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Granger Causality Test: A Useful Descriptive Tool for Time Series Data

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Abstract: *Interdependency of one or more variables on the other has been in the existence over long time when it was discovered that one variable has to move or regress toward another following the work done by Galton (1886); Pearson & Lee (1903); Kendall & Stuart, (1961); Johnston and DiNardo, (1997); Gujarati, (2004) etc. It was in the light of this dependency over time the researcher uses Granger Causality as an effective tool in time series Predictive causality using Nigeria GDP and Money Supply to know the type of causality in existence in the two time series variables under consideration and which one can statistically predicts the other.*

The research work aimed at testing for nature of causality between GDP and money supply for Federal Republic of Nigeria for the period of thirty years using the data sourced from Central Bank of Nigeria Statistical Bulletin. After observing the various conditions of Granger causality test such as ensuring stationarity in the variables under consideration; adding enough number of lags in the prescribed model before estimation as Granger causality test is sensitive to the number of lags introduced in the model; and as well as assuming the disturbance terms in the various models are uncorrelated, the result of the analysis indicates a bilateral relationship between Nigeria GDP and Money Supply. It implies Nigeria GDP Granger causes money Supply and vice versa. Based on the result of this study, both Nigeria GDP and money Supply can be successfully model using Vector Autoregressive Model since changes in one variable has a significant effect on the other variable.

Keywords: *Bilateral Causality, Time Series, VARMA Model, Stationarity, Wold representation GDP, Money Supply,*

I. Introduction

Two or more variables can be interdependence on one another if the occurrence of one causes the other to take place or vice versa, then one can talk of unidirectional causality and feedback causality. Changes in the first variable precede changes in the other. That is one Granger causes another knowing fully that the present with its lagged values can only predict the future but the future cannot predict the past.

A statistical relationship in itself cannot logically imply causation, to ascribe causality; one must appeal to a priori or theoretical considerations (Gujarati, 2004 pp23). A statistical relationship, however strong and however suggestive, can never establish causal connection: our ideas of causation must not from mere statistics but ultimately from some theory or other (Kendall & Stuart, 1961).

II. Model Description and Notations

2.1 Overview of ARMA Models

A set of repeated observations of the same variable such as Stock Returns, GDP, Money Supply, Interest Rates etc each one being recorded at a specific time are termed Time Series. These set of variables in their stationarity form can be modelled using ARMA model espoused in Box Jenkins &Reinsel, (1994).ARMA model of order (p,q) can be viewed as linear filter from of digital signal processing perspective,(Fangge and Peixian,(2011).

Consider the linear combination of the lagged variables in the equation (1) given below

$$\sum_{j=0}^{p}\alpha_j y_{t-j} = \sum_{j=0}^{q}\theta_j \mu_{t-j},\ \phi_0 = \theta_0 = 1 \qquad (1)$$

where

$E(\mu_t) = 0 ; E\{\mu_s \mu_t\} = \sigma_{st}^2$, σ>0

μ_t are shocks or innovation series
and that

$$E(y_t) = E(y_s) \text{ for all } t \text{ and } s$$

$$E(y_t y_{t-j}) = E(y_s y_{s-j}) \text{ for all } t \text{ and } s$$

The letter **E** is use to denote the expectation operator.
The ARMA (p,q) in equation (1) is a combination of the autoregressive structure of the residues (moving average MA) and a linear relationship between the value predicted by the model at time t and the past values of the time series(autoregressive AR).
Construction of ARMA model requires four iterative steps which are made explicit in Box, Jenkins, & Reinsel ,(1994); Hamilton,(1994); Salau, (2003); Oguntade,(2010); Oguntade and Ogunfiditimi (2013) etc.

2.2 Runs Test for Stationarity

ARMA models can be used directly if the time series is wide sense stationary, (Fangge & Peixian, 2011).The stationary assumption of any given time series need be checked for proper identification and estimation of the model. This is achieved by Runs Test for randomness. The stationarity assumption is rejected if Asymp. Sig $\prec$ 0.05.Otherwise time series should be differenced or transformed using various time series transformation techniques until stationarity is achieved.

2.3 Granger Causality Test

If variable Y contains useful information for predicting variable X , then Y causes X .That is Y is Granger / Predictive causality of X .Then Y causes X is denoted as $(Y \to X)$ and X causes Y is denoted as $(X \to Y)$ where the arrow points to the direction of Causality. The Granger Causality Test assumes that the information relevant to the prediction of variables Y and X is contained in the time series data on these variables.
The bilateral causality are related by the following regression models

$$X_t = \sum_{i=1}^{n} \alpha_i X_{t-i} + \sum_{j=1}^{n} \beta_j Y_{i-j} + \ell_{t1} \quad (1)$$

$$Y_t = \sum_{i=1}^{n} \phi_i X_{t-i} + \sum_{j=1}^{n} \varphi_j Y_{i-j} + \ell_{t2} \quad (2)$$

Where ℓ_{t1} and ℓ_{t2} are orthogonal disturbances

From the regression equations, we test the hypothesis H^1_0 that lagged Y terms do not belong in the regression ($\sum_{j=1}^{n} \beta_j = 0$). That is Y do not cause X using F statistic given below with m and $(n-k)$ degrees of freedom

$$F = \frac{(RSS_R - RSS_{UR})/m}{RSS_{UR}/(n-k)} \quad (3)$$

Also, $H^2_0 : \sum_{i=1}^{n} \phi_i = 0$ versus $H^2_1 : \sum_{i=1}^{n} \phi_i \neq 0$ is also tested for the model above

Note: rejection of H_0 at certain critical value of F implies one variable Granger causes another
See Gujarati (2004) and Cochrane (2005)

2.4 VAR as a time series Econometric Model

Multivariate causality among dependent or response variables is made possible through Vector Autoregressive technique .Causality, Cointegration and VAR as time Series Econometrics terms are powerful tools in estimation and prediction of Vector Autoregressive Moving Average Models which takes its root from the popular Box Jenkins methodology. See for example Brockwell and Davis, (1990); Gujarati, (2004); Cochrane, (2005) Consider the VAR given in (1) Y_t Granger causes X_t if Y_t helps to forecast X_t, given past X_t. But, Y_t does not Granger causes X_t if $\beta_j = 0$

That is if VAR in (1) is equivalent to

$$X_t = \sum_{i=1}^{n} \alpha_i X_{t-i} + 0\sum_{j=1}^{n} Y_{i-j} + \ell_{t1} \quad (3)$$

$$Y_t = \sum_{i=1}^{n} \phi_i X_{t-i} + \sum_{j=1}^{n} \varphi_j Y_{i-j} + \ell_{t2}$$

Equation (1) in matrix notations becomes

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} X_{t-i} \\ Y_{t-j} \end{bmatrix} + \begin{bmatrix} \ell_{t1} \\ \ell_{t2} \end{bmatrix}$$

$$\begin{bmatrix} I - La & -Lb \\ -Lc & -Ld \end{bmatrix} \begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} \ell_{t1} \\ \ell_{t2} \end{bmatrix}$$

$$\begin{bmatrix} a^*(L) & b^*(L) \\ c^*(L) & d^*(L) \end{bmatrix} \begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} \ell_{t1} \\ \ell_{t2} \end{bmatrix} \quad (4)$$

Inverting the autoregressive representation, we have (5)

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \frac{1}{a^*(L)d^*(L) - b^*(L)c^*(L)} \begin{bmatrix} d^*(L) & -b^*(L) \\ -c^*(L) & a^*(L) \end{bmatrix} \begin{bmatrix} \ell_{t1} \\ \ell_{t2} \end{bmatrix} \quad (5)$$

Equation (5) is moving average representation called Bivariate Wold representation.

From (5) Y does not Granger cause X Iff $b^*(L) = 0$, or if the autoregressive matrix lag polynomial is lower triangle.

Thus, Y does not Granger cause X if and only if the Wold moving average matrix lag polynomial is lower triangular. That is Y does not Granger cause X if and only if X "s bivariate Wold representation is the same as its univariate Wold representation. The projection of X on past X and Y is the same as projection of X on the past X, and that X is a function of its shocks only and does not respond to Y shocks while Y is a function of both X shocks and Y shocks. Details of these and others are available in Cochran, (2005).

III. Numerical Example

3.1 Data Description

The data for this research paper were extracted from the bulletin published by the Central Bank of Nigeria (CBN) (as shown in figure 1).The data on GDP and Money supply were used in the predicting multivariate model. The preliminary transformations were not left out as trends were discovered in the Figure 1

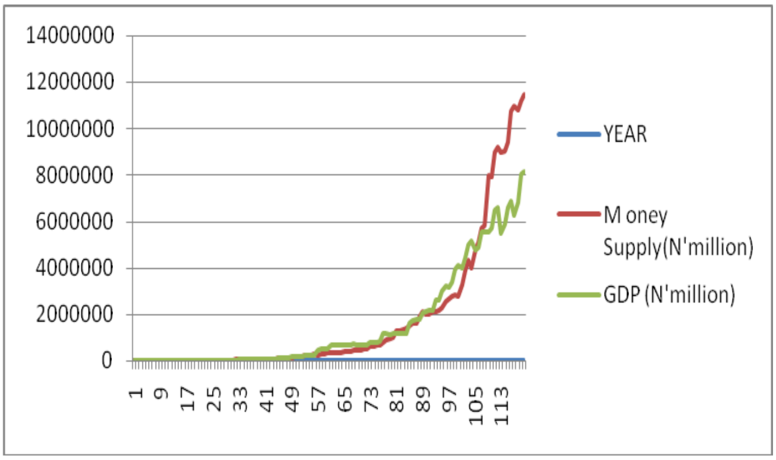

Fig1:Time Plot of Nigeria GDP and Money Supply

3.2 Empirical Results

Table 3.1 Null Unit Root Test for GDP

Hypothesis: D(GDP,2) has a unit root
Exogenous: Constant
Lag Length: 7 (Automatic - based on SIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-8.234916	0.0000
Test critical values:	1% level	-3.490772	
	5% level	-2.887909	
	10% level	-2.580908	

*MacKinnon (1996) one-sided p-values.

Table 3.2 Null Unit Root Test for MS

Null Hypothesis: D(money supply,2) has a unit root
Exogenous: Constant
Lag Length: 12 (Automatic - based on SIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.736734	0.0002
Test critical values:	1% level	-3.493747	
	5% level	-2.889200	
	10% level	-2.581596	

*MacKinnon (1996) one-sided p-values.

Table 3.3 Granger Causality Tests

Pairwise Granger Causality Tests
Date: 10/17/13 Time: 05:40
Sample: 1981Q1 2010Q4
Lags: 2

Null Hypothesis:	Obs	F-Statistic	Prob.
GDP does not Granger Cause Money supply	116	48.0658	9.E-16
money supply does not Granger Cause GDP		47.6875	1.E-15

Table 3.4: Outcome of Unit Root Tests

ADF TEST	F	P-VALUE	DECISION
GDP	**8.234916**	**0.0000**	**SIGNIFICANT AT 5%** α
MS	**4.736736**	**0.0002**	**SIGNIFICANT AT 5%** α

3.2.1: Result of Stationarity Test

Both GDP and money supply are not stationary in their level form but the desired level of stationarity was achieved after second difference with significant ADF values of 8.234916 and 4.736736 in absolute value respectively. We reject the Null hypothesis of presence of unit roots in both cases at **5%** α as the P –values in table 3.1 above are significant.

Table 3.5: Results of Causality Test

DIRECTION OF CAUSALITY	F	DECISION
MS $\rightarrow$ **GDP**	**48.0658**	**REJECT**
GDP $\rightarrow$ **MS**	**47.6875**	**REJECT**

3.2.2: Discussion of Results

The result suggests that the direction of causality is bi-direction in nature, since the estimated f values are significant at 5% level of significant; the critical F values are 48.0658 and 47.6875 respectively as elicited in Table 3.2

The granger causality test under the null hypotheses (H_o) GDP does not granger cause money supply and vice-versa are statistically significant, which implies that there is bilateral/feedback causality between GDP and money supply. The significant value of the test in the table above Implies the set of coefficients of GDP and Money Supply are statistically and significantly different from zero in both regressions.

Changes in either GDP or money supply causes changes in the other variable and hence changes in the economic growth and development of Nigeria. Investing more money into the economy leads to increase GDP.

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