

Heat Source Effects in Heat and Mass Transfer Of Nano Fluid Flow past a Sheet

G. V. P. N. Srikanth¹, B. Suresh Babu², Dr. G. Srinivas¹

¹Department of Mathematics, Guru Nanak Institute of Technology, Hyderabad, India

²Department of Mathematics, Sreyas Institute of Engineering & Technology, Hyderabad, India

ABSTRACT: The Heat source/sink and suction/injection effects are studied during the Heat and Mass transfer through copper, water nano-fluid along an inclined permeable oscillating flat sheet. The governing equations are solved and the influence of various parameters is analyzed. The Rate of heat transfer for volume fraction against heat source is also analyzed.

Keywords: Nano - fluid, MHD, Inclined plate, Method of line.

List of symbols:

- B_0 Constant applied magnetic field (Wb m^{-2})
 C_p Specific heat at constant pressure ($\text{J kg}^{-1} \text{K}^{-1}$)
 g Gravity acceleration (m s^{-2})
 J Current density
 M Dimensionless magnetic field parameter
 n Dimensionless frequency
 Nu Local Nusselt number
 Pr Prandtl number
 Q Dimensional heat source (kJ s^{-1})
 Q_H Dimensionless heat source parameter (kJ s^{-1})
 s Dimensionless suction parameter
 t Dimensionless time (s)
 T Local temperature of the nano-fluid (K)
 T_w Wall temperature (K)
 T_∞ Temperature of the ambient nano-fluid (K)
 u, w Dimensionless velocity components (m s^{-1})
 U_0 Characteristic velocity (m s^{-1})
 w_0 Mass flux velocity
 k Thermal conductivity
 D_f Diffusivity of water
 D_s Diffusivity of copper

Greek symbols:

- α Thermal diffusivity ($\text{m}^2 \text{s}^{-1}$)
 β_T Thermal expansion coefficient (K^{-1})
 β_c Molecular expansion coefficient
 ε Dimensionless small quantity ($\ll 1$)
 ϕ Solid volume fraction of the nano-particles
 μ Dynamic viscosity (Pa s)

ψ	Kinematic viscosity ($\text{m}^2 \text{s}^{-1}$)
θ	Dimensionless temperature
σ	Electrical conductivity ($\text{m}^2 \text{s}^{-1}$)
σ_1	Stefan-Boltzmann constant
δ	Mean absorption Coefficient
γ	Inclination angle of the plate
ρ	Density
C	Dimensionless diffusion

Superscript

– Dimensional quantities

Subscripts

f Fluid
s Solid
nf Nano-fluid

I. INTRODUCTION

Research in the field of Heat and Mass transfer challenging the cooling of many systems used in day to day life of mankind. The heat and mass transfer enhances enormously when nano-particles are suspended in liquids like water, ethylene glycol etc. This has substantiated by Das, Choi and Patel (2006) in their review paper. In this scenario cooling systems demand the very low heat and mass transfer rate through nano – fluids and heat mass energy systems like automobiles demanding the high heat and mass transfer rate through nano – fluids.

Kuznetsov and Nield (2010) studied the classical problem of free convection boundary layer flow of a viscous and incompressible fluid (Newtonian fluid) past a vertical flat plate to the case of nano-fluids. In these papers the authors have used the nano-fluid model proposed by Buongiorno (2006). Although this author discovered that seven slip mechanisms take place in the convective transport in nano-fluids, it is only the Brownian diffusion and the thermophoresis that are the most important when the turbulent flow effects are absent. More recently, Khan and Aziz (2011) studied Natural convection flow of a nano-fluid over a vertical plate with uniform surface heat flux. M. A. A. Hamad and I. Pop (2011) presented in their recent paper that the solid volume and heat source enhances the heat and mass transfer rate. This brief survey clearly indicates that a definitive conclusion regarding the role of nano-particles in enhancing natural convective transport is yet to be reached. Recently Anwar et.al (2012) studied the conjugate effects of heat and mass transfer of nano-fluids over a non-linear stretching sheet.

In this paper we aim to investigate the MHD Cu – water nano-fluid flow and the heat and mass transfer past a vertical infinite permeable inclined oscillating flat plate under heat and mass source and suction.

II. MATHEMATICAL FORMULATION

Consider the unsteady three dimensional free convection flow of a nano-fluid past a vertical permeable semi-infinite plate in the presence of an applied magnetic field with constant heat source. We consider a Cartesian coordinate system $(\bar{x}, \bar{y}, \bar{z})$. The flow is assumed to be in the \bar{x} direction, which is taken along the plate, and \bar{z} - axis is normal to the plate. We assume that the plate has an oscillatory movement on time \bar{t} and frequency \bar{n} with the velocity $u(0,t)$, which is given $u(0,t) = U_0(1 + \varepsilon \cos(nt))$, where ε is a small constant parameter ($\varepsilon \ll 1$) and U_0 is the characteristic velocity. We consider that initially ($t < 0$) the fluid as well as the plate is at rest. A uniform external magnetic field B_0 is taken to be acting along the \bar{z} -axis. We consider the case of a short circuit problem in which the applied electric field $E = 0$, and also assume that the induced magnetic field is small compared to the external magnetic field B_0 . The surface temperature is assumed to have the constant value T_w while the ambient temperature has the constant value T_∞ , where $T_w > T_\infty$. The conservation equation of current density $\nabla \cdot J = 0$ gives $J_z = \text{constant}$. Since the plate is electrically non-conducting, this constant is zero. It is assumed that the plate is infinite in extent and hence all physical quantities do not depend on \bar{x} and \bar{y} but depend only on \bar{z} and \bar{t} ,

$$\text{i.e.} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

It is further assumed that the regular fluid and the suspended nano-particles are in thermal equilibrium and no slip occurs between them. Under Bossinesq and boundary layer approximations, the boundary layer equations governing the flow and temperature are,

$$\frac{\partial w}{\partial z} = 0 \dots\dots\dots(1)$$

$$\frac{\partial u}{\partial t} + w \frac{\partial u}{\partial z} = \frac{1}{\rho_{nf}} \left[\mu_{nf} \frac{\partial^2 u}{\partial z^2} + (\rho \beta_T)_{nf} g (T - T_\infty) \cos \gamma + (\rho \beta_c)_{nf} g (c - c_\infty) \cos \gamma - \sigma B_0^2 u \right] \dots\dots\dots(2)$$

$$\frac{\partial T}{\partial t} + w \frac{\partial T}{\partial z} = \alpha_{nf} \frac{\partial^2 T}{\partial z^2} - \frac{Q}{(\rho c_p)_{nf}} (T - T_\infty) \dots\dots\dots(3)$$

$$\frac{\partial c}{\partial t} + w \frac{\partial c}{\partial z} = D_{nf} \frac{\partial^2 c}{\partial z^2} \dots\dots\dots(4)$$

The appropriate initial and boundary conditions for the problem are given by

$$\left. \begin{aligned} u(z,t) = 0, T = T_\infty, c = c_\infty \quad \text{for } t < 0 \quad \forall z \\ u(0,t) = U_0 \left[1 + \frac{\varepsilon}{2} (e^{i n t} + e^{-i n t}) \right], T(0,t) = T_w, c(0,t) = c_w \\ u(\infty,t) \rightarrow 0, T(\infty,t) \rightarrow T_\infty, c(\infty,t) \rightarrow c_\infty, \varepsilon \ll 1 \end{aligned} \right\} t \geq 0 \dots\dots\dots(5)$$

Thermo-Physical properties are related as follows:

$$\begin{aligned} \rho_{nf} &= (1-\phi) \rho_f + \phi \rho_s, \quad \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \quad \alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}} \\ (\rho c_p)_{nf} &= (1-\phi) (\rho c_p)_f + \phi (\rho c_p)_s \\ (\rho \beta)_{nf} &= (1-\phi) (\rho \beta)_f + \phi (\rho \beta)_s, \\ k_{nf} &= k_f \left[\frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \right] \dots\dots\dots(6) \end{aligned}$$

The thermo-physical properties (values) of the materials used are as follows.

Table I

Physical Properties	Water	Copper(Cu)
C_p (J / kg K)	4,179	385
ρ (kg / m ³)	997.1	8,933
κ (W / m K)	0.613	400
$\beta_T \times 10^{-5}$ (1/K)	21	1.67
$\beta_c \times 10^6$ (m ² /h)	298.2	3.05

We consider the solution of Esq. (1) as $w = -w_0 \dots\dots\dots(7)$

Where the constant w_0 represents the normal velocity at the plate which is positive for suction ($w_0 > 0$) and negative for blowing or injection ($w_0 < 0$). Thus, we introduce the following dimensionless variables:

$$z = \left(\frac{\psi_f}{U_0}\right)Z, \quad t = \left(\frac{\psi_f}{U_0^2}\right)t^*, \quad n = \left(\frac{U_0^2}{\psi_f}\right)\eta, \quad u = UU_0, \quad \theta = \frac{T-T_\infty}{T_w-T_\infty}, \quad c = \frac{c-c_\infty}{c_w-c_\infty} \quad \dots\dots\dots (8)$$

Using equations 5,6,7,8 the Eq. 2,3&4 can be written in the following dimensionless form:

$$\left[1-\phi + \phi \left(\frac{\rho_s}{\rho_f}\right)\right] \left(\frac{\partial U}{\partial \tau} - S \frac{\partial U}{\partial Z}\right) = \frac{1}{(1-\phi)^{2.5}} \frac{\partial^2 U}{\partial Z^2} + \left[1-\phi + \phi \frac{(\rho \beta_T)_s}{(\rho \beta_T)_f}\right] \theta \cos \gamma + \left[1-\phi + \phi \frac{(\rho \beta_c)_s}{(\rho \beta_c)_f}\right] c \cos \gamma - MU \quad \dots\dots\dots (9)$$

$$\left[1-\phi + \phi \frac{(\rho c_p)_s}{(\rho c_p)_f}\right] \left(\frac{\partial \theta}{\partial \tau} - S \frac{\partial \theta}{\partial Z}\right) = \frac{1}{p_r} \left[\frac{k_{nf}}{k_f} \frac{\partial^2 \theta}{\partial Z^2}\right] - \frac{1}{p_r} Q_H \theta \quad \dots\dots\dots (10)$$

$$\left[1-\phi + \phi \frac{(\rho c_p)_s}{(\rho c_p)_f}\right] \left(\frac{\partial c}{\partial \tau} - S \frac{\partial c}{\partial Z}\right) = \frac{1}{Sc} \left[\frac{\partial^2 c}{\partial Z^2}\right] \quad \dots\dots\dots (11)$$

Where the corresponding boundary conditions (5) can be written in the dimensionless form as:

$$\left. \begin{aligned} U(z,t) = 0, \quad \theta(z,t) = 0, \quad c(z,t) = 0 \quad \text{for } t < 0 \quad \forall z \\ U(0,t) = U_0 \left[1 + \frac{\varepsilon}{2}(e^{int} + e^{-int})\right], \quad \theta(0,t) = 1, \quad c(0,t) = 1 \\ U(\infty,t) \rightarrow 0, \quad \theta(\infty,t) \rightarrow 0, \quad c(\infty,t) \rightarrow 0 \end{aligned} \right\} \forall t \geq 0 \quad \dots\dots\dots (12)$$

Here p_r is the Prandtl number, S is the suction ($S>0$) or injection ($S<0$) parameter, M is the magnetic parameter, $Q_H (> 0)$ is the heat source parameter or $Q_H (< 0)$ is the heat sink parameter, Sc is the Schmidt number, which are defined as:

$$p_r = \frac{\psi_f}{\alpha_f}, \quad S = \frac{w_0}{U_0}, \quad M = \frac{\sigma B_0^2 \psi_f}{\rho_f U_0^2}, \quad Q_H = \frac{Q \psi_f^2}{k_f U_0^2}, \quad Sc = \frac{\psi_f}{d_f}$$

Where the velocity characteristic U_0 is defined as

$$U_0 = \left[g \beta_f (T_w - T_\infty) \psi_f\right]^{1/3} \quad \dots\dots\dots (13)$$

The local Nusselt number Nu in dimensionless form:

$$Nu = - \frac{k_{nf}}{k_f} \theta'(0) \quad \dots\dots\dots (14)$$

III RESULTS AND DISCUSSIONS

The governing equations are solved by using Method of lines with the help of Mathematica package. The variations of velocity U and temperature θ are graphically exhibited and the Heat Transfer rate (Nu) is exhibited in Table – II for various values of ϕ , S , M , α , Q_H by keeping $Pr = 6.2$, $nt = \pi/2$ and $\varepsilon = 0.02$. The effect of various parameters is as follows.

The increase of solid volume fraction reduces the velocity Fig.1 and enhances the temperature Fig.6. The thickness of momentum and the thermal boundary layers decreases with increase in ϕ . From Fig.2&7 the momentum and thermal boundary layers decreases for injection or suction. From Fig. 3 & 8 the momentum and thermal boundary layers decreasing for heat sink or source Q_H . The variations of velocity and temperature with magnetic parameter M are depicted in Figs. 4 & 9. The effects of inclination angle α on velocity and temperature

are exhibited in Figs. 5 & 10. The increase in inclination reduces the velocity and enhances the temperature. From Fig.11 it is observe that the increase in diffusivity or the decrease in viscosity increases the velocity. The same is observed in diffusion with variation of Sc Fig.12.

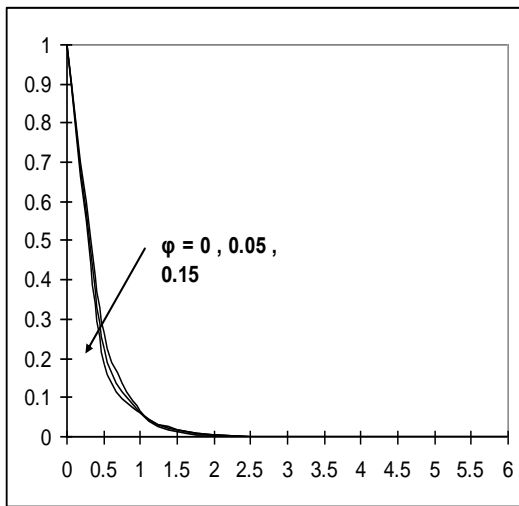


Fig.1 Variation of U with ϕ

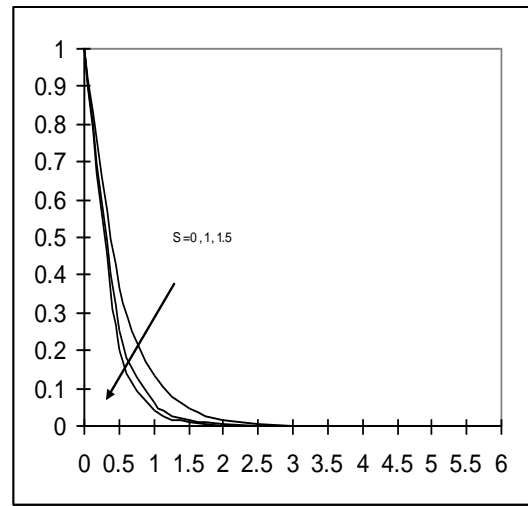


Fig.2 Variation of U with S

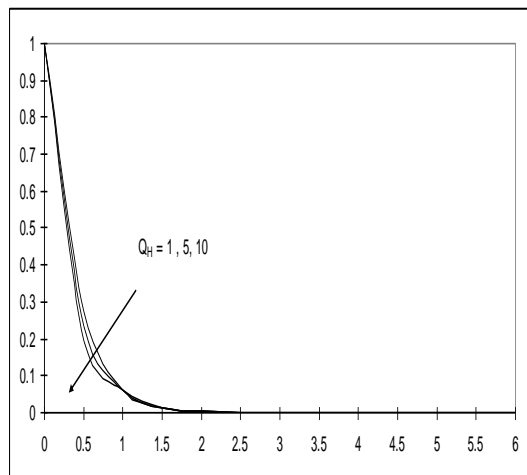


Fig.3 Variation of U with Q_H

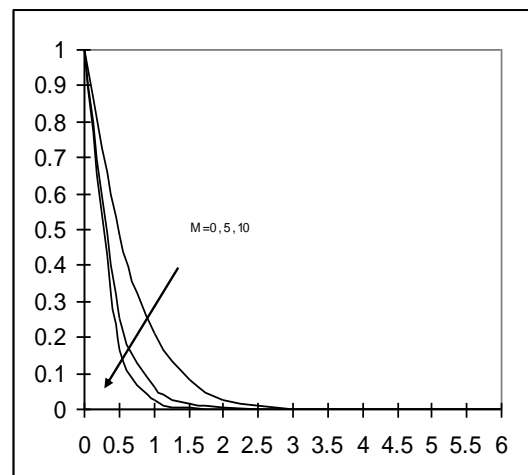


Fig.4 Variation of U with M

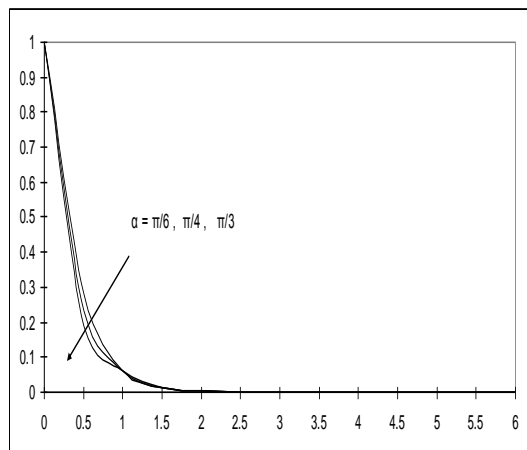


Fig.5 Variation of U with α

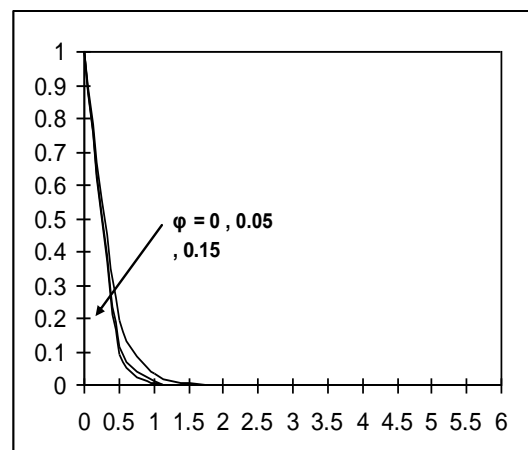


Fig.6 Variation of θ with ϕ

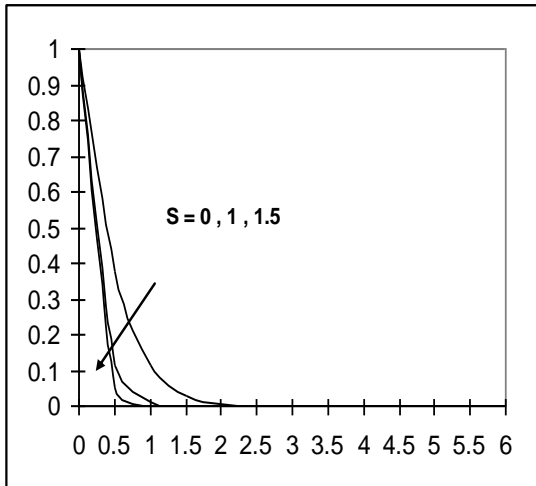


Fig.7 Variation of θ with S

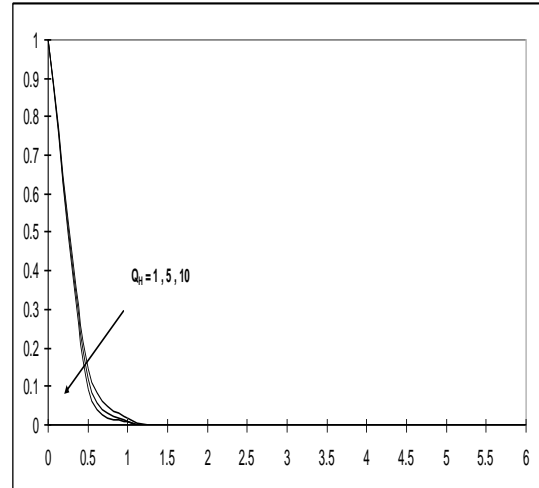


Fig.8 Variation of θ with Q_H

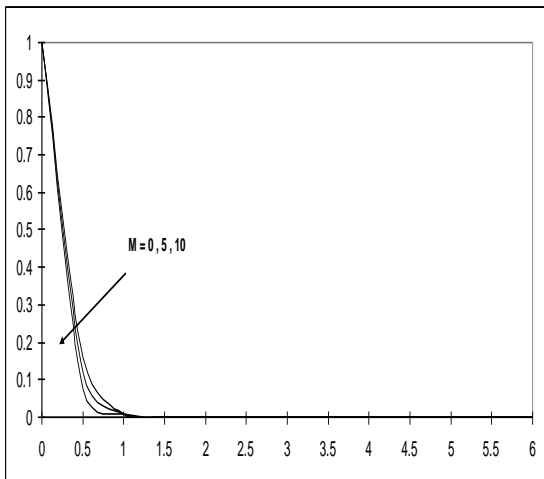


Fig.9 Variation of θ with M

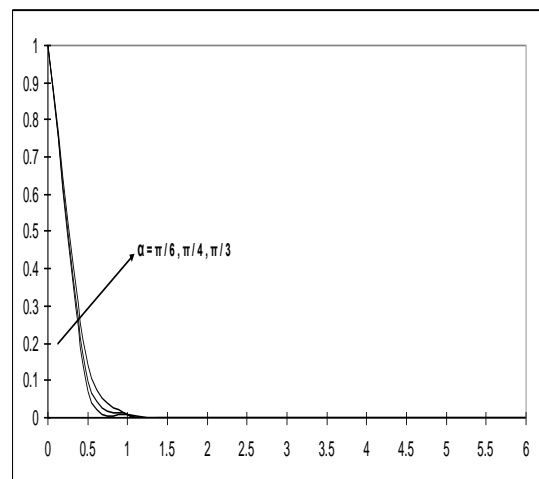


Fig.10 Variation of θ with α

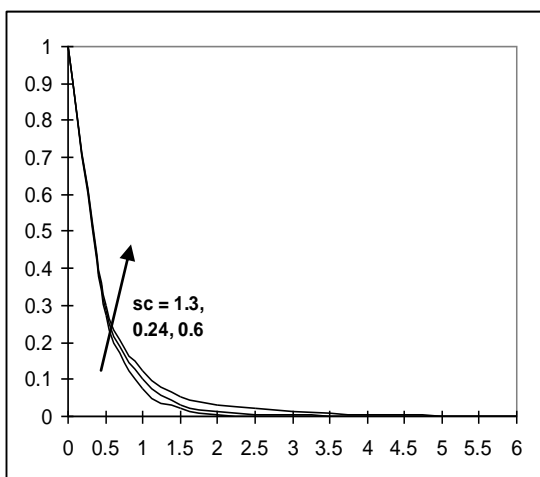


Fig.11 Variation of U with Sc

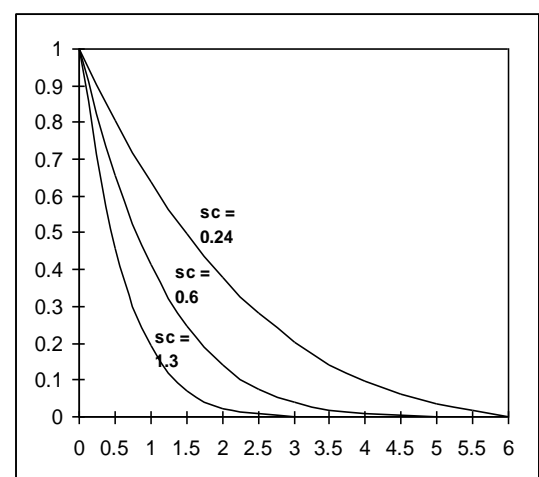


Fig.12 Variation of C with Sc

Nusselt Number:

Table II

$$S = 1; M = 5; \alpha = \frac{\pi}{3}; Sc = 0.6$$

ϕ	$Q_H = 5$	$Q_H = 10$
0	4.5034	4.8933
0.05	4.9525	5.4462
0.15	5.8815	6.6397
0.2	6.3913	7.3178

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