OPEN ACCESS
International
Journal
Of Modern Engineering Research (IJMER)

A production - Inventory model with JIT setup cost incorporating inflation and time value of money in an imperfect production process

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Abstract: *A production inventory model with Just-In-Time (JIT) set-up cost has been developed in which inflation and time value of money are considered under an imperfect production process. The demand rate is considered to be a function of advertisement cost and selling price. Unit production cost is considered incorporating several features like energy and labour cost, raw material cost and development cost of the manufacturing system. Development cost is assumed to be a function of reliability parameter.*
Considering these phenomena, an analytic expression is obtained for the total profit of the model. The model provides an analytical solution to maximize the total profit function.A numerical example is presented to illustrate the model along with graphical analysis. Sensitivity analysis has been carried out to identify the most sensitive parameters of the model.

I. Introduction

In the manufacturing system, a production process is not completely perfect. Producer may produce defective items from the very beginning or it may happen that at the beginning of the production process, all the produced items are non-defective. Consumers in the super market judge the goods on the basis of both quality depending upon defective or non-defective goods and price. The choice of quality is often an important factor to an industry. A lower quality product results in lower effectiveness with which a manufacturer meets consumers' demands. Productivity of a manufacturer is also a measure of transformation of efficiency and is considered as the inventory turnover ratio. The higher turnover causes much more productivity with which a manufacturer uses its inventory. Considering all the above factors , our attention is to determine the quality factor and maximum stock level so that the integrated total profit is maximized.

Volume flexibility is a major factor of flexible manufacturing systems (FMS) which helps to meet the market-demand at the optimal level. Many researchers have published their article in this direction. Sethi and Sethi (1990) presented five flexibilities at the system level and these are based on producers, routing, product and volume flexibility. Yan and Chang (1998) analyzed a general production inventory model in which there were production rate, deterioration rate and demand rate as a function of time. Skouri and Papachristos (2003) extended the model of Yan and Chang (1998) considering the backlogging rate which is a time dependent function and they introduced an algorithm for the solution of this problem. Rosenblatt and Lee (1986) and Porteus (1986) proposed an EMQ (economic manufacturing quantity) model. In these models, they assumed that the manufacturing process is in control state at the beginning of the production run and it may be shifted to an out-of-control state after a certain time. These defective products are repaired/reworked at a cost. Khouja and Mehrez (1994) extended a classical inventory model to the production model in which the production rate is a variable under managerial control and production cost is a function of production rate. Sana et al. (2007 a,b) discussed the EMQ model in an imperfect production system in which the defective items are sold at a reduced price. In this model, the demand function of defective items is a non-linear function of reduction rate. Some of the works in this direction are due to Khouja (1995), Mondal and Maiti (1997), Bhandari and Sharma (1971) and Bhunia and Maiti (1997). Recently Sana (2010) discussed a production-inventory model in an imperfect production process over a finite time horizon was considered. The production rate varies with time. The unit production cost is a function of production rate and product reliability parameter.

In present competitive market situation, the selling of product and the marketing policies depend on display of stock and advertisement. Advertisement through T.V, Newspaper, Radio, etc., and also through salesman have motivational effect on the customer to buy more. It is seen in the market that lesser selling price causes increase in demand whereas higher selling price decreases the demand. So, we can conclude that the demand of an item is a function of selling price and advertisement cost. Kolter(1971) analyzed marketing policies into inventory decisions and discussed the relationship between economic order quantity and decision. Other notable papers are due to Ladany and Sternleib (1974), Urban(1992), Goyal and Gunasekaran (1997), Leo(1998), etc., inventory models were discussed incorporating the effects of price variations and advertisement cost in demand.

Most of the articles do not take into account the effect of inflation and time-value of money. In the present economic situation, many countries have worsened due to large scale inflation and consequent sharp decline in the purchasing power of money. Therefore, effects of inflation and time-value of money can no longer be ignored in the present economy. The first article in this direction was presented by Buzacott (1975), taking inflation into account. Misra (1975) also analyzed an article incorporating inflationary effects. Many other researchers extended their ideas to other inventory situations by considering time-value of money, different inflation rates for the internal and external costs, finite replenishment rate, shortage, etc.. Other articles in this direction come from Bierman and Thomas (1977), Datta and Pal(1991), Bose et al.(1995), Ray and Chaudhuri (1997), Roy and Chaudhuri (2006), Roy and Chaudhuri (2007), Sana (2010), Roy and Chaudhuri (2011). Taking the above features into consideration, we develop a production-inventory model with variable set-up cost incorporating inflation and time value of money in an imperfect production process. Demand rate is a linear decreasing function of selling price and the advertisement cost. The unit production cost depends upon labour, raw material charges, advertisement cost and product reliability parameter. In addition, effect of inflation and time-value of money without lead time in finite time horizon are also considered. The total profit function is maximized analytically in this model. The model has been illustrated with a numerical example along with graphical analysis and sensitivity analysis of parameters of the total profit are presented.

II. Notations and Assumptions

This paper is developed with the following *Notations* and *Assumptions*.

Notations :

s : the selling price per unit

A : advertisement cost per unit item

c_h : the holding cost per unit per unit time

c_s : the shortage cost per unit per unit time

c_0 : the set-up cost per production run

c_r : the raw material cost per unit which is fixed

t_1 : the time upto which the production is made and after $t = t_1$ the production is stopped

T : one cycle time

L : fixed cost like energy and labour

D : demand rate

P : production rate which is fixed , $P \geq D$ always.

ψ : product reliability factor

$f(\psi)$: developement cost of the production system

$c(\psi)$: unit production cost

ψ_{max} : maximum value of ψ

ψ_{min} : minimum value of ψ

Q : maximum stock level

$q(t)$: stock level at time t

R : disposal or rework cost per unit defective item

a : cost of resource, technology and design complexity for the product when $\psi = \psi_{max}$

b : represents the difficulties in increasing reliability which depends upon the technological design complexity and resource limitation etc

γP : represents die or tool cost which is proportional to the production rate

γ : $r - i$,where r is the interest rate per unit currency and i is the inflation rate per unit currency

g, h : constant values

Assumptions :

- The demand rate D depends on the sum of the advertisement cost and is a decreasing function of the selling price, i.e $D(A,s) = A + g - hs$, where g > 0 and h > 0
- The unit production cost $c(\psi) = L + c_r + \dfrac{f(\psi)}{P} + \gamma P$
- During the production period , the defective items are produced.The smaller value of ψ provides better quality product.The development cost of the production system is given by $f(\psi) = ae^{b\frac{(\psi_{max}-\psi)}{\psi-\psi_{min}}}$, $\psi \in [\psi_{min}, \psi_{max}]$
- Selling price s is determined by a mark-up over a unit production cost $c(\psi)$ i.e. $s = \mu c(\psi)$, μ is the mark-up.
- $JIT set\ up\ cost = \dfrac{c_0 D}{Q}$ has been considered which depends upon the demand rate.
- Shortages are not allowed.
- Lead time is assumed to be zero.

III. Development of the model

During the interval $[0,t_1]$, the stock-level q(t) gradually increases due to production and demand until the production is stopped and stock-level q(t) is at maximum level Q at $t = t_1$. In the interval $[t_1, T]$ only demand occurs.So stock level q(t) gradually decreases due to demand in the interval $[t_1, T]$ until it is zero at $t = T$. The cycle repeats itself again. The pictorial representation of the model is given in the Fig.1. Insert Fig-1 here
The stock level $q(t)$ at any time t can therefore be represented by the following differential equations :

$$\frac{dq(t)}{dt} = P - D, \quad 0 \le t \le t_1 \qquad (1)$$

and

$$\frac{dq(t)}{dt} = -D, \quad t_1 \le t \le T \qquad (2)$$

with initial and boundary conditions are $q(t) = 0$, when $t = 0$ and $t = T$ and $q(t) = Q$, when $t = t_1$
The solution of the differential equations (1) and (2) are

$$q(t) = (P - D)t, \qquad 0 \le t \le t_1 \qquad (3)$$

$$= (T - t)D, \qquad t_1 \le t \le T \qquad (4)$$

From (3), we have

$$t_1 = \frac{Q}{P - D}. \qquad (5)$$

From (4), we have

$$T = \frac{QP}{D(P - D)}. \qquad (6)$$

The present value of total revenue is

$$C_{REV} = \int_0^T sDe^{-\gamma t} dt$$

$$= \frac{sD}{\gamma}(1 - e^{-\gamma T}). \qquad (7)$$

The present value of production cost is

$$C_{PRO} = \int_0^T c(\psi) De^{-\gamma t} dt$$

$$= \frac{c(\psi)D}{\gamma}(1-e^{-\gamma T}). \qquad (8)$$

The present value of holding cost is

$$C_{HOL} = \int_0^T c_h Q e^{-\gamma t} dt$$

$$= \frac{c_h Q}{\gamma}(1-e^{-\gamma T}). \qquad (9)$$

The present value of set up cost is

$$C_{SET} = c_0 \int_0^T \frac{D}{Q} e^{-\gamma t} dt$$

$$= \frac{c_0 D}{\gamma Q}(1-e^{-\gamma T}). \qquad (10)$$

The present value of reworked cost is

$$C_{REW} = \int_0^T R\psi P e^{-\gamma t} dt$$

$$= \frac{R\psi P}{\gamma}(1-e^{-\gamma T}). \qquad (11)$$

The total profit incorporating inflation and time value of money is given by

$$\pi(Q,\psi) = [C_{REV} - C_{PRO} - C_{HOL} - C_{SET} - C_{REW}]$$

$$= \frac{1}{\gamma}\{sD - c_h Q - Dc(\psi) - R\psi P - \frac{c_0 D}{Q}\} \times (1-e^{-\gamma T}). \qquad (12)$$

Now, the problem is to determine the optimal values for Q and ψ such that π in (12) is maximized. However, it is a two dimensional decision-making problem for a retailer.

IV. Optimal solution and theoretical results

The necessary conditions for $\pi(Q,\psi)$ to be maximum are

$$\frac{\delta\pi(Q,\psi)}{\delta Q} = \frac{1}{\gamma}(-c_h + \frac{c_o D}{Q^2})(1-e^{-\gamma T})$$

$$+\frac{1}{\gamma}\{sD - c_h Q - Dc(\psi) - R\psi P - \frac{c_0 D}{Q}\}\gamma e^{-\gamma T} = 0. \qquad (13)$$

and

$$\frac{\delta\pi(Q,\psi)}{\delta\psi} = \frac{1}{\gamma}\{(\mu-1)\frac{\delta c(\psi)}{\delta\psi}\}D(1-e^{-\gamma T}) + \frac{1}{\gamma}\{(\mu-1)c(\psi)$$

$$-\frac{c_0}{Q}\}\frac{\delta D}{\delta\psi}(1-e^{-\gamma T}) + \frac{1}{\gamma}\{(\mu-1)c(\psi) - \frac{c_0}{Q}\}D\gamma$$

$$e^{-\gamma T}\frac{\delta T}{\delta\psi} - (c_h Q + R\psi P)e^{-\gamma T}\frac{\delta T}{\delta\psi} - \frac{RP}{\gamma}(1-e^{-\gamma T}) = 0. \qquad (14)$$

After rearranging the terms in (13) and (14), we get

$$-\gamma(sD - c_h Q - Dc(\psi) - R\psi P - \frac{c_o D}{Q})e^{-\gamma T}\frac{\delta T}{\delta Q} = (\frac{c_0 D}{Q^2} - c_h)(1-e^{-\gamma T}). \qquad (15)$$

and

$$\frac{1}{\gamma}\{(\mu-1)\frac{\delta c(\psi)}{\delta\psi}\}D(1-e^{-\gamma T}) + \frac{1}{\gamma}\{(\mu-1)c(\psi) - \frac{c_0}{Q}\}$$

$$\frac{\delta D}{\delta\psi}(1-e^{-\gamma T})+\frac{\gamma e^{-\gamma T}}{1-e^{-\gamma T}}\frac{\delta T}{\delta\psi}\pi(Q,\psi)-\frac{RP}{\gamma}(1-e^{-\gamma T})=0. \qquad (16)$$

Applying (16), we have $J_1(\psi)$ =0 where

$$J_1(\psi)=(\psi-\psi_{min})^2\frac{1-e^{-\gamma T}}{\gamma}\frac{\delta c(\psi)}{\delta\psi}\{(\mu-1)D-h\mu(\mu-1)c(\psi)+h\mu\frac{c_0}{Q}\}$$
$$-(\psi-\psi_{min})^2 RP\frac{1-e^{-\gamma T}}{\gamma}+(\psi-\psi_{min})^2\frac{\gamma e^{-\gamma T}}{1-e^{-\gamma T}}\frac{\delta T}{\delta\psi}\pi(Q,\psi). \qquad (17)$$

Applying (15) and (17), we have the following propositions:

Proposition 1. If $c_hQ^2-c_0D>o$, then $\frac{\delta^2\pi}{\delta Q^2}$ is negative .

Proof. We first obtain second-order derivative of $\pi(Q,\psi)$ from (13) and using (15), we have

$$\frac{\delta^2\pi}{\delta Q^2}=-\frac{2c_0D}{\gamma Q^3}(1-e^{-\gamma T})+\frac{1}{\gamma}(\frac{c_0D}{Q^2}-c_h)\gamma e^{-\gamma T}\frac{\delta T}{\delta Q}+(\frac{c_0D}{Q^2}-c_h)e^{-\gamma T}\frac{\delta T}{\delta Q}$$
$$-\gamma(sD-c_hQ-Dc(\psi)-R\psi P-\frac{c_0D}{Q})e^{-\gamma T}(\frac{\delta T}{\delta Q})^2+(sD-c_hQ$$
$$-Dc(\psi)-R\psi P-\frac{c_0D}{Q})e^{-\gamma T}\frac{\delta^2T}{\delta Q^2}$$
$$=\frac{-2c_0D}{\gamma Q^3}(1-e^{-\gamma T})+2(\frac{c_0D}{Q^2}-c_h)e^{-\gamma T}\frac{\delta T}{\delta Q}+(\frac{c_0D}{Q^2}-c_h)(1-e^{-\gamma T})\frac{\delta T}{\delta Q}$$
$$=\frac{-2c_oD}{\gamma Q^3}(1-e^{-\gamma T})-\frac{2}{Q^2}(c_hQ^2-c_0D)e^{-\gamma T}\frac{\delta T}{\delta Q}-$$
$$\frac{1}{Q^2}(c_hQ^2-c_0D)(1-e^{-\gamma T})\frac{\delta T}{\delta Q}<0$$

Proposition 2. As $\psi\to\psi_{min}$, $J_1(\psi)\to\infty$.

Proof. From (17), we have ,

$$J_1(\psi)=\frac{1-e^{-\gamma T}}{\gamma}\frac{ab}{P}e^{\frac{b(\psi_{max}-\psi)}{\psi-\psi_{min}}}(\psi_{min}-\psi_{max})$$
$$\{(\mu-1)D-h\mu(\mu-1)c(\psi)+\frac{h\mu c_0}{Q}\}+\frac{\gamma e^{-\gamma T}}{1-e^{-\gamma T}}$$
$$\pi(Q,\psi)\frac{QP(P-2D)}{(PD-D^2)^2}\frac{h\mu ab}{P}e^{\frac{b(\psi_{max}-\psi)}{\psi-\psi_{min}}}(\psi_{min}-\psi_{max})$$
$$-(\psi-\psi_{min})^2\frac{RP}{\gamma}(1-e^{-\gamma T}) \qquad (18)$$

From (18), we have $J_1(\psi)\to\infty$ as $\psi\to\psi_{min}$. So we may formulate a lemma as follows.

Lemma. If $J_1(\psi_{max})<0$, then $J_1(\psi)=0$ must have at least one solution in $[\psi_{min},\psi_{max}]$, otherwise $J_1(\psi)=0$ may have or may not have a solution in $[\psi_{min},\psi_{max}]$. Also the solution gives a maximum value of

π , since $\dfrac{\delta^2\pi}{\delta\psi^2}$ is negative at that solution.

V. Numerical example

To illustrate the proposed model , we consider the following parameter values of some product in appropriate units: c_h =\$2, c_r =\$15, ψ_{max} =2.2, ψ_{min} =.01, γ =\$.19, P =160 units, L =\$2000, c_0 =\$50, a =\$20, b =.25, R =\$15, A =\$2000, g =55, h =.25, μ =5. We obtain the optimal solution of the model as π^* =402585, Q^* =314.639, ψ^* =2.00375.

Insert Fig-2 here

VI. Sensitivity analysis

Using the Numerical Example, the sensitivity of each of the decision variables Q^* , ψ^* , and the maximum total profit $\pi^*(Q^*,\psi^*)$ to changes in each of the 8 parameters P , a , b , c_0 , γ , g , ψ_{max} , and ψ_{min} is examined in Table 1. The sensitivity analysis is performed by changing each of the parameters by -25%, -10%, +10% and +25%, taking one parameter at a time and keeping the remaining parameters unchanged.
From Table 1, we can analyze the following cases.

- When P increases by 10 % , then Q^* decreases and the total average profit π decreases by 31.88 %. When P decreases by 10 % and 25 % respectively , then Q^* increases but ψ^* decreases and the total average profit increases by 31.48 % and 79.26 % respectively. The above discussion illustrates that the production rate parameter is highly sensitive.
- The parameters a , b , c_0 , ψ_{min} are insensitive.
- When γ is decreased by 10 % and 25 % respectively , then Q^* increases but ψ^* decreases and the total average profit π is increased by 46.45 % and 138.95 % respectively. So γ is highly sensitive.
- When g increases , then Q^* also increases but ψ^* decreases. The total average profit is increased by 92.52 % and 58.14 % due to change of g by 25 % and 10 % respectively. So it is clear that g is highly sensitive.
- When ψ_{max} is increased by 10 % , then Q^* increases and the total average profit π increases by 1.74 %.

Insert Table 1 here

VII. Conclusions

The following features are observed in the present model.

1. Since demand for the product in an industry is dependent on several features like price, time and advertisement cost , so in this model, demand has been considered as a sum of two functions, advertisement cost and linear decreasing function of selling price.
2. In this model, product quality factor is an important variable which determines the unit production cost and development cost of the production system. Also selling price is dependent on unit production cost.
3. Since effects of inflation and time-value of money can no longer be ignored in the present economy, so we consider here effects of inflation and time value of money.
4. Deterioration of inventory which is a common feature in the inventory of consumer goods, has been considered in this model.

We solve this problem analytically and numerically.The sensitivity of the solution to changes in different parameters has been discussed.

The model can be extended in several ways:

a. We might extend the proposed profit function stochastically.
b. We could extend the model by considering the production rate which is either variable or linear increasing function of demand.
c. The model can also be extended by taking into consideration shortages and lead time.

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Table 1: Sensitivity analysis

Effects of P , a , b and c_0 on profit (Example 1)

para-	% change	Q^*	ψ^*	%
meter	in the			change
	parameter			in π
	+25	-	-	-
P	+10	218.59	-	-31.88
	-10	404.57	1.8577	31.78
	-25	522.564	1.75612	79.26
	+25	313.743	-	-.332
a	+10	314.28	2.09023	-.134
	-10	314.994	1.91292	.136
	-25	315.551	1.76669	.343
	+25	-	-	-
b	+10	314.671	2.09024	-.0025
	-10	314.606	1.91086	.0042
	-25	314.571	1.75728	.014
	+25	314.667	2.00376	-.0005
c_0	+10	314.65	2.00376	-.0002
	-10	314.627	2.00375	.0002
	-25	314.611	2.00376	.0004

Table 1: Continued

Effects of γ , g , ψ_{max} and ψ_{min} on profit (Example 1)

para-	% change	Q^*	ψ^*	%
meter	in the			change
	parameter			in π
	+25	-	-	-
γ	+10	-	-	-
	-10	446.426	1.77377	46.45
	-25	691.7	1.55078	138.95
	+25	574.883	1.51213	92.52
g	+10	481.597	1.64376	58.14
	-10	-	-	-
	-25	-	-	-
	+25	-	-	-
ψ_{max}	+10	318.832	-	1.74
	-10	314.697	1.88921	.038
	-25	314.758	1.70794	.099
	+25	318.832	-	1.74
ψ_{min}	+10	314.639	2.00423	0
	-10	314.639	2.00327	0
	-25	314.638	2.00253	0

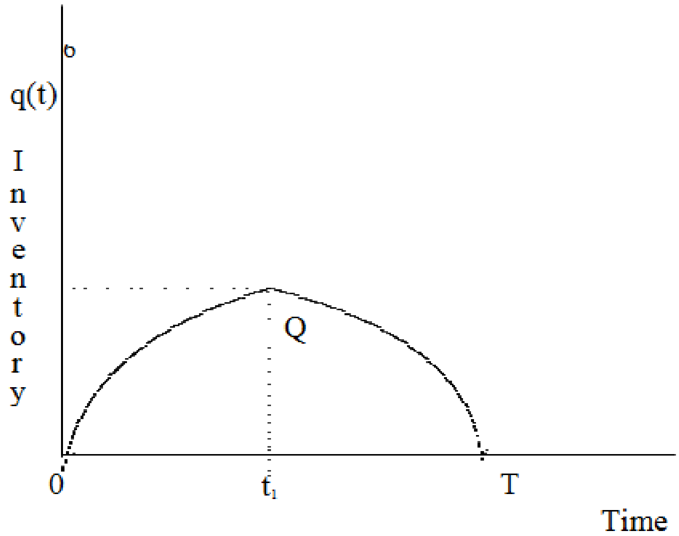

Fig 1: Graphical representation of Model

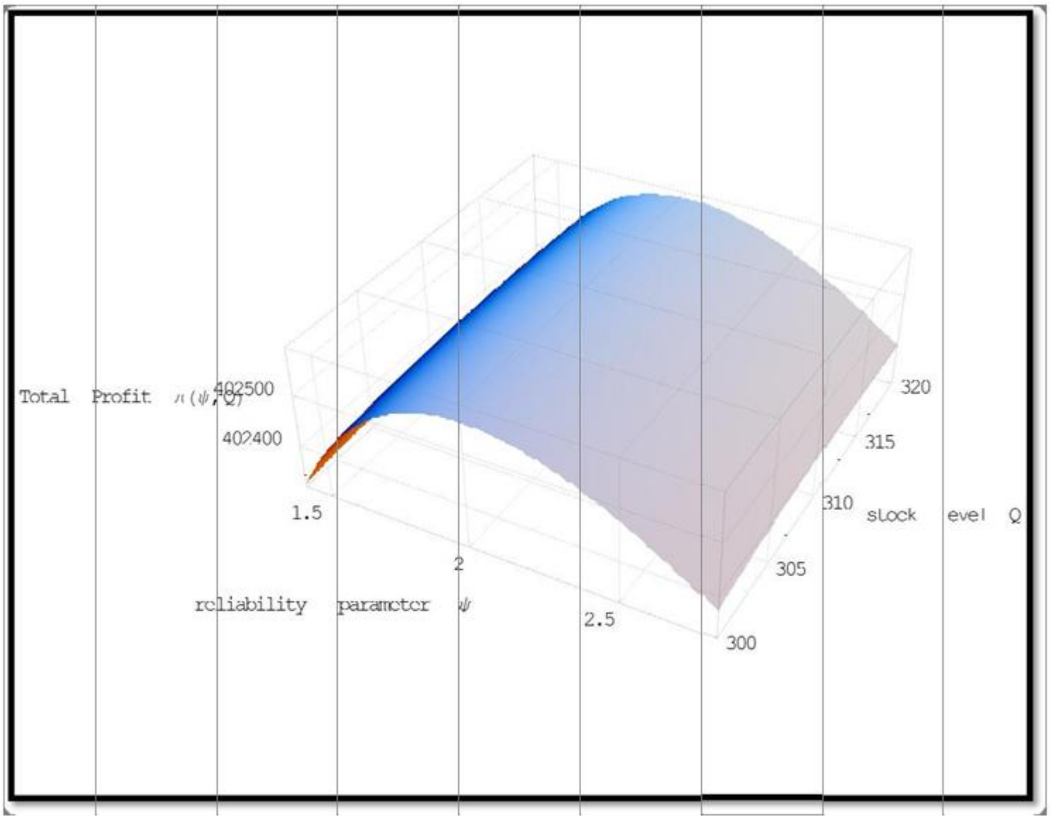

Fig. 2: Maximum total profit π (Q, ψ) versus *Q* and ψ of Example