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Exact Solutions of Convection Diffusion Equation by Modified F-Expansion Method

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Abstract: *In this paper, the modified F-expansion method is proposed for constructing more than one exact solutions of nonlinear convection diffusion equation with the aid of symbolic computation Mathematica. By using this method, some new exact travelling wave solutions of the convection diffusion equation are successfully obtained. These exact solutions include the soliton-like solutions, trigonometric function solutions and rational solutions. Also, it is shown that the proposed method is efficient for solving nonlinear partial differential equations arising in mathematical physics.*

Keywords: *convection diffusion equation, modified F-expansion method, travelling wave solutions*

I. Introduction

It is well known that nonlinear partial differential equations (NLPDEs) play a major role in describing many phenomena arising in many fields of sciences and engineering. The investigation of exact solutions of NLPDEs plays an important role in the study of nonlinear physical phenomena. In recent years, direct search for exact solutions to NLPDE has become more and more attractive partly due to availability of computer symbolic system like Mathematica or Maple which allows us to perform some complicated and tedious algebraic calculation on computer. Different methods have been invented to obtain exact solutions of NLPDEs, such as homotopy perturbation method [3], homotopy analysis method [9,14], variational iteration method [11], tanh method [17] and so on. In the recent past F- expansion method [19,21] was proposed to construct exact solutions of NLPDEs. This method was later further extended in different manners, like the generalized F-expansion method [20], modified F-expansion method [6,7,8], improve F-expansion method [18], etc. in this study, the modified F-expansion method is used.

The objective of this paper is to apply modified F-expansion method to construct the exact travelling wave solutions of the following nonlinear convection-diffusion equation [10,12,15] for two cases.

$$u_t = \left(u^m\right)_{xx} + \left(u^n\right)_x \tag{1}$$

This equation arises in many physical phenomena for different values of m and n. For example, $(m,n)=(1,2)$ eq.(1) converted into classic Burgers' equation [1,13,14,16] which has many application in gas dynamics, traffic flow, shock wave phenomena etc. Eq.(1) also arises in theory of infiltration of water under gravity through a homogeneous and isotropic porous medium for $n \geq m > 1$ [4]. In the case of $(m,n)=(4,3)$, eq.(1) models the flow of a thin viscous sheet on an inclined bed [5]. Act as a foam drainage equation for $(m,n)=(3/2,2)$ [8].

II. Outline Of The Modified F-Expansion Method

In this section, the description of the method is given.

Consider a given nonlinear partial differential equation with independent variables x,t and dependent variable u,

$$P\left(u,u_t,u_{xx},u_{xt},u_{tt},....\right)=0 \tag{2}$$

Generally speaking, the left-hand side of eq.(2) is a polynomial in u and its various partial derivatives. The main points of the modified F-expansion method for solving eq.(2) are as follows:

[1]. Seek travelling wave solutions to eq.(2) by taking

$$u(x,t)=u(\xi) \text{ where } \xi = h(x+vt) \tag{3}$$

where $h \neq 0$ and v are constants to be determined, inserting eq.(3) into eq.(2) yields an ordinary differential equation (ODE) for $u(\xi)$

$$P(u, u', u'', ...) = 0 \tag{4}$$

where prime denotes the derivative with respect to ξ.

[2]. Suppose that $u(\xi)$ can be expressed as

$$u(\xi) = \sum_{i=-N}^{N} a_i F^i(\xi) \text{ where } a_N \neq 0 \tag{5}$$

where $a_i \,(i = -N, ..., -1, 0, 1, ...N)$ are all constants to be determined and $F(\xi)$ satisfies following Riccati equation

$$F'(\xi) = A + BF(\xi) + CF^2(\xi) \tag{6}$$

where A, B, C are constants and N is a integer which can be determined by considering the homogeneous balance between the governing nonlinear term(s) and highest order derivative of $u(\xi)$ in eq.(4) and

(i) when $N = \left(p/q \right)$ is fraction, let $u(\xi) = w^{p/q}(\xi)$

(ii) when N is negative integer, let $u(\xi) = w^N(\xi)$

By using (i) and (ii), we change eq.(4) into another ODE for $w(\xi)$ whose balancing number will be a positive integer.

[3]. Substitute eq.(5) into eq.(4) and using eq.(6) then left-hand side of eq.(4) can be converted into a finite series in $F^p(\xi)$, $(p = -N, ..., -1, 0, 1, ..., N)$. Equating each coefficient of $F^p(\xi)$ to zero yields a system of algebraic equations.

[4]. Solve the system of algebraic equations probably with the aid of Mathematica, a_i, h, v can be expressed by A, B, C. Substituting these results into eq.(5), we can obtain the general form of travelling wave solutions to eq.(4).

[5]. From the general form of travelling wave solutions listed in appendix, we can give a series of soliton-like solutions, trigonometric function solutions and rational solutions of eq.(2).

III. Implementation Of The Method

To solve eq.(1), following two cases are considered:

Case [I]: $n = m > 1$

Case [II]: $n = 2m - 1,\ m > 1$

Case [I]: $n = m > 1$

Considering $n = m$ in eq.(1), we get

$$u_t = (u^m)_{xx} + (u^m)_x \tag{7}$$

Let us start our analysis by using the transformation,

$$u(x,t) = u(\xi) \text{ where } \xi = h(x + vt) \tag{8}$$

where $h \neq 0$ and v are constants to be determined later. Substituting eq.(8) into eq.(7), we get ODE

$$hvu' - h^2 m(m-1) u^{m-2} (u')^2 - h^2 m u^{m-1} u'' - hmu^{m-1} u' = 0 \tag{9}$$

where prime denotes differentiation with respect to ξ. Balancing the orders of u' and $u^{m-1}u''$ in eq.(9), we have $N = -\dfrac{1}{m-1}$. Now, we use $u(\xi) = w^{-1/m-1}(\xi)$ in eq.(9), we get ODE in the following form,

$$vww' + \frac{m(2m-1)}{m-1} h w^{-1} (w')^2 - mhw'' - mw' = 0 \tag{10}$$

Balancing the orders of ww' and w'' in eq.(10), we get $N = 1$. So we can write solution of eq.(7) in the form,

$$u(x,t) = w^{-1/m-1}(\xi) \text{ where } w(\xi) = a_0 + a_{-1}F^{-1}(\xi) + a_1 F(\xi) \tag{11}$$

Substituting eq.(11) into eq.(10) and using eq.(6), the left-hand side of eq.(10) can be converted into a finite series in $F^p(\xi)$, $(p = -4,-3,-2,-1,0,1,2,3,4)$. Equating each coefficient of $F^p(\xi)$ to zero, we obtained the following set of algebraic equations with the aid of Mathematica:

$$\left.\begin{aligned}
&F^{-4}(\xi): -2A^2hma_{-1}^2 - \frac{A^2hma_{-1}^2}{m-1} + \frac{2A^2hm^2a_{-1}^2}{m-1} - Aa_{-1}^3 v = 0 \\
&F^{-3}(\xi): -2A^2hma_0a_{-1} + Ama_{-1}^2 - 3ABhma_{-1}^2 - \frac{2ABhma_{-1}^2}{m-1} + \frac{4ABhm^2a_{-1}^2}{m-1} \\
&\qquad - 2Aa_0a_{-1}^2 v - Ba_{-1}^3 v = 0 \\
&F^{-2}(\xi): Ama_0a_{-1} - 3ABhma_0a_{-1} + Bma_{-1}^2 - B^2hma_{-1}^2 - 2AChma_{-1}^2 - \frac{B^2hma_{-1}^2}{m-1} \\
&\qquad - \frac{2AChma_{-1}^2}{m-1} + \frac{2B^2hm^2a_{-1}^2}{m-1} + \frac{4AChm^2a_{-1}^2}{m-1} - 2A^2hma_{-1}a_1 + \frac{2A^2hma_{-1}a_1}{m-1} \\
&\qquad - \frac{4A^2hm^2a_{-1}a_1}{m-1} - Aa_0^2a_{-1}v - 2Ba_0a_{-1}^2v - Ca_{-1}^3v - Aa_{-1}^2a_1v = 0 \\
&F^{-1}(\xi): Bma_0a_{-1} - B^2hma_0a_{-1} - 2AChma_0a_{-1} + Cma_{-1}^2 - BChma_{-1}^2 - \frac{2BChma_{-1}^2}{m-1} \\
&\qquad + \frac{4BChm^2a_{-1}^2}{m-1} - 4ABhma_{-1}a_1 + \frac{4ABhma_{-1}a_1}{m-1} - \frac{8ABhm^2a_{-1}a_1}{m-1} - Ba_0^2a_{-1}v \\
&\qquad - 2Ca_0a_{-1}^2v - Ba_{-1}^2a_1v = 0 \\
&F^{0}(\xi): Cma_0a_{-1} - BChma_0a_{-1} - \frac{C^2hma_{-1}^2}{m-1} + \frac{2C^2hm^2a_{-1}^2}{m-1} - Am\ a_0a_1 - ABhma_0a_1 \\
&\qquad - 2B^2hma_{-1}a_1 - 4AChma_{-1}a_1 + \frac{2B^2hma_{-1}a_1}{m-1} + \frac{4AChma_{-1}a_1}{m-1} - \frac{4B^2hm^2a_{-1}a_1}{m-1} \\
&\qquad - \frac{8AChm^2a_{-1}a_1}{m-1} - \frac{A^2hma_1^2}{m-1} + \frac{2A^2hm^2a_1^2}{m-1} - Ca_0^2a_{-1}v + Aa_0^2a_1v - Ca_{-1}^2a_1v \\
&\qquad + Aa_{-1}a_1^2v = 0 \\
&F^{1}(\xi): -Bma_0a_1 - B^2hma_0a_1 - 2AChma_0a_1 - 4BChma_{-1}a_1 + \frac{4BChma_{-1}a_1}{m-1} \\
&\qquad - \frac{8BChm^2a_{-1}a_1}{m-1} - Ama_1^2 - ABhma_1^2 - \frac{2ABhma_1^2}{m-1} + \frac{4ABhm^2a_1^2}{m-1} + Ba_0^2a_1v \\
&\qquad + 2Aa_0a_1^2v + Ba_{-1}a_1^2v = 0 \\
&F^{2}(\xi): -Cma_0a_1 - 3BChma_0a_1 - 2C^2hma_{-1}a_1 + \frac{2C^2hma_{-1}a_1}{m-1} - \frac{4C^2hm^2a_{-1}a_1}{m-1} \\
&\qquad - Bma_1^2 - B^2hma_1^2 - 2AChma_1^2 - \frac{B^2hma_1^2}{m-1} - \frac{2AChma_1^2}{m-1} + \frac{2B^2hm^2a_1^2}{m-1} \\
&\qquad + \frac{4AChm^2a_1^2}{m-1} + Ca_0^2a_1v + 2Ba_0a_1^2v + Ca_{-1}a_1^2v + Aa_1^3v = 0 \\
&F^{3}(\xi): -2C^2hma_0a_1 - Cma_1^2 - 3BChma_1^2 - \frac{2BChma_{-1}^2}{m-1} + \frac{4BChm^2a_1^2}{m-1} \\
&\qquad + 2Ca_0a_1^2v + Ba_1^3v = 0 \\
&F^{4}(\xi): -2C^2hma_1^2 - \frac{C^2hma_1^2}{m-1} + \frac{2C^2hm^2a_1^2}{m-1} + Ca_1^3v = 0
\end{aligned}\right\} \tag{12}$$

Solving the above algebraic equations by Mathematica, we considered different cases as follows:

Case 1: $A = 0$, we have

$$a_0 = 0, a_{-1} = 0, a_1 = a_1, v = -\frac{Chm}{a_1(m-1)} \quad (13)$$

Case 2: $B = 0$, we have

$$a_0 = a_0, a_{-1} = 0, a_1 = a_1, v = \frac{2Chma_0 + ma_1}{2a_0 a_1} \quad (14)$$

Using cases 1,2 and appendix, we obtained the following exact generalized solutions of eq.(7)

<u>Soliton-like solutions:</u>

(1) $A = 0, B = 1, C = -1$

Assuming, $h = \frac{m-1}{m}$ in eq.(13), we get

$$a_0 = 0, a_{-1} = 0, a_1 = a_1, v = \frac{1}{a_1} \quad (15)$$

From appendix and eq.(15), we have

$$u_1(x,t) = \left\{ a_1 \left(\frac{1}{2} + \frac{1}{2} \tanh \left[\frac{m-1}{2m} \left(x + \frac{1}{a_1} t \right) \right] \right) \right\}^{-1/m-1} \quad (16)$$

(2) $A = 0, B = -1, C = 1$

Assuming, $h = -\frac{m-1}{m}$ in eq.(13), we get

$$a_0 = 0, a_{-1} = 0, a_1 = a_1, v = \frac{1}{a_1} \quad (17)$$

From appendix and eq.(17), we have

$$u_2(x,t) = \left\{ a_1 \left(\frac{1}{2} + \frac{1}{2} \coth \left[\frac{m-1}{2m} \left(x + \frac{1}{a_1} t \right) \right] \right) \right\}^{-1/m-1} \quad (18)$$

The solution given by eq.(18) and that obtained in [2], shows good agreement.

(3) $A = 1, B = 0, C = -1$

Assuming, $a_0 = a_1$ and $h = \frac{m-1}{2m}$ in eq.(14), we get

$$a_0 = a_1, a_{-1} = 0, a_1 = a_1, v = \frac{1}{2a_1} \quad (19)$$

From appendix and eq.(19), we have

$$u_3(x,t) = \left\{ a_1 + a_1 \tanh \left[\frac{m-1}{2m} \left(x + \frac{1}{2a_1} t \right) \right] \right\}^{-1/m-1} \quad (20)$$

Again assuming $a_0 = -a_1$ and $h = -\frac{m-1}{2m}$ in eq.(14), we get

$$a_0 = -a_1, a_{-1} = 0, a_1 = a_1, v = -\frac{1}{2a_1} \quad (21)$$

From appendix and eq.(21), we have

$$u_4(x,t) = \left\{ -a_1 - a_1 \coth \left[\frac{m-1}{2m} \left(x - \frac{1}{2a_1} t \right) \right] \right\}^{-1/m-1} \quad (22)$$

(4) $A = \frac{1}{2}, B = 0, C = -\frac{1}{2}$

Assuming, $a_0 = a_1$ and $h = \frac{m-1}{m}$ in eq.(14), we get

$$a_0 = a_1, a_{-1} = 0, a_1 = a_1, v = \frac{1}{2a_1} \quad (23)$$

From appendix and eq.(23), we have

$$u_5(x,t) = \left\{ a_1 + a_1 \left(\coth\left[\frac{m-1}{m}\left(x + \frac{1}{2a_1} t \right) \right] \pm \operatorname{csc}h\left[\frac{m-1}{m}\left(x + \frac{1}{2a_1} t \right) \right] \right) \right\}^{-1/_{m-1}} \quad (24)$$

$$u_6(x,t) = \left\{ a_1 + a_1 \left(\tanh\left[\frac{m-1}{m}\left(x + \frac{1}{2a_1} t \right) \right] \pm i \operatorname{sec}h\left[\frac{m-1}{m}\left(x + \frac{1}{2a_1} t \right) \right] \right) \right\}^{-1/_{m-1}} \quad (25)$$

<u>Trigonometric function solutions:</u>

(5) $A = \frac{1}{2}, B = 0, C = \frac{1}{2}$

Assuming, $a_0 = -ia_1$ and $h = \frac{m-1}{im}$ in eq.(14), we get

$$a_0 = -ia_1, a_{-1} = 0, a_1 = a_1, v = \frac{i}{2a_1} \quad (26)$$

From appendix and eq.(26), we have

$$u_7(x,t) = \left\{ -ia_1 + a_1 \left(\sec\left[\frac{m-1}{i\,m}\left(x + \frac{i}{2a_1} t \right) \right] + \tan\left[\frac{m-1}{i\,m}\left(x + \frac{i}{2a_1} t \right) \right] \right) \right\}^{-1/_{m-1}} \quad (27)$$

$$u_8(x,t) = \left\{ -ia_1 + a_1 \left(\csc\left[\frac{m-1}{i\,m}\left(x + \frac{i}{2a_1} t \right) \right] - \cot\left[\frac{m-1}{i\,m}\left(x + \frac{i}{2a_1} t \right) \right] \right) \right\}^{-1/_{m-1}} \quad (28)$$

(6) $A = -\frac{1}{2}, B = 0, C = -\frac{1}{2}$

Assuming, $a_0 = -ia_1$ and $h = -\frac{m-1}{im}$ in eq.(14), we get

$$a_0 = -ia_1, a_{-1} = 0, a_1 = a_1, v = \frac{i}{2a_1} \quad (29)$$

From appendix and eq.(29), we have

$$u_9(x,t) = \left\{ -ia_1 - a_1 \left(\sec\left[\frac{m-1}{i\,m}\left(x + \frac{i}{2a_1} t \right) \right] - \tan\left[\frac{m-1}{i\,m}\left(x + \frac{i}{2a_1} t \right) \right] \right) \right\}^{-1/_{m-1}} \quad (30)$$

$$u_{10}(x,t) = \left\{ -ia_1 - a_1 \left(\csc\left[\frac{m-1}{i\,m}\left(x + \frac{i}{2a_1} t \right) \right] + \cot\left[\frac{m-1}{i\,m}\left(x + \frac{i}{2a_1} t \right) \right] \right) \right\}^{-1/_{m-1}} \quad (31)$$

Case [II]: $n = 2m-1,\ m > 1$

We consider $n = 2m-1$ in eq.(1), we get,

$$u_t = \left(u^m\right)_{xx} + \left(u^{2m-1}\right)_x,\ m > 1. \quad (32)$$

Differentiating eq.(32) with respect to x we have following partial differential equation,

$$u_t - m(m-1)u^{m-2}\left(u_x\right)^2 - mu^{m-1}u_{xx} - (2m-1)u^{2m-2}u_x = 0 \quad (33)$$

Suppose the solution of eq.(33) is of the form

$$u(x,t) = u(\xi) \text{ where } \xi = h(x + vt) \quad (34)$$

where $h \neq 0$ and v are constants to be determined. Substituting eq.(34) into eq.(33), we get ODE,

$$vu^{-(m-1)}u' - m(m-1)hu^{-1}\left(u'\right)^2 - mhu'' - (2m-1)u^{m-1}u' = 0 \quad (35)$$

where prime denotes differentiation with respect to ξ. Now, balancing the order of $u^{m-1}u'$ with u'' in eq.(35), we obtained integer $N=\frac{1}{m-1}$. Again we use the transformation $u(\xi)=w^{1/m-1}(\xi)$ into the eq.(35), we get following ODE

$$v\,w^{-1}w' - m\left(\frac{1}{m-1}\right)h\,w^{-1}\left(w'\right)^2 - m\,h\,w'' - (2m-1)\,w\,w' = 0 \tag{36}$$

According to homogeneous balance between ww' and w'', we have $N=1$. So we can write solution of eq.(32) in the following form,

$$u(\xi)=w^{1/m-1}(\xi) \text{ where } w(\xi)=a_0+a_{-1}F^{-1}(\xi)+a_1F(\xi) \tag{37}$$

Substituting eq.(37) into eq.(36) and using eq.(6), the left-hand side of eq.(36) can be converted into a finite series in $F^p(\xi)$, ($p=-4,-3,-2,-1,0,1,2,3,4$). Equating each coefficient of $F^p(\xi)$ to zero yields the following set of algebraic equations:

$$\left.\begin{aligned}
&F^{-4}(\xi): -2A^2hma_{-1}^2 - \frac{A^2hma_{-1}^2}{m-1} - Aa_{-1}^3 + 2Ama_{-1}^3 = 0\\
&F^{-3}(\xi): -2A^2hma_0a_{-1} - 3ABhma_{-1}^2 - \frac{2ABhma_{-1}^2}{m-1} - 2Aa_0a_{-1}^2\\
&\quad + 4Ama_0a_{-1}^2 - Ba_{-1}^3 + 2Bma_{-1}^3 = 0\\
&F^{-2}(\xi): -3ABhma_0a_{-1} - Aa_0^2a_{-1} + 2Ama_0^2a_{-1} - B^2hma_{-1}^2 - 2AChma_{-1}^2\\
&\quad - \frac{B^2hma_{-1}^2}{m-1} - \frac{2AChma_{-1}^2}{m-1} - 2Ba_0a_{-1}^2 + 4Bma_0a_{-1}^2 - Ca_{-1}^3 + 2Cma_{-1}^3\\
&\quad - 2A^2hma_{-1}a_1 + \frac{2A^2hma_{-1}a_1}{m-1} - Aa_{-1}^2a_1 + 2Ama_{-1}^2a_1 - Aa_{-1}v = 0\\
&F^{-1}(\xi): -B^2hma_0a_{-1} - 2AChma_0a_{-1} - Ba_0^2a_{-1} + 2Bma_0^2a_{-1} - BChma_{-1}^2\\
&\quad - \frac{2BChma_{-1}^2}{m-1} - 2Ca_0a_{-1}^2 + 4Cma_0a_{-1}^2 - 4ABhma_{-1}a_1 + \frac{4ABhma_{-1}a_1}{m-1}\\
&\quad - Ba_{-1}^2a_1 + 2Bma_{-1}^2a_1 - Ba_{-1}v = 0\\
&F^{0}(\xi): -BChma_0a_{-1} - Ca_0^2a_{-1} + 2Cma_0^2a_{-1} - \frac{C^2hma_{-1}^2}{m-1} - ABhma_0a_1 + Aa_0^2a_1\\
&\quad - 2Ama_0^2a_1 - 2B^2hma_{-1}a_1 - 4AChma_{-1}a_1 + \frac{2B^2hma_{-1}a_1}{m-1} + \frac{4AChma_{-1}a_1}{m-1}\\
&\quad - Ca_{-1}^2a_1 + 2Cma_{-1}^2a_1 - \frac{A^2hma_1^2}{m-1} + Aa_{-1}a_1^2 - 2Ama_{-1}a_1^2 - Ca_{-1}v + Aa_1v = 0\\
&F^{1}(\xi): -B^2hma_0a_1 - 2AChma_0a_1 + Ba_0^2a_1 - 2Bma_0^2a_1 - 4BChma_{-1}a_1\\
&\quad + \frac{4BChma_{-1}a_1}{m-1} - ABhma_1^2 - \frac{2ABhma_1^2}{m-1} + 2Aa_0a_1^2 - 4Ama_0a_1^2 + Ba_{-1}a_1^2\\
&\quad - 2Bma_{-1}a_1^2 + Ba_1v = 0\\
&F^{2}(\xi): -3BChma_0a_1 + Ca_0^2a_1 - 2Cma_0^2a_1 - 2C^2hma_{-1}a_1 + \frac{2C^2hma_{-1}a_1}{m-1} - B^2hma_1^2\\
&\quad - 2AChma_1^2 - \frac{B^2hma_1^2}{m-1} - \frac{2AChma_1^2}{m-1} + 2Ba_0a_1^2 - 4Bma_0a_1^2 + Ca_{-1}a_1^2\\
&\quad - 2Cma_{-1}a_1^2 + Aa_1^3 - 2Ama_1^3 + Ca_1v = 0\\
&F^{3}(\xi): -2C^2hma_0a_1 - 3BChma_1^2 - \frac{2BChma_1^2}{m-1} + 2Ca_0a_1^2 - 4Cma_0a_1^2 + Ba_1^3\\
&\quad - 2Bma_1^3 = 0\\
&F^{4}(\xi): -2C^2hma_1^2 - \frac{C^2hma_1^2}{m-1} + Ca_1^3 - 2Cma_1^3 = 0
\end{aligned}\right\} \tag{38}$$

Solving the above algebraic equations by Mathematica, we considered different cases as follows:
Case 1: $A=0$, we have

$$a_0=-\frac{Bhm}{2(m-1)},a_{-1}=0,a_1=-\frac{Chm}{m-1},v=\frac{B^2h^2m^2}{4(m-1)^2} \tag{39}$$

Assuming, $h=2\left(\frac{m-1}{m}\right)$ in eq.(39), we get,

$$a_0=-B,a_{-1}=0,a_1=-2C,v=B^2 \tag{40}$$

Case 2: $B=0$, we have

$$a_0=0,a_{-1}=0,a_1=-\frac{Chm}{m-1},v=-\frac{ACh^2m^2}{(m-1)^2} \tag{41}$$

Assuming, (1) $h=2\left(\frac{m-1}{m}\right)$ (2) $h=\frac{m-1}{m}$ and (3) $h=-\frac{m-1}{m}$ respectively in eq.(41), we get,

(1) $a_0=0,a_{-1}=0,a_1=-2C,v=-4AC$ (42)

(2) $a_0=0,a_{-1}=0,a_1=-C,v=-AC$ (43)

(3) $a_0=0,a_{-1}=0,a_1=C,v=-AC$ (44)

Case 3: $A=B=0$, we have

$$a_0=0,a_{-1}=0,a_1=-\frac{Chm}{m-1},v=0 \tag{45}$$

Using cases 1 to 3 and appendix, we obtained the following exact generalized travelling wave solutions of eq.(32).

<u>Soliton-like solutions:</u>

(1) $A=0,B=1,C=-1$, from eq.(40) and appendix, we have

$$u_1(x,t)=\left\{\tanh\left[\frac{m-1}{m}(x+t)\right]\right\}^{1/m-1} \tag{46}$$

(2) $A=0,B=-1,C=1$, from eq.(40) and appendix, we have

$$u_2(x,t)=\left\{\coth\left[\frac{m-1}{m}(x+t)\right]\right\}^{1/m-1} \tag{47}$$

(3) $A=\frac{1}{2},B=0,C=-\frac{1}{2}$, from eq.(42) and appendix, we have

$$u_3(x,t)=\left\{\coth\left[\frac{2(m-1)}{m}(x+t)\right]\pm\operatorname{csc}h\left[\frac{2(m-1)}{m}(x+t)\right]\right\}^{1/m-1} \tag{48}$$

$$u_4(x,t)=\left\{\tanh\left[\frac{2(m-1)}{m}(x+t)\right]\pm i\operatorname{sec}h\left[\frac{2(m-1)}{m}(x+t)\right]\right\}^{1/m-1} \tag{49}$$

(4) $A=1,B=0,C=-1$, from eq.(43) and appendix, we have same solutions given by eq.(46) and eq.(47).

<u>Trigonometric function solutions :</u>

(5) $A=\frac{1}{2},B=0,C=\frac{1}{2}$, from eq.(44) and appendix, we have

$$u_5(x,t)=\left\{-\frac{1}{2}\left(\sec\left[\frac{m-1}{m}\left(x-\frac{t}{4}\right)\right]+\tan\left[\frac{m-1}{m}\left(x-\frac{t}{4}\right)\right]\right)\right\}^{1/m-1} \quad (50)$$

$$u_6(x,t)=\left\{-\frac{1}{2}\left(\csc\left[\frac{m-1}{m}\left(x-\frac{t}{4}\right)\right]-\cot\left[\frac{m-1}{m}\left(x-\frac{t}{4}\right)\right]\right)\right\}^{1/m-1} \quad (51)$$

(6) $A=-\frac{1}{2}, B=0, C=-\frac{1}{2}$, from eq.(44) and appendix, we have

$$u_7(x,t)=\left\{\frac{1}{2}\left(\sec\left[\frac{m-1}{m}\left(x+\frac{t}{4}\right)\right]-\tan\left[\frac{m-1}{m}\left(x+\frac{t}{4}\right)\right]\right)\right\}^{1/m-1} \quad (52)$$

$$u_8(x,t)=\left\{\frac{1}{2}\left(\csc\left[\frac{m-1}{m}\left(x+\frac{t}{4}\right)\right]+\cot\left[\frac{m-1}{m}\left(x+\frac{t}{4}\right)\right]\right)\right\}^{1/m-1} \quad (53)$$

(7) $A=1, B=0, C=1$, from eq.(44) and appendix, we have

$$u_9(x,t)=\left\{-\tan\left[\frac{m-1}{m}(x-t)\right]\right\}^{1/m-1} \quad (54)$$

(8) $A=-1, B=0, C=-1$, from eq.(44) and appendix, we have

$$u_{10}(x,t)=\left\{\cot\left[\frac{m-1}{m}(x-t)\right]\right\}^{1/m-1} \quad (55)$$

<u>Rational solution :</u>

(9) $A=0, B=0, C\neq 0$, from eq.(45) and appendix, we have

$$u_{11}(x,t)=\left\{\frac{Chm}{m-1}\left(\frac{1}{Chx+b}\right)\right\}^{1/m-1} \quad (56)$$

where b is an arbitrary constant.

The solutions given by eq.(46), eq.(47), eq.(54) and eq.(55) obtained by modified F-expansion method and those obtained in [12,15], shows good agreement.

IV. Conclusion

In this paper, the modified F-expansion method is implemented to produce new exact travelling wave solutions of the convection diffusion equation. The used method is straightforward and concise. Moreover, the obtained solutions satisfy given convection diffusion equation and reveal that this method is promising mathematical tool because it can provide a different class of new travelling wave solutions which may be of important significance for the explanation of some relevant physical problems in mathematical physics and engineering.

Appendix: Relations between values of (A, B, C) and corresponding $F(\xi)$ in Riccati equation

$$F'(\xi)=A+BF(\xi)+CF^2(\xi)$$

A	B	C	$F(\xi)$
0	1	−1	$\frac{1}{2}+\frac{1}{2}\tanh\left(\frac{\xi}{2}\right)$
0	−1	1	$\frac{1}{2}-\frac{1}{2}\coth\left(\frac{\xi}{2}\right)$

$\frac{1}{2}$	0	$-\frac{1}{2}$	$\coth(\xi) \pm \operatorname{csc}h(\xi), \tanh(\xi) \pm i \operatorname{sec}h(\xi)$
1	0	-1	$\tanh(\xi), \coth(\xi)$
$\frac{1}{2}$	0	$\frac{1}{2}$	$\sec(\xi)+\tan(\xi), \csc(\xi)-\cot(\xi)$
$-\frac{1}{2}$	0	$-\frac{1}{2}$	$\sec(\xi)-\tan(\xi), \csc(\xi)+\cot(\xi)$
$1(-1)$	0	$1(-1)$	$\tan(\xi), \cot(\xi)$
0	0	$\neq 0$	$-\frac{1}{C\xi+b}$ (b is an arbitrary constant)

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