defined with range -1 to 1. Other value indicates partial degree of correlation.

**TABLE 1. Correlation values**

Correlation value	Interpretation
1	The second sequence m sequence matches the first sequence exactly
0	There is no relation between sequence
-1	The two sequences are mirror image of each other

**III.1.1 Auto correlation:** Autocorrelation is mainly of two types periodic autocorrelation and aperiodic autocorrelation In periodic autocorrelation The m-sequence have the best periodic autocorrelation in terms of minimizing the maximum value of the out-of-phase Autocorrelation. It is best utilized if the synchronization window is longer than on Period. In aperiodic autocorrelation If the synchronization window is only one period long or less, then the correlation is aperiodic.

The formal definition of aperiodic autocorrelation is

$x = x_0, x_1, \dots, \dots, x_{N-1}$  and  $y = y_0, y_1, \dots, \dots, y_{N-1}$  is given by

$$C_{x,y}(i) = \begin{cases} \sum_{j=0}^{N-1-i} x_j y_{j+i}^* & 0 \leq i \leq N-1 \\ \sum_{j=0}^{N-1+i} x_{j-i} y_j^* & -(N-1) \leq i < 0 \end{cases}$$

Auto correlation is the similarity of sequence with all phase shifts of itself[6]. pure random data should have a correlation value close to 0 for all auto correlations with a phase shift other than 0.

$$R(\tau) = \frac{1}{N} \sum_{k=1}^N B_k B_{k-\tau} \quad \text{where } \tau = 0, N, 2N$$

**III.1.2 Cross correlation:** In this case the comparison is made between two sequences from rather than a shifted copy of sequence with itself. In general cross correlation value produced by matching sequence with random sequence is low [6].the cross correlation between two sources A and B, is defined as

$$R_{A,B}(\tau) = \frac{1}{N} \sum_{k=1}^N A_k B_{k-\tau} \quad \text{where } \tau = 0, N, 2N$$

**III.2 Transmitter block:** It Gives output as spread signal means spreading of bandwidth. The spread signal is generated by modulating message signal with the gold code. Message signal is of 20 bit type and the bits are in the fashion of 0's and 1's. each and every code bit is mapped to string of six 1's and six 0's wherever 1's and 0' appears in the message signal. This is the key point in CDMA which increases the bandwidth of the message signal by keeping symbol per rate constant. Next step is to generate the gold code, it can be generated with the help of two PN sequences i.e. PN sequence 1 and PN sequence 2.

**Generation of Gold Code**

- At first there are two pseudo noise sequences produced named PN 1 and PN 2.
- The PN1 and PN2 are 20 bit long by XORing specific pattern and shifting each of them continuously.
- For PN1, the XORing pattern is obtained by taking bits 1, 4,6,7,13,19.
- For PN1,the XORing pattern is obtained by taking bits 1,2,3,5,7,9,10,13,14,16,17
- From the two sequences each output bit is XORed and stored in an array called Gold.

**Modulation of message signal with Gold code:** The main idea for modulation is that replacing each 0 in the 20 bit message signal with the help of original gold codes. In the message signal each 1 is inverted to 20 bit gold codes. Therefore, 20 bit message signal is converted into (20 × 20) =400 bit spreaded signal.

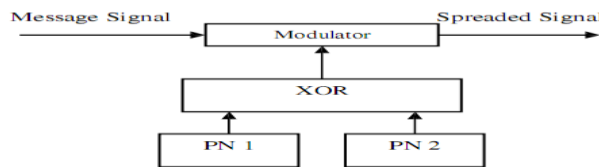


Figure4: Transmitter Block

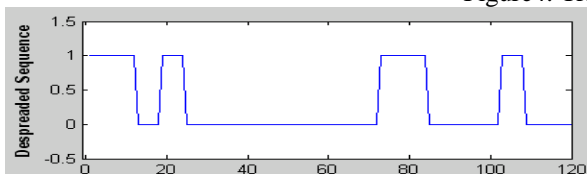


Figure5: Message signal

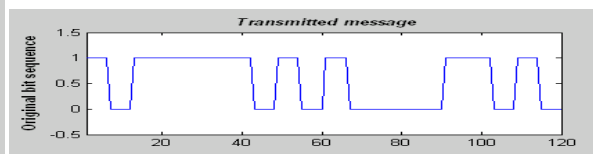


Figure6: Gold code

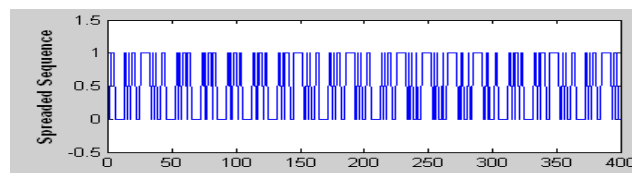


Figure7: Spread signal

**III.3 Demodulation:** In the demodulation process the transmitted signal is demodulated with the carrier signal like to get the original message signal at the end point.

In this section the received spreaded signal is demodulated with the gold codes for producing the original signal.

- As original message signal and gold code are 20 bit long, we have XORed each 20 bit of the received signal.
- The XORing results of the gold codes are either 0 or 1, as the message signal consists of either gold codes or inverted codes.
- The resulting 0 is stored in an array simply called despreaded signal and 1 is stored 1.
- This process is repeated up to 400 bits after each 20 bits.

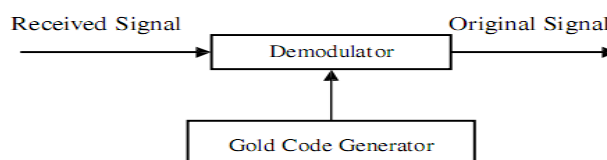


Figure8: Receiver block diagram

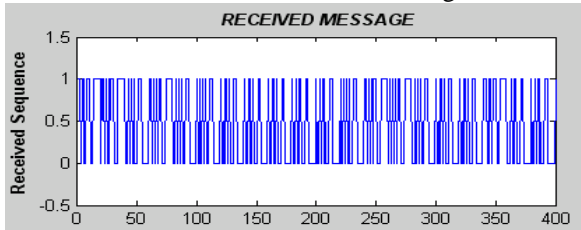


Figure9: Received signal

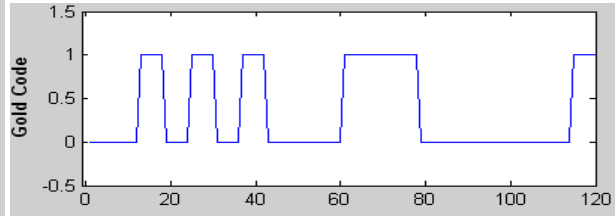


Figure10: Gold code

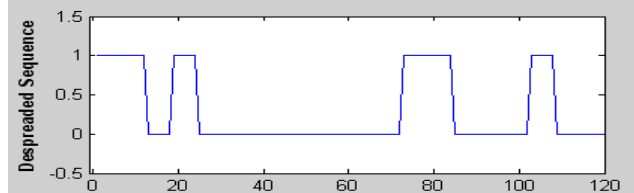


Figure11: Gold code

**1.4 Walsh Hadmard code generation:**

- We first generate 8 length Walsh codes using generateHadamardMatrix function in Matlab.
- To showcase auto correlation and cross correlation properties, we first perform Cross correlation of Walsh code 1 with the rest of the Walsh codes
- Auto Correlation is performed only using the first Walsh code.
- Results prove that Walsh codes have excellent cross-correlation property and poor autocorrelation property. Excellent cross-correlation property (zero cross-correlation) implies orthogonality, which makes it suitable for CDMA applications.

$$H_{2N} = \begin{bmatrix} H_N & H_N \\ H_N & -H_N \end{bmatrix}$$

Figure12: Walsh Hadmard Matrix

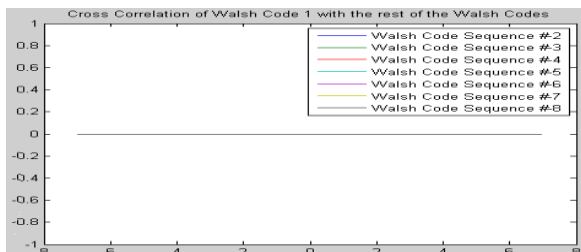


Figure13: Cross Correlated Walsh hadmard code

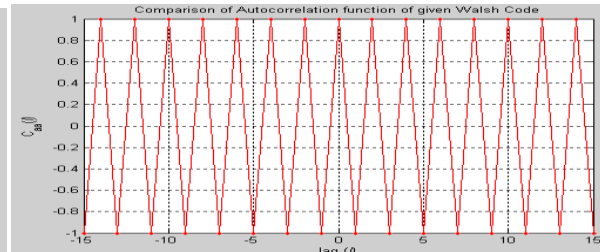


Figure14: Comparison of Auto correlated Walsh Hadmard Code

**IV. SIMULATION RESULTS FOR CORRELATION**

**IV.1 Verification of Autocorrelation:** Auto Correlation is the similarity of sequence with all phase shifts of itself. Pure random data should have a Correlation value close to 0 for all auto Correlations with a phase shift other than 0.

$$R(\tau) = \frac{1}{N} \sum_{k=1}^N B_k B_{k-\tau} \text{ where } \tau = 0, N, 2N$$

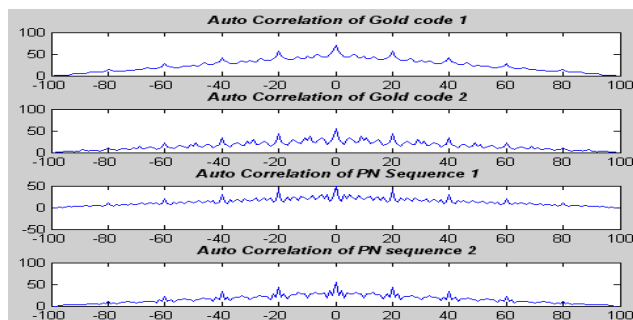


Figure15: Autocorrelation of spread codes

**IV.2 Cross correlation:** In this case the comparison is made between two sequences from rather than a shifted copy of sequence with itself. In general cross correlation value produced by matching sequence with random sequence is low. The cross correlation between two sources A and B, is defined as

$$R_{A,B}(\tau) = \frac{1}{N} \sum_{k=1}^N A_k B_{k-\tau} \quad \text{where } \tau = 0, N, 2N$$

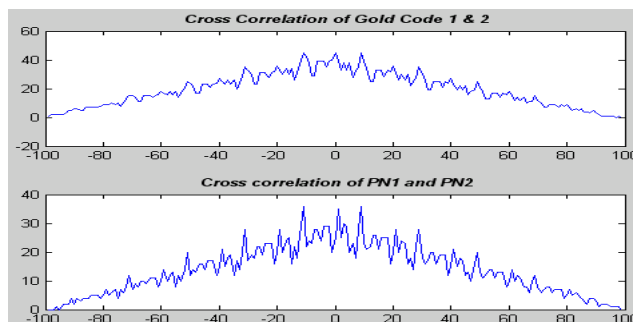


Figure16: Cross correlation of spread codes

## V. RESULTS AND CONCLUSIONS

Correlations are tested which gives better result. From the graphs and results we can said that we have generated a 20 bit long gold code, modulated a message signal, produced 400 bits long spreaded signal and for producing the original message signal demodulated the spreaded signal by successfully. And from other graphs we can also said that the gold codes are better than PN sequences, therefore, gold codes are more appropriate for modulation process and to spread a message signal than PN sequences. Also these properties are verified for another type of code known as Walsh Hadamard code.

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