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A New Bi-level Program Based on Unblocked Reliability for a Continuous Road Network Design

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ABSTRACT: *With increasing demands for better and more reliable service, developing a method for designing a highly-reliability road network has become a critical issue. A Network Design Problem is used to determine a set of design parameters leading to the creation of an optimal road network. Moreover, the performance of an Origin-Destination pair with high traffic demands is a top priority in the optimizing process. A bi-level programming technique, that is upper-level and lower-level programs, can be used to formulate the Continuous Network Design Problem. This study establishes a new type of bi-level program based on unblocked reliability. The proposed bi-level program is applied to analyze a real local road network that has 22 nodes and 62 links. A set of link capacity expansions is determined by solving the proposed bi-level program using the Hooke-Jeeves algorithm. This kind of capacity expansion results in optimization by maximizing the balance between the unblocked reliability of the entire road network and the road network expansion ratio. The new, proposed bi-level program can comply with the various limits on environment and financial resources.*
Keywords: *Bi-level Program, Continuous Network Design Problem, Link capacity expansion, Unblocked Reliability, User Equilibrium*

I. INTRODUCTION

The Network Design Problem (NDP) is used to determine a set of design parameters that leads the road network to an optimal state. Typically, the design parameters are link capacity expansions. There are two forms of the problem, the Continuous Network Design Problem (CNDP) and the Discrete Network Design Problem (DNDP). The CNDP takes the network topology as given and is concerned with the parameterization of the network. The DNDP is concerned with the topology of the network, such as the construction of a new road, bridge, tunnel or bypass.

The CNDP can be formulated as a bi-level program[1]. Planners and managers in the upper level determine the parameters of the road network, and travelers in the lower level respond to the change in the road network. The upper-level program expresses the expectation of the planners and managers, that is the minimization of the total cost function. In general, the total cost is the sum of the total travel cost in the network and investment cost of link capacity expansions for fixed traffic demands. A User Equilibrium (UE) can be formulated as an equivalent minimization problem, which is considered as a lower-level program in the CNDP[2]. The UE describes a stable state when all drivers attempt to minimize the respective travel time. In other words, the drivers' route choice behavior is voluntary rather than controlled by managers of the road network system.

With increasing demands for better and more reliable service, the road network system has incorporated a reliability analysis as an integral part of its planning, design and operation. Then, it is necessary to introduce a reliability index of road network system performance to the CNDP. There are several reliability concepts in the road network system, such as connectivity reliability[3], travel time reliability[4], capacity reliability[5], and unblocked reliability. Unblocked reliability is the probability during one day of the road unit or system's ability to maintain an unblocked state at peak hours in which the highest traffic volumes are observed. The length of the road was regarded as a factor affecting route choice behavior when unblocked reliability was applied to analyze the investment optimization of the road network.

This paper proposes a new model of the bi-level program based on unblocked reliability for solving the CNDP in order to supply a set of optimal link capacity expansions, which improve the road network to a higher reliability level. The upper-level program that consists of the unblocked reliability index and cost, which is expressed as the road network expansion ratio, is described. The UE traffic assignment is regarded as the lower-level program. The Hooke-Jeeves (HJ) algorithm[6] is applied to solve the proposed bi-level program. Then, the proposed bi-level program is tested by a local road network, which has 22 nodes and 62 links.

II. Basic Idea Of Bi-Level Programming Based On Reliability Index

The CNDP can be formulated as a bi-level program. Planners and managers can influence the drivers' route choice behavior by changing a road network, but they cannot control their route choice behavior. When the Origin-Destination (OD) demand is inelastic, the objective function of the upper-level program is the system cost and the investment cost. Generally, system cost is expressed by travel times of the total network, and minimum travel time is expected. In this paper, the optimization objective is obtained by subtracting the investment cost from the unblocked reliability when the performance of a road network is indexed by unblocked reliability. The maximum value of unblocked reliability is expected.

If the link flows, x, is fixed, optimization objective is maximized, where $x=(x_1,\ \cdots,\ x_a,\ \cdots,\ x_G)$ is a vector and G is the total number of links in a road network. When the solution of link capacity expansions, y, is given, where $y=(y_1,\ \cdots,\ y_a,\ \cdots,\ y_G)$ is a vector, an assignment algorithm can be performed to obtain new link flows, x. This forms the basis of the iterative design-assignment bi-level programming algorithm [7].

Upper-level program: Solve the network design problem for y given x. Proceed to the lower-level program.
Lower-level program: Given y find new x. Return to the upper-level program.

Only one variable's optimization failed to optimize the entire system performance, which reflects the essence of the bi-level programming.

III. Upper-Level Program Based On Unblocked Reliability

Objective function F of the upper-level program is expressed as follows:

$$\max F(y) = R - d\lambda\phi \tag{1}$$

where,

$$\phi = \sum_{a\in A}(L_a y_a) \tag{2}$$

From the viewpoint of a planner, the first term on the right-hand side of Eq. (1), R, is the entire network unblocked reliability that represents the performance of the road network system, and R is a function of link flows. The link flows are obtained by solving the lower-level program when a set of link capacity expansions is fixed. The second term on the right-hand side of Eq. (1) presents the cost associated with improving the road network. The second term consists of a product of three parts, such as a control factor of the road network expansion scale, d, unit conversion factor, λ, sum of the product of each link's length, L_a, and link capacity expansion, y_a, and, ϕ, which indicates the road network expansion scale. The ratio of the road network expansion scale to the existing network scale, $\lambda\phi$, equals the road network expansion ratio.

3.1 Model of unblocked reliability

Nodes and links are the elements composing a road network. A node is a junction of two or more links and a link connects two nodes. A link may be regarded as a conduit for the flow between two nodes; all links referred to in this paper are directed. A road network is aggregated as several associated levels depending on classification and viewpoint. A governor considers the performance of the whole network, whereas a driver focuses his attention on the performance of an OD pair. A path is a sequence of nodes connected by links in one direction so that a movement is feasible from the origin to a given destination. For a fundamental analysis of traffic network, the capacity of a node is regarded as infinite. When either queue behavior or turning movement is of interest, the extra nodes are introduced to represent each explicit direction. The problem of node capacity will be a transformed link capacity problem between extra nodes. Along these lines, a four-level model of the road network unblocked reliability (link, path, OD pair and the entire network) is introduced[8, 9]. Here, the unblocked reliabilities of the link, the path m connecting origin i and destination j, the OD pair connecting origin i and destination j, and the entire network are presented by R_a, R_{ij}^m, R_{ij}, and R, respectively.

3.2 Cost Associated with Improving Road Network

The second term on the right-hand side of Eq. (1), $d\lambda\phi$, is called the penalty term. The conversion factor for physical dimension, λ, is formulated as follows:

$$\lambda = \frac{1}{\sum_{a \in A} \left(L_a C_a \right)} \tag{3}$$

The physical meaning of a denominator of Eq. (3), which is the sum of the product of each link's length and capacity, is an index of an existing road network scale. For a given road network, λ is a constant. Then, the penalty term in the objective function of upper-level program, $d\lambda\phi$, becomes a dimensionless factor. This penalty term ensures the network can be expanded to a reasonable degree. Also, the road network expansion will reach the optimal degree when the bi-level program has been solved.

IV. Lower-Level Program

The lower-level model of the bi-level program for the CNDP based on unblocked reliability is a fixed demand UE problem in this paper. UE is the state in which there is no motivation to change the system. UE means that travel times are identical along any used routes connecting an OD pair and less than or equal to the travel time on all unused routes, if every traveler attempts to choose the path with the shortest travel time[10]. The UE assignment problem is to find the link flows that satisfy the UE criterion when all OD entries have been appropriately assigned.

This link flow pattern can be obtained by solving the following mathematical problem[8, 11]. The objective function of the lower-level program is given by $Z(x)$, where $x = \left(x_1, \ \cdots, \ x_a \cdots, \ x_G \right)$ is a vector variable of link flows.

$$\min Z(x) = \sum_{a \in A} \int_0^{x_a} t_a(\omega, y_a) d\omega \tag{4}$$

The objective function is the sum of the integrals of the link travel time functions for a given link capacity expansion, y. The link travel time, $t_a(x_a)$, which represents the relationship between the flow and the travel time for link a, is calculated using the standard Bureau of Public Roads function[12]. Of course, the total capacity of each link is composed of the link capacity expansion and the existing link capacity when travel time, t_a, is calculated by Eq. (5).

$$t_a(x_a) = t_a^e \left[1 + 0.15 (x_a / C_a)^4 \right] \tag{5}$$

The Frank-Wolfe (FW) algorithm is used to solve the mathematical problem[13]. This algorithm is an iteration process to find the feasible flows that can reduce the value of the objective function[8].

V. Using Hooke-Jeeves Algorithm To Solve The Continuous Network Design Problem Based On Un-Blocked Reliability Model

The Hooke-Jeeves (HJ) algorithm is a fundamental algorithm for solving the Continuous Network Design Problem (CNDP)[6]. It was used first to solve the CNDP by Abdulaal and LeBlanc as a direct search method in the road network[14].

Consider the problem of maximizing the objective function of the upper-level program without constraints. The HJ algorithm proceeds by a sequence of exploratory and pattern moves. If an exploratory move leads to an increase in the value of the objective function of the upper-level program, it is called a success; otherwise it is called a failure. The purpose of solving the proposed bi-level program is to find a set of link capacity expansions to maximize the objective function of the upper-level program. The flow chart of the Hooke-Jeeves algorithm employed in this paper is shown in Appendix A.

Reviewing the exploratory move procedure, the capacity change of a link will be accepted only when the increase in the penalty term is smaller than that of the entire road network unblocked reliability. After the total links have been tested, if the current objective function value of the upper-level program is greater than the corresponding objective function value of the upper-level program of success link capacity expansions, the new success link capacity expansions are obtained. The purpose of the pattern move is to speed up the process of road network expansion through multiplying a coefficient with the capacity expansion of every improving link; this coefficient is greater than unity.

The bi-level program that is used to describe the CNDP is a non-convexity program. Even the solution of the bi-level program can be found, although it is usually a local optimum, not a global optimum. Hence, the HJ algorithm, as one of the traditional heuristic algorithms for solving a bi-level program, also has a similar local optimum for a solution. There are four parameters, such as the initial step length of exploratory move, s, the reduction factor of the step length of exploratory move, θ, the convergence step length of exploratory move, η, and the step length of pattern move, v, in the HJ algorithm. They will mainly influence the iteration times, but hardly influence the objective function of the upper-level program when the four parameters fall within a reasonable range. Even though it is possible to obtain a large objective function value of an upper-level program for a given road network using certain parameters in the HJ algorithm, these parameters do not have any universality. The four parameters provided for the case study in this paper are empirically defined as 1,000, 0.5, 100, and 2, respectively.

VI. Case Study Of A Real Local Road Network

6.1. Features of existing road network

A trunk road network in a local area is being employed as a case study. This road network was constructed based on an actual road network belonging to the author's province. Congestion often occurs at peak hours in the morning on this road network. First, the actual road network was expressed by a graph consisting of nodes and links; a code expresses a node, and a link is denoted by codes of two adjacent nodes. The topology and distance of the existing road network is shown in Fig. 1. There are a national road and a parallel motorway in the north-south direction, as well as some national roads and some access roads. The access roads provide access to and from the motorway. There are 22 nodes and 62 links in the local road network. A traffic zone is the unit of geography most commonly used in road network planning models. It is represented with a single point from the graph theory viewpoint and regarded as the area of the origins and destinations of trips. The 1st to 11th nodes express traffic zones that are the cities in this area. The 12th to the 21st nodes express the interchange of the motorway. The 22nd node is an important intersection between two national roads. The two side-by-side links have the same parameters including length, free-flow travel time and capacity between each node pair in the existing road network. The link capacity on the motorway is 4,600 pcu/h. For a national road, the capacity of each link between node pairs such as (5, 16), (8, 16), (6, 8), (6, 18), (7, 20) and (9, 20), is 1,600 pcu/h, and between other node pairs is 3800 pcu/h. For the access road, the capacity of each link between node pairs such as (1, 12), (2, 13) and (3, 14) is 3,800 pcu/h, while that between other node pairs is 1,600 pcu/h.

An OD matrix shows the sources and objectives of traffic flows in a road network. The Ministry of Land, Infrastructure, Transport and Tourism of Japan (MLIT) conducts a nationwide traffic census. The OD matrix of a local area trunk road network is shown in Table 1, which was calculated according to the traffic census data provided by MLIT. The OD matrix is regarded as a fixed demand for traffic assignment in the local area of the trunk road network.

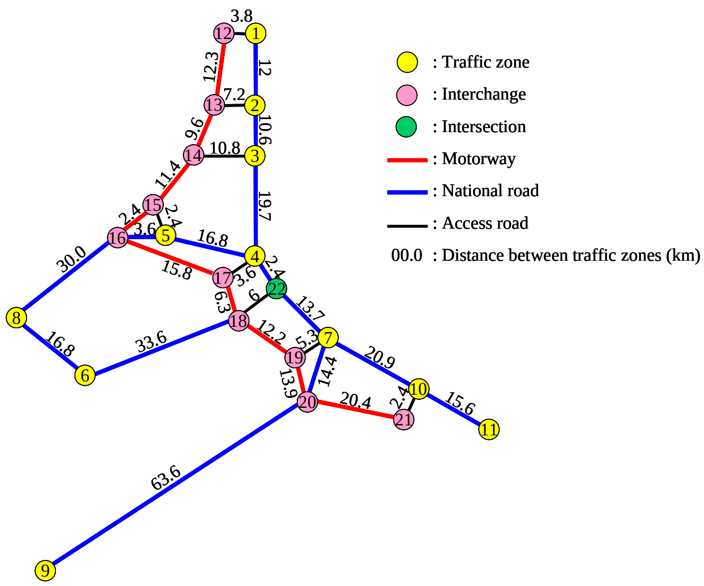


Fig. 1 Road network of a local area

Table 1 Origin-Destination matrix of traffic zones

Unit: pcu/h

Origin	Destination											
	1	2	3	4	5	6	7	8	9	10	11	Sum.
1	-	195	239	1,184	1,842	0	357	0	245	141	136	4,339
2	1,600	-	0	5	0	0	1	0	0	0	0	1,606
3	1,608	0	-	0	0	357	0	0	0	0	0	1,965
4	165	0	0	-	2,540	71	0	59	0	0	0	2,835
5	1,254	0	52	1,405	-	0	425	0	0	168	162	3,466
6	0	0	29	290	0	-	0	1,035	0	0	0	1,354
7	15	0	0	0	251	0	-	278	0	0	0	544
8	0	0	0	586	0	804	243	-	0	97	91	1,821
9	613	0	0	0	0	0	0	0	-	167	158	938
10	0	1,144	1,286	0	0	0	0	0	0	-	0	2,430
11	7	0	0	0	137	0	0	152	227	0	-	523
Sum.	5,262	1,339	1,606	3,470	4,770	1,232	1,026	1,524	472	573	547	21,821

6.2. Used path of existing road network

The used paths under the UE state can be obtained when the OD matrix is assigned to the road network by the lower-level program based on the FW algorithm. The travel time of any possible path can be calculated by the link travel times shown in Table 2. The link travel times of two directions may not be the same since the link flows in the two directions are not in balance. For example, here the travel time of path from 4 to 9 is estimated. The travel time of path 4-22-7-20-9 is 1.90 hour, which is greater than the travel time of the used path, 4-17-18-19-20-9, between the OD pair 4-9 as shown in Fig. 2. The travel times of other paths are also greater than the travel time of the used path, 4-17-18-19-20-9. This case study has verified that the lower-level program can determine the used paths in accord with the UE criterion. Of course, the link flows have also been calculated. Then, unblocked reliability in the upper-level program can be calculated by the proposed model.

Table 2 Link travel time in existing road network

Unit: h

Origin	Destination																					
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1		0.20										0.10										
2	0.22		0.18										0.18									
3		0.19		0.34										0.27								
4			0.34		0.29												0.09					0.04
5				0.28											0.10	0.09						
6								0.43										0.84				
7										0.35									0.13	0.36		0.24
8						0.42										0.61						
9																				1.29		
10							0.36				0.39										0.06	
11										0.39												
12	0.10												0.15									
13		0.18										0.15		0.12								
14			0.27										0.12		0.11							
15					0.11									0.11		0.06						
16					0.09			0.60							0.06		0.16					
17				0.09												0.16		0.06				
18						0.84											0.06		0.12			0.15
19							0.13											0.12		0.17		
20							0.36		1.27										0.17		0.29	
21										0.06										0.29		
22				0.04			0.23											0.15				

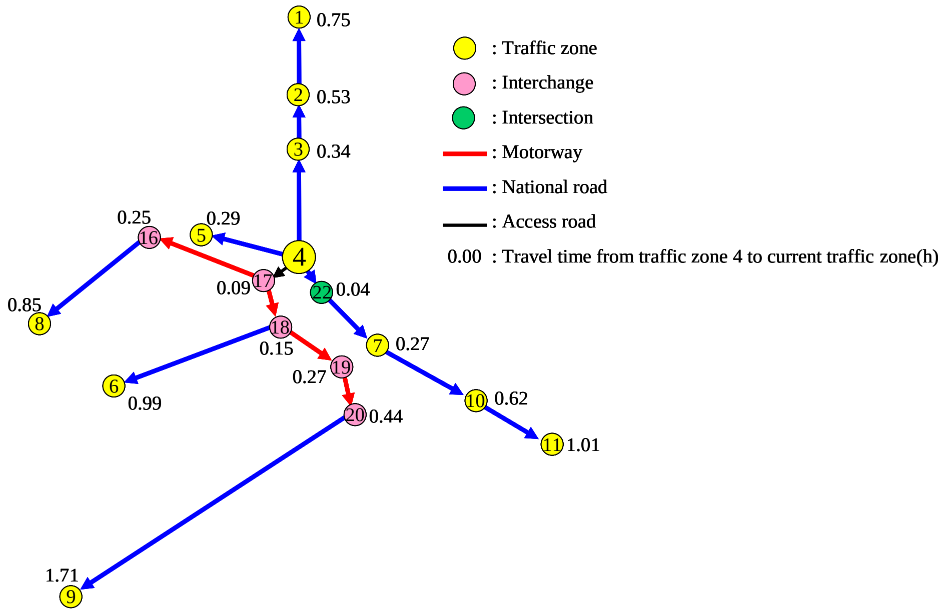


Fig. 2 Tree of used paths of traffic zone 4 originating in existing road network

6.3. Optimization result

Iteratively solving the lower-level and upper-level program in the HJ algorithm can optimize the road network in our case study. When the special constraints of the environment and financial resources are not considered, the controllable factor d = 1. The initial solution, $y^0 = 0$, is given initially. Then, the HJ algorithm is applied, and the result is a set of most worthy links and their capacity expansions as shown in Fig. 3. In Fig. 3, only links that will be improved are shown, and the other link capacities are not changed in the planning. The network expansion ratio, $\lambda\phi$, equals 7.9% in the existing road network scale by applying the HJ algorithm to solve the proposed bi-level program when the objective function of the upper-level program reaches maximum.

The links have been graded according to the unblocked reliability, as shown in Fig. 4. The brown, pink, blue and green arrows express the grade of reliability from low to high, corresponding to R_a< 25%, 25% ≤R_a < 50%, 50%≤R_a < 75% and 75%≤R_a, respectively. In the existing road network, the links of the four grades are 4, 15, 21, and 22, respectively. In the improved road network, the group of most unreliable links disappeared and all of the brown links were promoted to blue links; and six pink links were also promoted to blue links. No links were promoted to 75% or more because it would be too costly to improve links to an excessively high level of unblocked reliability. The proposed bi-level program can avoid this kind of unreasonable proposal. Therefore, in the improved road network, the links of four grades were 0, 9, 31, and 22. This case shows that the links of lower reliability were improved.

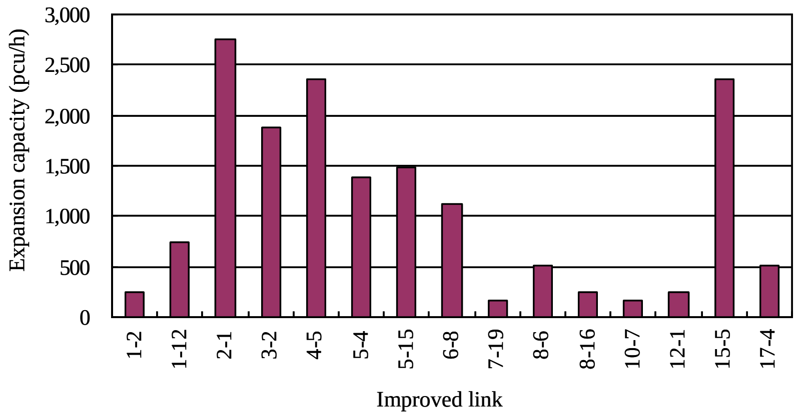
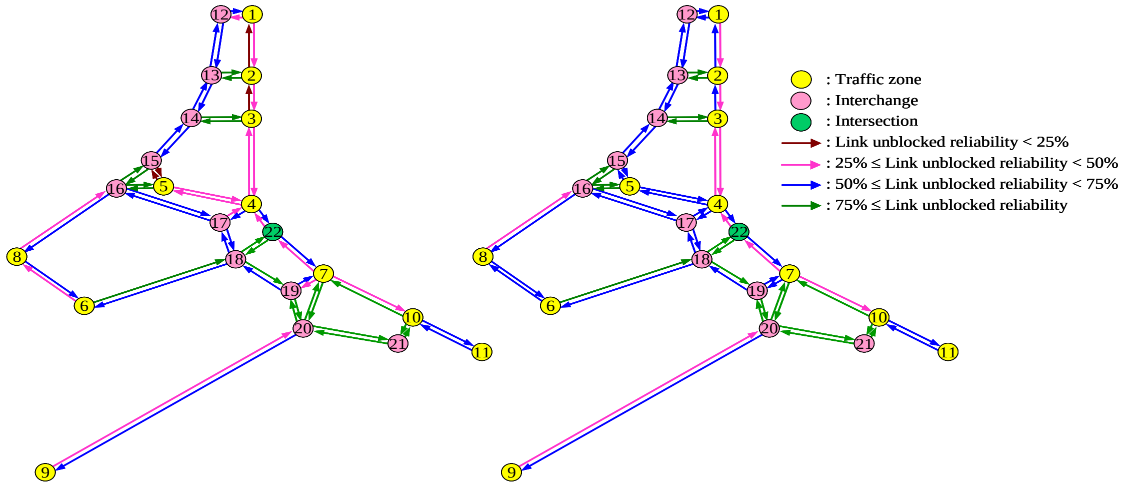


Fig. 3 Link capacity expansions

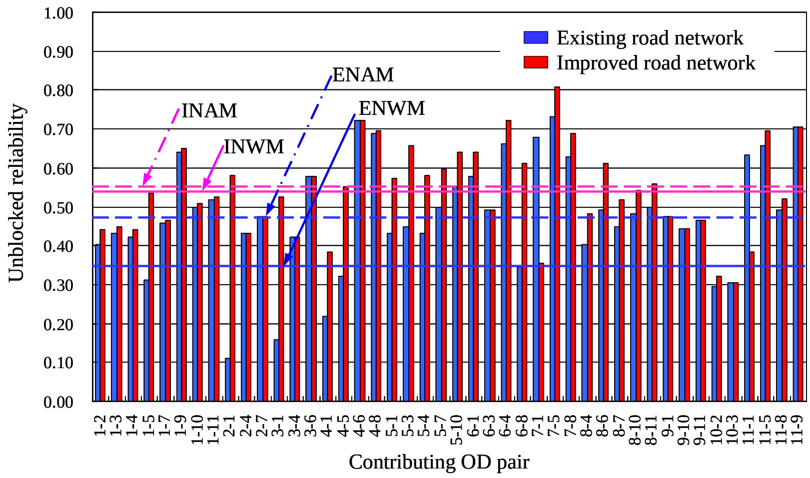


(a) Existing road network (b) Improved road network

Fig. 4 Grading links by unblocked reliability

The OD pair's unblocked reliabilities of the existing road network and the improved road network are shown in Fig. 5. In Fig. 5, only 44 contributing OD pairs are displayed, which have positive OD flows in Table 2. The unblocked reliability of the entire road network, *R*, is the weighted mean of the contributing OD pair's unblocked reliability, since the other OD pair's weight is zero. Here, the weight is the proportion of the OD flow to the total traffic demand. There is a distinct improvement in the unblocked reliability of the entire road network from 35.1% of the existing network to 53.4% of the improved network.

For studying the influence of improvement planning, the arithmetic mean of the unblocked reliability of the contributing OD pairs was also calculated. The arithmetic mean is the sum of the contributing OD pair's unblocked reliability divided by the number of items. The arithmetic mean reports the central tendency. In the existing road network, the number of OD pairs, in which each unblocked reliability is less than the arithmetic mean, is 21, and the traffic demands account for 78.2% of the total. This case indicates that the unblocked reliabilities of OD pairs with high traffic demands, which have high weight, are at a lower level. Hence, the arithmetic mean and the weighted mean, 47.6% and 35.1%, respectively, show an extraordinary disparity in the existing road network. For the improved road network, the unblocked reliability of 21 OD pairs accounting for 52.9% of all traffic demands was greater than the arithmetic mean. In the improved road network, the arithmetic mean and weighted mean became 54.6% and 53.4%, respectively, obviously close to each other. These results showed that the performance of an OD pair with high traffic demands has been considered in the proposed model of the bi-level program since the increased scale of the weighted mean was greater than the arithmetic mean.



INAM: Improved network arithmetic mean INWM: Improved network weighted mean
ENAM: Existing network arithmetic mean ENWM: Existing network weighted mean

Fig. 5 Unblocked reliability of OD pair in improved network contrasts with existing network

6.4. Significant influence on objective function of upper-level program from control factor of road network expansion scale

The trend in the objective function of the upper-level program with changes in the control factor of the road network expansion scale is shown in Fig. 6. The abscissa indicates the control factor of the road network expansion scale, *d*, and the ordinate expresses the three dimensionless values whose relationship is formulated by Eq. (1). When *d* increases, the results of the bi-level program in different simulation processes show the reducing objective function value of the upper-level program, lower unblocked reliability and network expansion ratio. The curve of the road network expansion ratio is steeper with a decrease in *d*, which signifies a reduction in the contribution of more network expansions to the road network performance improvement. In other words, the proposed bi-level program reflected the nature of road network improvement that satisfies a law of diminishing marginal utility. If *d* approaches zero, the expansion ratio will approach infinity, and the unblocked reliability will approach unity. When *d* is given a sufficiently large value, no link is expanded and the unblocked reliability is also unchanged.

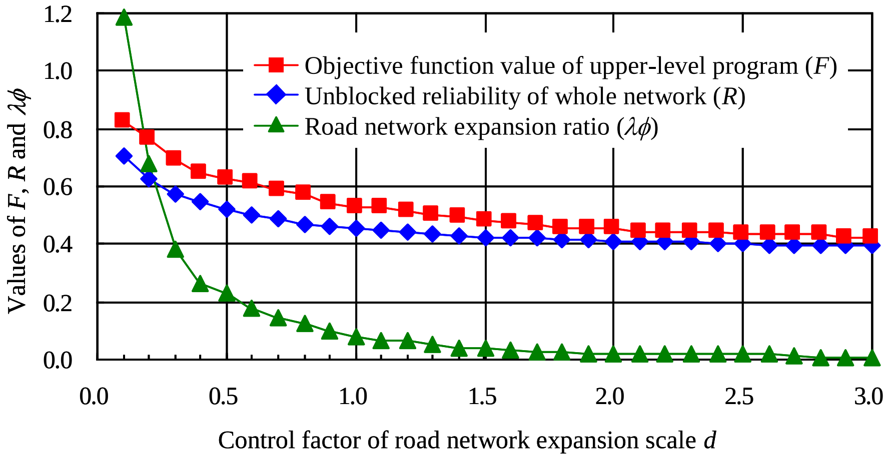


Fig. 6 Trend of objective function of upper-level program with changing control factor of road network expansion scale

VII. Conclusion

In this paper, a new bi-level program model based on the unblocked reliability for solving the continuous network design problem was proposed. A set of link capacity expansions that can best improve the road network was found using the Hooke-Jeeves algorithm to solve the proposed bi-level program. These link capacity expansions reach an optimization result by maximizing the surplus of the unblocked reliability of the entire road network minus the road network expansion ratio, where the user equilibrium is followed. Also, the performance of an Origin-Destination pair with high traffic demands is a top priority. The penalty term on improvement costs is established to ensure that the road network expansion reaches a reasonable scale and only the links that can greatly improve unblocked reliability of the entire road network will be selected when using the Hooke-Jeeves algorithm. The control factor of the road network expansion scale is introduced in the objective function of the upper-level program. When planners and managers set the control factor to different degrees, the proposed bi-level program can comply with various limits of environmental and financial resources. The proposed bi-level program is a useful tool for design of a high-reliability road network system with optimization link capacity expansions.

Nomenclature

A: set of links in network
a: link in network, $a \in A$
C: capacity of a link, pcu/h
d: control factor of road network expansion scale
F: objective function of upper-level program
g: counter for exploratory move in Hooke-Jeeves algorithm
G: total number of links in road network
h: counter for pattern move in Hooke-Jeeves algorithm
I: set of all origin nodes
J: set of all destination nodes

L: length of a link, km
M: set of used routes between Origin-Destination pair
m: used path between Origin-Destination pair, $m \in M$
R: unblocked reliability index
s: step length of exploratory move for link capacity expansion in Hooke-Jeeves algorithm, pcu/h
t: travel time, h
v: step length of pattern move in Hooke-Jeeves algorithm
x: link flow, pcu/h
y: link capacity expansion, pcu/h
Z: objective function of lower-level program
β : direction factor in Hooke-Jeeves algorithm, $\beta = 1$ for positive capacity expansion, and $\beta = -1$ for negative capacity expansion
η : convergence step length of exploratory move for link capacity expansion in Hooke-Jeeves algorithm, pcu/h
θ : reduction factor of step length of exploratory move in Hooke-Jeeves algorithm
λ : conversion factor for physical dimension, h/(km·pcu)
μ : expansion direction in Hooke-Jeeves algorithm
ϕ : road network expansion scale
ω : variable of integration in objective function of User Equilibrium problem

Superscripts
e: free-flow state

Subscripts
i: origin node, $i \in I$
j: destination node, $j \in J$

Abbreviations
CNDP: Continuous Network Design Problem
DNDP: Discrete Network Design Problem
FW: Frank-Wolfe
HJ: Hooke-Jeeves
NDP: Network Design Problem
OD: Origin-Destination
UE: User Equilibrium

REFERENCES

[1] S. Chiou, Bilevel programming for the continuous transport network design problem, *Transportation Research Part B*, *39*, 2005, 361-383.
[2] J. Magnus and P. Michael, Sensitivity analysis of separable traffic equilibrium equilibria with application to bilevel optimization in network design, *Transportation Research Part B*, *41*, 2007, 4-31.
[3] Y. Iida and H. Wakabayashi, An Approximation Method of Terminal Reliability of Road Network Using Partial Minimal Path and Cut Sets, *Proc. of the Fifth World Conference on Transport Research*, *4*, 1989, 367-380.
[4] Y. Iida, Basic Concepts and Future Directions of Road Network Reliability Analysis, *Journal of Advanced Transportation*, *33(2)*, 1999, 125-134.
[5] A. Chen, H. Yang, H. K. Lo and W. H. Tang, A Capacity Related Reliability for Transportation Networks, *Journal of Advanced Transportation*, *33(2)*, 1999, 183-200.
[6] R. Hooke and T. A. Jeeves, Direct Search Solution of Numerical and Statistical Problems, *Journal of the Association for Computing Machinery*, *8(2)*, 1961, 212-229.
[7] M. G. H. Bell and Y. Iida, *Transportation Network Analysis*, (John Wiley & Sons, 1997), 193-204.
[8] S. Hou, N. Maruyama and S. Kato, New Methodology to Calculate Unblocked Reliability to Assess Road Network Operation Performance, *Journal of Mechanical Systems for Transportation and Logistics*, *1(3)*, 2008, 240-251.
[9] S. Hou, N. Maruyama, M. Hirota and S. Kato, Optimization Framework for Road Network Directed by Unblocked Reliability for Given Network Topology and Inelastic Demand with Stochastic User Equilibrium, *WSEAS Transactions on Business and Economics*, *6(6)*, 2009, 292-301.
[10] J. G. Wardrop, Some Theoretical Aspects of Road Traffic Research, *Proc. of the Institution of Civil Engineers*, Part II, *1(1)*, 1952, 325-362.

[11] M. Beckmann, C. B. McGuire and C. B. Winsten, *Studies in the Economics of Transportation*, 3, (Yale University Press, 1956), 59-79.
[12] A. Chen, H. Yang, H. K. Lo and W. H. Tang, Capacity reliability of a road network: an assessment methodology and numerical results, *Transportation Research Part B, 36*, 2002, 225-252.
[13] Y. Sheffi, *Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods,* (Prentice-Hall, 1984), 95-107 and 116-122.
[14] M. Abdulaal and L. J. LeBlanc, Continuous equilibrium network design models, *Transportation Research part B, 13* 1979, 19-32.

Appendix A

The Hooke-Jeeves (HJ) algorithm is a fundamental algorithm for solving the Continuous Network Design Problem (CNDP)[6, 14]. The HJ algorithm proceeds by a sequence of exploratory and pattern moves. The flow chart of the HJ algorithm is shown in Fig. A1.

In order to express different values of capacity expansion in the iteration process, three vectors are named, including the test link capacity expansion, $\hat{y}$, benchmark link capacity expansion, $\bar{y}$, and the success link capacity expansion, y^h. The implementation steps for solving a bi-level program based on unblocked reliability are given as follows:

Step 0: Initialization.

Given an initial solution of y^0, solve the lower-level program with y^0. That is, the current capacity is the sum of the initial capacity expansion, y^0, and the capacity of the existing road network. Calculate the objective function of the upper level to obtain $F(y^0)$. Set $\bar{y} = y^0$ and $F(\bar{y}) = F(y^0)$. Give the step length of exploratory move, *s*, reduction the factor of the step length of exploratory move, *θ*, convergence step length of exploratory move, η, and step length of pattern move, *v*. Set direction factor, $\beta = 1$, counter, *g* = 1 (from first link), for exploratory move and counter, *h* =0, for the pattern move.

Step 1: Exploratory moves.

Step 1-1: If *g* > *G*, each link has been tested, so go to step 2. Let μ_g be a vector that contains a '1' in the *g*th position and '0' elsewhere.

Step 1-2: Set $\hat{y} = \bar{y} + \beta s \mu_g$. Solve the lower-level program with $\bar{y}$. Calculate the objective function of the upper level to obtain $F(\bar{y})$.

Step 1-3: If $F(\hat{y}) > F(\bar{y})$ renew the benchmark link capacity expansions and objective function of the upper level, $\bar{y} = \hat{y}$ and $F(\bar{y}) = F(\hat{y})$, respectively. Then, set *g* = *g*+1 and go to step 1-1. Otherwise perform the next step.

Step 1-4: If $\beta = 1$, set $\beta = -1$ and go to step 1-2; otherwise *g* = *g*+1, $\beta = 1$ and go to Step 1-1.

Step 2: Pattern moves.

Step 2-1: If $F(\bar{y}) > F(y^h)$, renew the success solution and objective function value of the upper-level program to $y^{h+1} = \bar{y}$ and $F(y^{h+1}) = F(\bar{y})$, respectively. Perform pattern move, $\bar{y} = y^h + v(y^{h+1} - y^h)$. Solve the lower-level program with $\bar{y}$. Calculate the objective function of the upper level to obtain $F(\bar{y})$. Set *h* = *h*+1, *g* = 1 and go to step 1. Otherwise, perform the next step.

Step 2-2: If the convergence criterion, $s < \eta$, is met, stop (the current solution, y^h, is the optimal link capacity expansion); otherwise, reduce the step length of exploratory, $s = \theta s$, return to the success solution and objective function value of the upper-level program, $\bar{y} = y^h$ and $F(\bar{y}) = F(y^h)$, respectively. Then, set $g = 1$ and go to step 1.

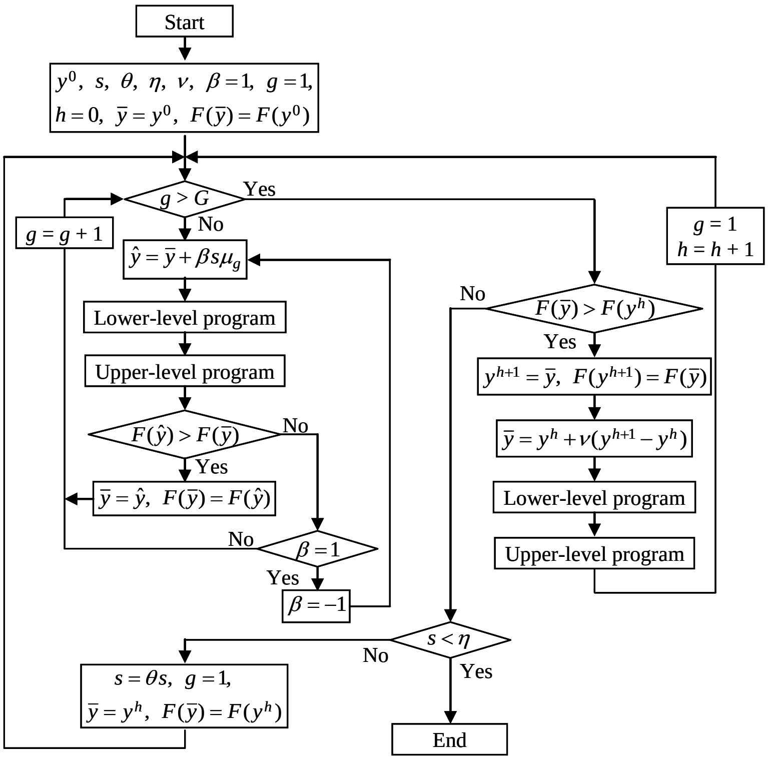


Fig. A1 Flowchart of Hooke-Jeeves algorithm to solve bi-level program with unblocked reliability.