

Effect of Sweep Angle on Rolling Moment Derivative of an Oscillating Supersonic/Hypersonic Delta Wing

Asha Crasta¹, S. A. Khan², Antony A. J³

¹Sr. Assistant Professor, Mangalore Institute of Technology and Engineering, Moodabidri, Karnataka, India,

²Principal, Z. H. College of Engineering & Technology, AMU, Aligarh, UP, India,

³Vice Principal, Sahyadri College of Engineering and Management, Mangalore, India.

Abstract: In the Present paper effect of sweep angle on roll of damping derivative of a delta wing with straight leading edges for an attached shock case in supersonic/hypersonic flow has been studied analytically. A Strip theory is used in which strips at different span wise location are independent. This combines with similitude which leads to give a piston theory. The Present theory is valid for attached shock case only. The results of the present study reveals that with the increase in the sweep angle; it results in continuous decrease in the roll damping derivative, it is also seen that the magnitude of the decrement for lower sweep angle is very large as compared to the higher values of the sweep angles due to the drastic change in the plan form area. Roll damping derivative progressively increases with the angle of attack, however, with the increase in the Mach number it results in the decrement in the damping derivative and later conforms to the Mach number principle. Effects of wave reflection, leading edge bluntness, and viscosity have not been taken into account. Results have been obtained for supersonic/hypersonic flow of perfect gases over a wide range of angle of attack, plan form area, and the Mach number.

Keywords: delta wing, Hypersonic flow, Piston theory, Rolling derivative, Supersonic Flow, sweep angle.

I. Introduction

The analysis of hypersonic and supersonic flow over flat deltas with straight leading edge over a wide incidence range is of current interest since the desire for high speed, maneuverability and efficiency has been dominating the evolution of high performance military aircrafts. The knowledge of aerodynamic load and stability for such types is a need for calculating simple but reasonably accurate methods for parametric calculations facilitating the design process. The computation of dynamic stability for these shapes at high incidence which is likely to occur during the course of reentry or maneuver is of current interest. Usually the shock waves are very strong when descending and they can either be detached or attached.

The theories for steady delta wings in supersonic/hypersonic flow with shock wave attached were given by Pike [1] and Hui [2]. Carrier [3] and Hui [4] gave exact solutions for 2-D flow in the case of an oscillating wedge and for an oscillating flat plate were given by Hui [5], which is valid uniformly for all supersonic Mach numbers and wedge angles or angles of attack with attached shock wave. Hui [5] also calculated pressure on the compression side of a flat delta.

The importance of dynamic stability at large incidence during re-entry or maneuver has been pointed out by Orlik-Ruckemann [6]. The shock attached relatively high aspect ratio delta is often preferred for its high lift to drag ratio.

Hui and Hemdan [7] have studied the unsteady shock detached case in the context of thin shock layer theory. Liu and Hui [8] have extended Hui's [5] theory to a shock attached delta wing in pitch. Light hill [9] has developed a "Piston Theory" for oscillating airfoils at high Mach numbers. A parameter δ is introduced, which is a measure of maximum inclination angle of Mach wave in the flow field. It is assumed that $M_\infty \delta$ is less than or equal to unity (i.e. $M_\infty \delta \leq 1$) and is of the order of maximum deflection of a streamline. Light hill [9] likened the 2-D unsteady problem to that of a gas flow in a tube driven by a piston and termed it "Piston Analogy".

Ghosh [10] has developed a large incidence 2-D hypersonic similitude and piston theory. It includes Light hill's [9] and Mile's [11] piston theories. Ghosh and Mistry [12] have applied this theory of order of ϕ^2 where ϕ is the angle between the attached shock and the plane approximating the windward surface. For a plane surface, ϕ is the angle between the shock and the body. The only additional restriction compared to small disturbance theory is that the Mach number downstream of the bow shock is not less than 2.5.

Ghosh [13] has obtained a similitude and two similarity parameters for shock attached oscillating delta wings at large incidence. Crasta and Khan have extended the Ghosh similitude to Hypersonic/supersonic flows past a planar wedge [14] and [18] and Non planar wedge [20], [21], and [22]. Crasta and Khan have obtained stability derivatives in pitch and roll of a delta wing with straight leading edge [23] and [24] and curved leading edges for supersonic flows [15] and Hypersonic flows [16]. Crasta and Khan have studied the effect of angle of incidence on pitching derivatives and roll of a damping derivative of a delta wing with curved leading edges for an attached shock case [17] and [27]. Further in all cases stability derivatives in Newtonian limit have been calculated by Crasta and Khan [19], [25], and [26]. In the present analysis the effect of Sweep angle on rolling moment derivative of an oscillating supersonic/hypersonic delta wing with straight leading edge is been studied and some of the results have been obtained.

II. ANALYSIS

A thin strip of the wing, parallel to the centerline, can be considered independent of the z dimension when the velocity component along the z direction is small. This has been discussed by Ghosh's [10]. Using this Ghosh's Hypersonic large incidence similitude the pressure distribution in Hypersonic flow is given by

$$\frac{P}{P_\infty} = 1 + AM_p^2 + AM_p(B + M_p^2)^{\frac{1}{2}}, \text{ Where } P_\infty \text{ is free stream pressure} \quad (1)$$

Since strips at different span wise location are assumed independent of each other, the strip can be considered as a flat plate at an angle of attack. The angle of incidence is same as that of wing. Angle ϕ is the angle between the shock and the strip. A piston theory which has been used in equation(1) has been extended to supersonic flow. The expression is given below.

$$\frac{p}{p_\infty} = 1 + A\left(\frac{M_p}{\cos \phi}\right)^2 + A\left(\frac{M_p}{\cos \phi}\right)\left(B + \left(\frac{M_p}{\cos \phi}\right)^2\right)^{\frac{1}{2}} \quad (2)$$

Where p_∞ is free stream pressure, $A = \frac{(\gamma + 1)}{4}$, $B = (4/(\gamma + 1))^2$, γ is the specific heat ratio and M_p = the local piston Mach number normal to the wedge surface.

Rolling Damping Derivative:

Let the rate of roll be \bar{p} and rolling moment be L, defined according to the right hand system of reference.

$$\therefore L = 2 \int_0^c \left(\int_0^{Z=f(x)} p \cdot z \cdot dz \right) dx \quad (3)$$

The piston Mach number is given by

$$M_p = M_\infty \sin \alpha - \frac{z}{a_\infty} \bar{p} \quad (4)$$

The roll damping derivative is non-dimensionalized by dividing with the product of dynamic pressure, wing area,

and span and characteristic time factor $\frac{C}{U_\infty}$

$$\therefore -C_{l_p} = \frac{1}{\rho_\infty U_\infty C^3 b \cot \varepsilon} \left(\frac{-\partial L}{\partial p} \right)_{\substack{\alpha = \alpha_0 \\ p=0}} \quad (5)$$

Combining through (1) to (5)

Rolling moment due to rate of roll of Hypersonic Flow is given by

$$-C_{l_p} = \sin \alpha_0 f(S_1) \left[\frac{\cot \varepsilon}{12} \right] \quad (6)$$

$$\text{Where } S_1 = M_\infty \sin \alpha_0 \quad (7)$$

$$f(S_1) = \frac{(\gamma + 1)}{2S_1} \left[2S_1 + \frac{(B + 2S_1^2)}{(B + S_1^2)^{\frac{1}{2}}} \right] \quad (8)$$

Rolling moment derivative can be expressed in terms of aspect ratio as follows.

$$-C_{l_p} = \sin \alpha_0 f(S_1) \left[\frac{AR}{48} \right] \tag{9}$$

Rolling moment derivative in Supersonic Flow is given by

$$\therefore -C_{l_p} = \frac{\sin \alpha_0 f(S_1)}{(\cos^2 \phi)} \left[\frac{\cot \varepsilon}{12} \right] \tag{10}$$

Where $f(S_1) = \frac{(r+1)}{2S_1} [2S_1 + (B + 2S_1^2)/(B + 2S_1^2)^{\frac{1}{2}}]$ and $S_1 = \frac{M_\infty \sin \alpha_0}{\cos \phi}$.

III. RESULTS AND DISCUSSIONS

The expressions obtained analytically using the present theory in supersonic/hypersonic flow are being shown in the figures 1 to 10.

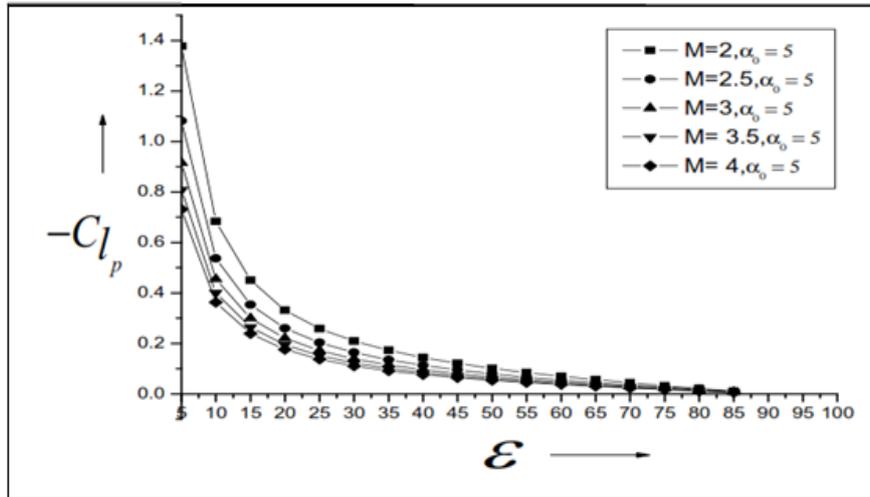


Fig. 1: Variation of Roll Damping derivative with sweep angle

Fig. 1 presents results for damping derivative in roll for sweep angles from five degrees to eighty five degrees, Mach numbers in the range from $M = 2$ to 4 at a fixed value of angle of attack of five degrees. It is seen that with the increase in Mach number the roll damping derivative decreases continuously, however, the magnitude is different for different Mach numbers. It is observed there is decrement of twenty one per cent, eighteen per cent, eleven per cent and thirteen per cent for the Mach number band 2 to 2.5, 2.5 to 3, 3 to 3.5 and 3.5 to 4 for a fixed value of sweep angle of five degrees. For higher sweep angles the magnitude of decrement is diminishing.

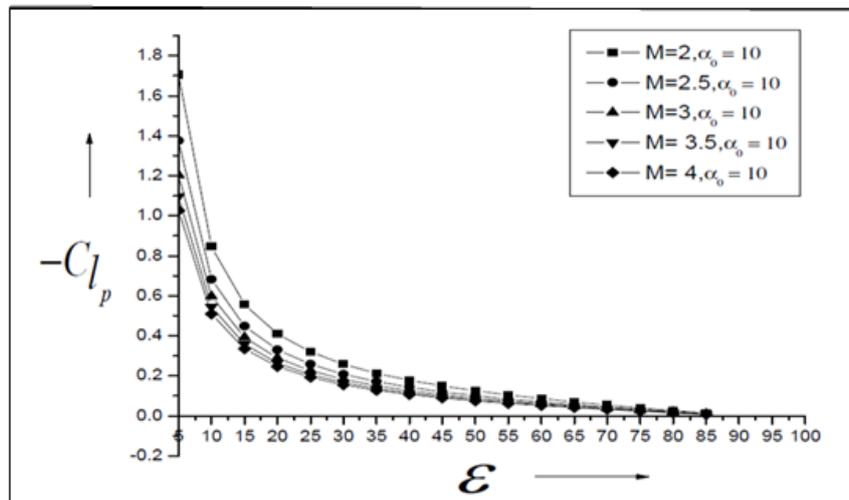


Fig. 2: Variation of Rolling Damping derivative with sweep angle

Fig. 2 presents results for damping derivative in roll for sweep angles from five degrees to eighty five degrees, Mach numbers in the same range of Mach numbers as discussed above and fixed value of angle attack of ten degrees. It is seen that with the increase in Mach number the roll damping derivative decreases continuously, however, the magnitude is different for different Mach numbers. It is observed there is decrement of eighteen per cent, fourteen per cent, eight per cent and five per cent for the Mach number band 2 to 2.5, 2.5 to 3, 3 to 3.5 and 3.5 to 4 for a fixed value of sweep angle of five degrees.

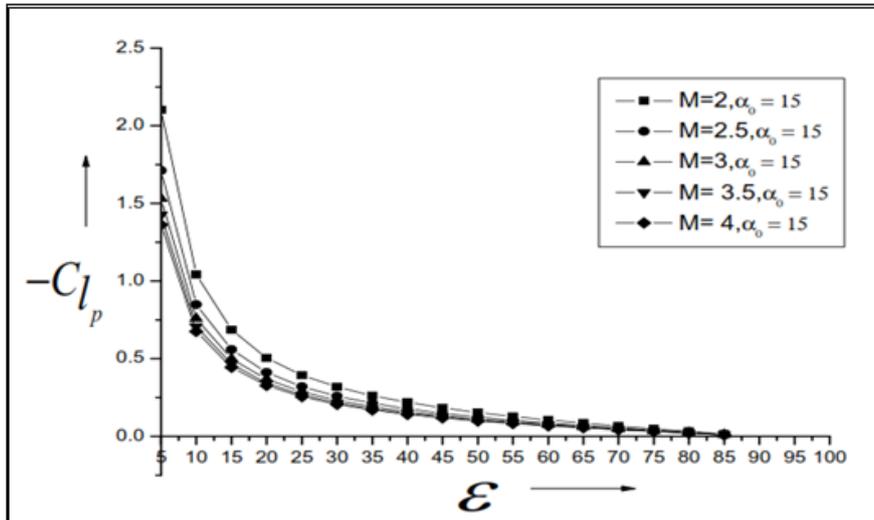


Fig. 3: Variation of Roll Damping derivative with sweep angle

Fig. 3 presents results for damping derivative in roll for sweep angles from five degrees to eighty five degrees, Mach numbers in the same range of Mach numbers as we have discussed earlier and for a fixed value of angle of attack of fifteen degrees. It is seen that with the increase in Mach number the roll damping derivative decreases continuously, however, the magnitude is different for different Mach numbers. It is observed there is decrement of eighteen per cent, twelve per cent, ten per cent and seven per cent for the Mach number band 2 to 2.5, 2.5 to 3, 3 to 3.5 and 3.5 to 4 for a fixed value of sweep angle of five degrees. For higher sweep angles the order of decrement is small.

Fig. 4 presents results for damping derivative in roll for sweep angles from five degrees to eighty five degrees, Mach numbers in the same range of Mach numbers as we have discussed earlier and for a fixed value of angle of attack of twenty degrees. It is seen that with the increase in Mach number the roll damping derivative decreases continuously, however, the magnitude is different for different Mach numbers. It is observed there is decrement of twenty one per cent, twelve per cent, six per cent and five per cent for the Mach number band 2 to 2.5, 2.5 to 3, 3 to 3.5 and 3.5 to 4 for a fixed value of sweep angle of five degrees.

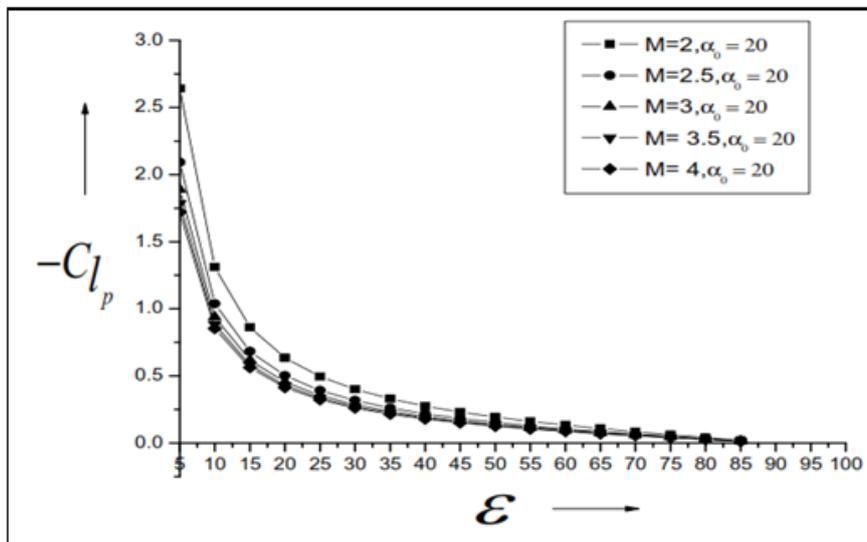


Fig. 4: Variation of Roll Damping derivative with sweep angle

Fig. 5 presents the similar results for damping derivative in roll for sweep angles from five degrees to eighty five degrees, Mach numbers in the same range of Mach numbers as we have discussed earlier and for a fixed value of angle of attack of twenty five degrees. It is seen that with the increase in Mach number the roll damping derivative decreases continuously, however, the magnitude is different for different Mach numbers. It is observed there is decrement of seventeen per cent, ten per cent, five per cent and three per cent for the Mach number band 2 to 2.5, 2.5 to 3, 3 to 3.5 and 3.5 to 4 for a fixed value of sweep angle of five degrees.

This variation in the magnitude of the roll damping derivatives may be attributed due to the variation in the angle of attack and all other parameters remains the same; due to the increase in the angle of attack will change the shock structure and its strength which will dictate the flow field in the vicinity of the wing surface area and the oblique shock wave.

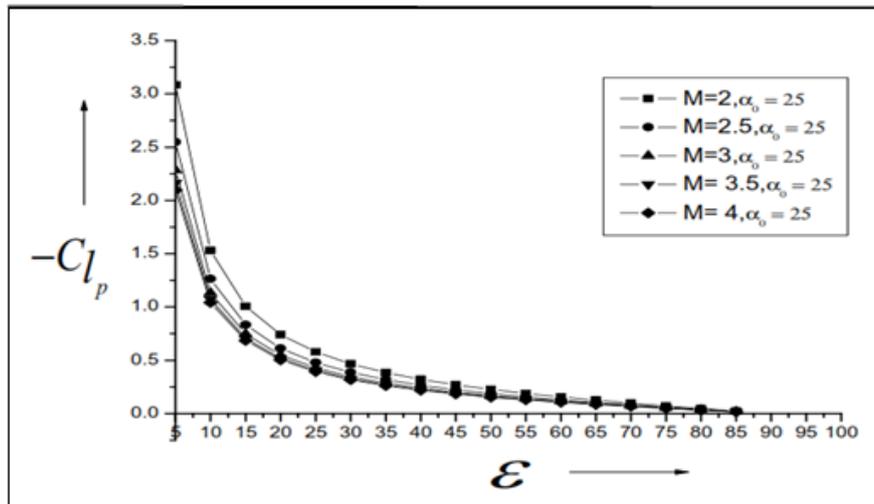


Fig. 5: Variation of Roll Damping derivative with sweep angle

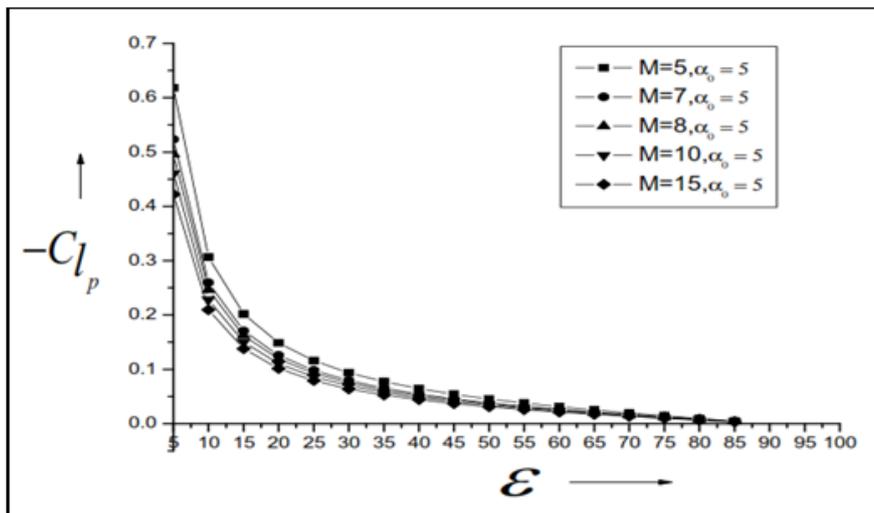


Fig. 6: Variation of Roll damping derivative with sweep angle

Fig. 6 presents results for damping derivative in roll for sweep angles from five degrees to eighty five degrees, Mach numbers in the range $M = 5$ to 15 for a fixed value of angle of attack of five degrees. It is seen that with the increase in Mach number the roll damping derivative decreases continuously, however, the magnitude is different for different Mach numbers. It is observed that this decrement is ten one per cent, seven per cent, six per cent and four per cent for the Mach number band 5 to 7, 7 to 8, 8 to 10 and 10 to 15 for a fixed value of sweep angle of five degrees.

Results for angle of attack ten, fifteen, twenty and twenty five degrees are shown in Figures 7 to 10. From the figures it is evident that for angle of attack ten degrees there is marginal change in the values of the damping derivatives, whereas, when the angle of attack was increased to fifteen, twenty, and twenty five they don't show any variation in the roll damping derivatives for the Mach numbers in the range five to fifteen degrees. This may be due to the very high Mach number the flow will be independent of Mach number, and the

shock wave angle will be very small and the shock wave will be very close to the body, this may be the reason for this behaviour.

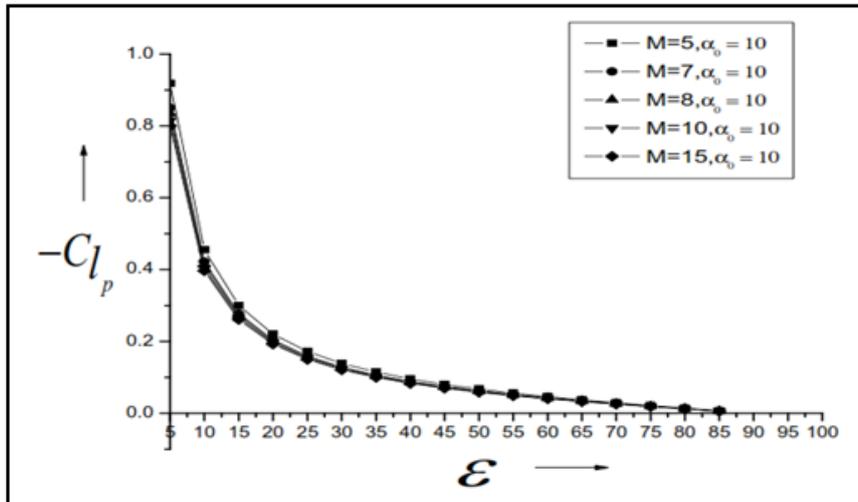


Fig. 7: Variation of Roll damping derivative with sweep angle

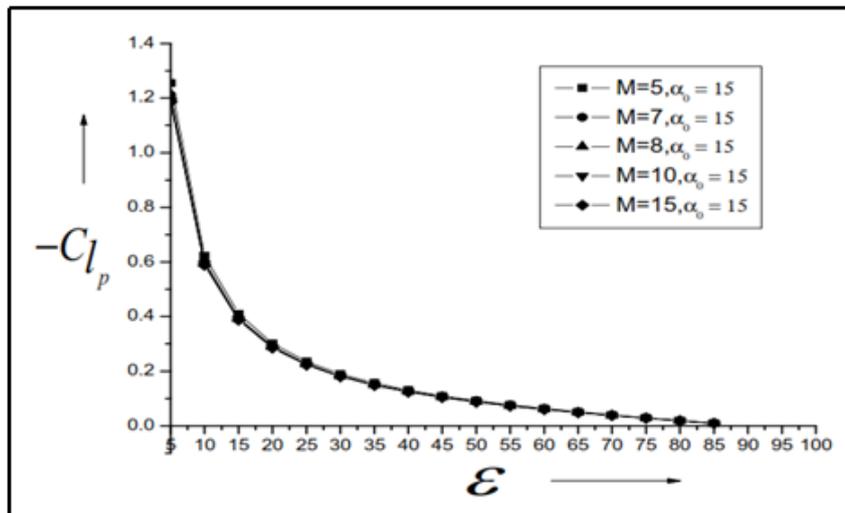


Fig. 8: Variation of Roll damping derivative with sweep angle

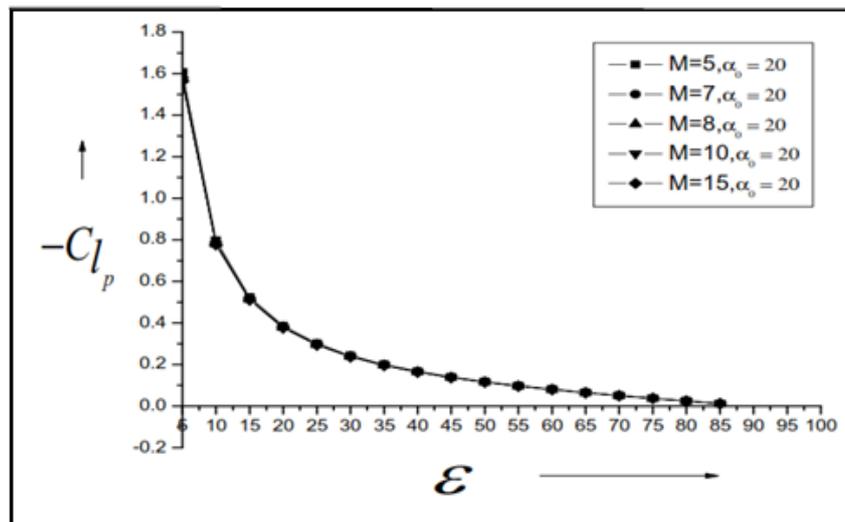


Fig. 9: Variation of Roll damping derivative with sweep angle

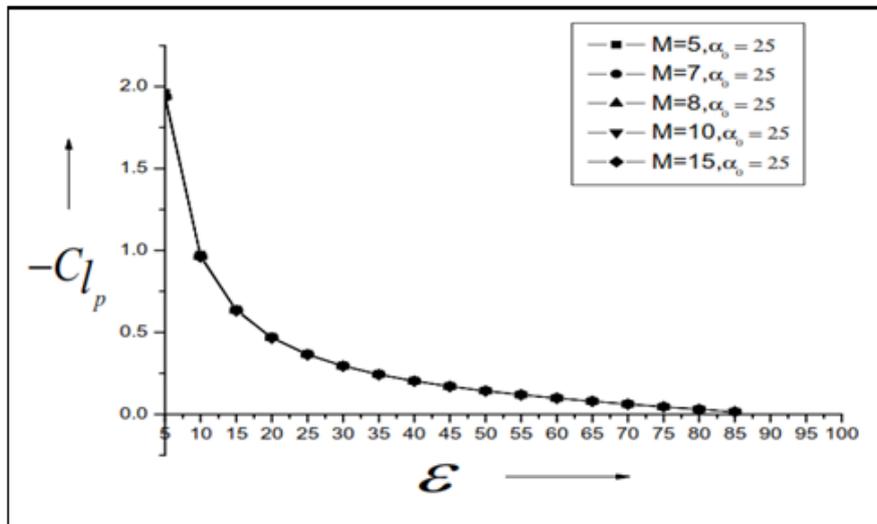


Fig. 10: Variation of Roll damping derivative with sweep angle

IV. CONCLUSION

Based on the above discussion we draw the following conclusions:

- Roll damping derivative decreases with Mach number in supersonic flow, whereas in hypersonic flow it becomes independent of Mach number after certain value of the Mach number.
- There is a considerable decrease in the roll damping derivatives with sweep angle. This decrease is due to the decrease in the plan form area of the wing. This decrease is highest for five degree sweep angle, and further increase in sweep angle does lead to decrement but the magnitude is small.
- Roll damping derivative linearly increases with angle of attack up to twenty five degrees.
- In hypersonic flow for angle of attack there is appreciable change in the roll damping derivative, however, for ten degrees angle of attack it is negligible and for angle of attack beyond ten degrees it remains constant. This may be due to increase in the angle of attack leading to decrease in the shock wave angle leading to the oblique shock coming very close to the surface of the wing.
- The present theory is simple and yet gives good result with enormous computational ease.
- The present is valid when the shock wave is attached with the leading edge of the wing.
- In present theory does not include the effect of viscosity, leading edge bluntness, and real gas effects.

REFERENCES

- [1] Pike, J., "The pressure on flat and anhydral delta wings with attached shock waves", The Aeronautical Quarterly, November 1972, XXIII, Part 4, pp. 253-262.
- [2] Hui, W.H., "Stability of Oscillating Wedges and Caret Wings in Hypersonic and Supersonic Flows", AIAA Journal, Vol. 7, Aug. 1969, pp.1524-1530.
- [3] Carrier, G.F. 1949, "The oscillating Wedge in Supersonic stream", Journal of Aeronautical Sciences, Vol. 16, No. 3, pp. 150-152, March.
- [4] Hui, W. H., "Supersonic/hypersonic flow past on oscillating flat plate at high angles of attack", ZAMP, Vol. 29, 1978, pp. 414-427.
- [5] Hui, W. H., "Supersonic and hypersonic flow with attached shock waves over delta wings", Proc of Royal Society, London, 1971, A. 325, pp. 251-268.
- [6] Orlik-Ruckemann, K. J., "Dynamic stability testing of aircraft needs versus capabilities", Progress in the Aerospace Sciences, Academic press, N.Y., 1975, 16, pp. 431-447.
- [7] Hui, W. H. and Hemdan, H. T., "Unsteady hypersonic flow over delta wings with detached shock waves", AIAA Journal, April 1976, 14, pp. 505-511.
- [8] Lui, D. D. and Hui W. H., "Oscillating delta wings with attached shock waves," AIAA Journal", June 1977, 15, 6, pp. 804-812.
- [9] Light Hill, M. J., "Oscillating Aerofoil at High Mach Numbers", Journal of Aeronautical Sciences, Vol. 20, June 1953, pp. 402-406.
- [10] Ghosh K., "A new similitude for aerofoil in hypersonic flow", Proc of the 6th Canadian congress of applied mechanics", Vancouver, 29th May 3rd June, 1977, pp. 685-686.
- [11] Miles, J. W., "Unsteady flow at hypersonic speeds, Hypersonic flow", Butter worths Scientific Publications, London, 1960, pp. 185-197.
- [12] Ghosh, K and Mistry B. K., "Large incidence hypersonic similitude and oscillating non-planar wedges", AIAA Journal, August 1980, 18, 8, pp. 1004-1006.

- [13] Ghosh K., "Hypersonic large deflection similitude for oscillating delta wings", The Aeronautical journal, October, 1984, pp. 357-361.
- [14] Asha Crasta and Khan S. A., "High Incidence Supersonic similitude for Planar wedge", International Journal of Engineering research and Applications, Vol. 2, Issue 5, September-October 2012, pp. 468-471.
- [15] Khan S. A. and Asha Crasta, "Oscillating Supersonic delta wings with curved leading edges", Advanced Studies in Contemporary mathematics, Vol. 20 (2010), No.3, pp. 359-372.
- [16] Asha Crasta and S. A. Khan, "Estimation of Stability derivatives of an Oscillating Hypersonic delta wings with curved leading edges", Vol.3, Issue 3, December 2012, pp. 483-492.,
- [17] Asha Crasta and S. A. Khan, "Effect of Angle of Incidence on Stability derivatives of a wing", International Journal for Research in Applied Science and Engineering Technology, Vol. 2, Issue V, May 2014, pp. 411-422.
- [18] Asha Crasta and S. A. Khan, "Hypersonic Similitude for Planar Wedges", International Journal of Advanced Research in Engineering and Technology, Volume 5, Issue2, February 2014, pp. 16-31.
- [19] Asha Crasta and S. A. Khan, "Estimation of Stability Derivatives for a Planar Wedge in the Newtonian Limit", IOSR Journal of Mathematics, Volume 10, Issue 2, Version I, March-April 2014, pp. 01-06.
- [20] Asha Crasta and S. A. Khan, "Effect of Mach number on Stiffness and Damping Derivatives for Oscillating Hypersonic Non-Planar Wedge", IOSR Journal of Mechanical and Civil Engineering , Volume 11, Issue 2, Ver. VIII, March- April 2014, pp. 04-11.
- [21] Asha Crasta and S. A. Khan, "Effect of Angle of Incidence on Stiffness and Damping derivatives for Oscillating Hypersonic Non-planar Wedge", International Journal for Research in Applied Science and Engineering Technology, Vol. 2, Issue IV, April 2014, pp. 229-242.
- [22] Asha Crasta and S. A. Khan, "Supersonic Similitude for Oscillating Non-Planar wedge", IOSR Journal of Mathematics, Volume 10, Issue 2, Ver.VI, March-April 2014, pp.15-24.
- [23] Asha Crasta, M. A. Baig, S. A. Khan , 'Estimation of Stability derivatives of a Delta wing in Hypersonic flow', International Journal of Emerging trends in Engineering and Developments , Vol. 6, Issue 2, September 2012, pp. 505-516.
- [24] Asha Crasta and S. A. Khan, "Oscillating Supersonic delta wing with Straight Leading Edges", International Journal of Computational Engineering Research, Vol. 2, Issue 5, September 2012, pp. 1226-1233.
- [25] Asha Crasta and S. A. Khan, 'Stability derivatives of a Delta Wing with Straight leading edge in the Newtonian Limit', International Journal of Engineering research and Applications, Vol. 4, Issue 3, Version 2, March 2014, pp. 32-38.
- [26] Asha Crasta and S. A. Khan, "Stability Derivatives in the Newtonian Limit", International Journal of Advanced Research in Engineering and Technology, Volume 4, Issue 7, November-December 2013, pp. 276-289.
- [27] Asha Crasta, S. A. Khan , 'Effect of angle of incidence on Roll damping derivative of a Delta Wing', International Journal of Emerging trends in Engineering and Developments Volume 2, Issue 4, March 2014, pp. 343-356.
- [28] Asha Crasta, S.A.Khan, 'Effect of sweep angle on Roll damping derivative of a Delta Wing', International Journal of Emerging trends in Engineering and Developments Volume 5, Issue 4, August-September 2014, pp. 45-55.