

# Mathematical Modelling of an 3 Phase Induction Motor Using MATLAB/Simulink

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**Abstract:** Mechanical energy is needed in the daily life use as well as in the industry. Induction motors play a very important role in both worlds, because of low cost, reliable operation, robust operation and low maintenance. To derive the mathematical model of a 3 phase Induction motor, the theory of reference frames has been effectively used as an efficient approach. Dynamic models (mathematical models) are employed in to better understand the behaviour of induction motor in both transient and steady state. The dynamic modelling sets all the mechanical equations for the inertia, torque and speed versus time. It also models all the differential voltage, currents and flux linkages between the stationary stator as well as the moving rotor. This paper presents a step by step Matlab/Simulink implementation of an induction machine using dq0 axis transformations of the stator and rotor variables in the arbitrary reference frame [1].

**Keywords:** Reference frames, Dynamic models, dq0 axis transformations, Matlab/Simulink.

## I. INTRODUCTION

The dynamic simulation is one of the key steps in the validation of the design process of the motor-drive system, which eliminates the designing mistakes and the resulting errors in the prototype construction and testing. The dynamic model of the induction motor in direct, quadrature, and zero-sequence axes can be derived from fundamental equations of transformation. The dynamic analysis of the symmetrical induction machines in the arbitrary reference frame has been intensively used as a standard simulation approach from which any particular mode of operation may then be developed. Matlab/Simulink has an advantage over other machine simulators in modelling the induction machine using dq0 axis transformation. Generally modelling of these equations is considered difficult so that in this paper they are presented in their simplified form. The transformations used at various steps are based on simple trigonometric relationship obtained as projections on a set of axes. The dynamic model is used to obtain transient responses, small signal equations, and a transfer function of induction motor. Dynamic models (mathematical models) are employed in to better understand the behaviour of induction motor in both transient and steady state.

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## II. THEORETICAL BACKGROUND

### A. Assumptions

There are few assumptions are to be made while deriving mathematical model of a 3-phase Induction Motor. They are listed below.

1. Uniform air gap.
2. Squirrel cage type construction.
3. Balanced stator and rotor windings, with sinusoidally distributed winding.
4. Saturation and parameter change are neglected.[1,2,3]

The steady state model and equivalent circuit of the Induction Motor is useful for studying the performance of the machine in steady state. This implies that all electrical transients are neglected during load changes or stator frequency variations. Such variations arise in application involving variable-speed drives. The variable-speed drives are converter fed from finite sources, unlike the utility sources, due to limitations of the switch ratings and filter sizes. This results in their incapability to supply large transient power. Hence, we need to evaluate the dynamics of converter-fed variable-speed drives to assess the adequacy of the converter switches and the converters for a given number of motor and their interaction to determine the excursions of currents & torque in the converter and motor.

**B. Equivalent Circuit of an Induction Motor**

The voltage and torque equations that describe the dynamic behaviour of an induction motor are time-varying. It is successfully used to solve such differential equations and it may involve some complexity. A change of variables can be used to reduce the complexity of these equations by eliminating all time-varying inductances, due to electric circuits in relative motion, from the voltage equations of the machine [1, 2, 3, 4].

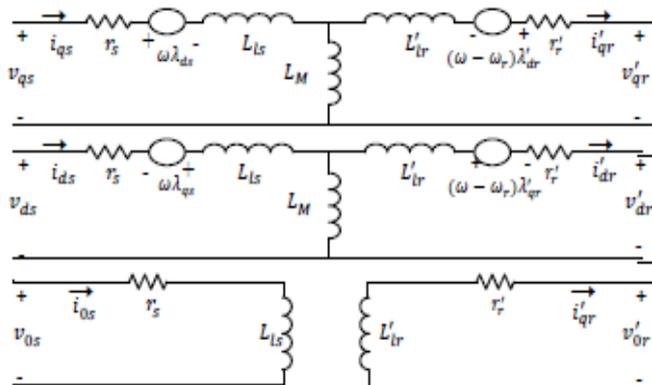


Fig. 1 A dq0 equivalent circuit of an Induction Motor

The above figure shows a dq0 equivalent circuit of an Induction motor. The circuit comprise of various time varying inductances which are to be simulated to analyse the dynamic performance of the 3-phase Induction motor.

**III. INDUCTION MOTOR MODEL**

**A. Generalised Concept in an Arbitrary Reference Frame**

Reference frame are very much like observer platforms, in that each of the platforms gives a unique view of the system at hand as well as a dramatic simplification of the system equation. For example, for the purpose of control, it is desirable to have the system variables as dc quantities, although the actual variables are sinusoidal. This could be accomplished by having a reference frame revolving at the same angular speed as that of the sinusoidal variable. As the reference frames are moving at an angular speed equal to the angular speed equal to angular frequency of sinusoidal supply, so that differential speed between them is reduced to zero, resulting in the sinusoid signal behaving as DC signal from the reference frames. So by moving that lane, it becomes easier to develop a small-signal equation out of a nonlinear equations, as the operating point is described only by DC values; this then leads to linearized system around operating point. Such advantages are many from using reference frame s. instead of deriving the transformation for each and every particular reference frame; it is advantageous to derive general transformation for an arbitrary rotating reference frame. Then any particular reference frame model can be derived by substituting the appropriate frame speed and position in the generalized reference model.

**B. Three phase to Two phase Transformation**

The dynamic model of induction motor can be derived if the equivalence between three-phase and two-phase machine is studied. The equivalence is based on the quantity of MMF produced in two-phase and three-phase windings along with equal current magnitudes. Assuming that each of the three-phase winding has  $N_s$  turns per phase and equal currents magnitudes, the two-phase winding will have  $3N_s/2$  turns per phase for MMF equality. The d and q axes MMF are found by resolving MMF of the three-phase along d and q axes. The common term, the number of turns is cancelled on both sides of equations leaving behind the current equalities. The three-phase stator voltages of an induction machine under balanced conditions can be expressed as, [2,3,4]

$$V_a = \sqrt{2}V_{rms} \sin(\omega t);$$

$$V_b = \sqrt{2}V_{rms} \sin(\omega t - 2\pi / 3);$$

$$V_c = \sqrt{2}V_{rms} \sin(\omega t + 2\pi/3);$$

Here  $V_a$  ,  $V_b$  &  $V_c$  are the three line voltages.

The relationship between  $\alpha\beta$  and abc is as follows.

$$\begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

Then, the direct and quadrature axes voltages are

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix}$$

The instantaneous values of the stator and rotor currents in three-phase system are ultimately calculated using the following transformation;

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix}$$

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$$

### C. Equations of Flux Linkages & Simulink Model

The dynamic model of induction motor in arbitrary reference frame can be represented by using flux linkages as variables. This involves the reduction of a number of variables in dynamic equations, which greatly facilitates their solution by using analogue and hybrid computers. Even when the voltages and currents are discontinuous, the flux linkages are continuous. This gives the advantage of differentiating these variables with numerical stability. In addition, the flux linkages representation is used in motor drives to highlight the process of the decoupling of the flux and torque channels in the induction and synchronous machine.

The various flux linkage equations are expressed as [4],

$$\frac{d\Psi_{qs}}{dt} = \omega_b \left[ V_{qs} - \left( \frac{\omega_e}{\omega_b} \right) \Psi_{ds} - \left( \frac{R_s}{X_{ls}} \right) (\Psi_{mq} - \Psi_{qs}) \right]$$

$$\frac{d\Psi_{ds}}{dt} = \omega_b \left[ V_{ds} - \left( \frac{\omega_e}{\omega_b} \right) \Psi_{qs} - \left( \frac{R_s}{X_{ls}} \right) (\Psi_{md} - \Psi_{ds}) \right]$$

$$\frac{d\Psi_{qr}}{dt} = \omega_b \left[ V_{qr} - \left( \frac{\omega_e - \omega_r}{\omega_b} \right) \Psi_{dr} - \left( \frac{R_r}{X_{lr}} \right) (\Psi_{mq} - \Psi_{qr}) \right]$$

$$\frac{d\Psi_{dr}}{dt} = \omega_b \left[ V_{dr} - \left( \frac{\omega_e - \omega_r}{\omega_b} \right) \Psi_{qr} - \left( \frac{R_r}{X_{lr}} \right) (\Psi_{md} - \Psi_{dr}) \right]$$

Where

$$\Psi_{mq} = X_{ml} \left[ \frac{\Psi_{qs}}{X_{ls}} + \frac{\Psi_{qr}}{X_{lr}} \right]$$

$$\Psi_{md} = X_{ml} \left[ \frac{\Psi_{ds}}{X_{ls}} + \frac{\Psi_{dr}}{X_{lr}} \right]$$

Then substituting the values of the flux linkages to find the currents,

$$i_{qs} = \frac{1}{X_{ls}} (\Psi_{qs} - \Psi_{mq}) X_{ml} = 1 / \left[ \frac{1}{X_m} + \frac{1}{X_{ls}} + \frac{1}{X_{lr}} \right]$$

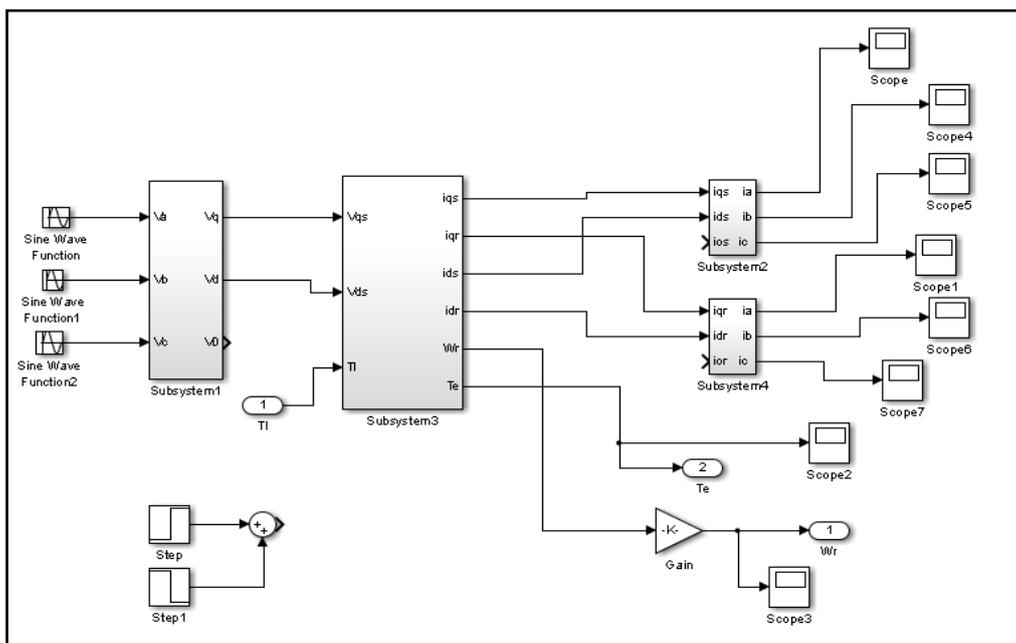


Fig. 2 Overview of the complete dynamic model in Matlab/Simulink environment

$$i_{qr} = \frac{1}{X_{lr}} (\Psi_{qr} - \Psi_{mq})$$

$$i_{ds} = \frac{1}{X_{ls}} (\Psi_{ds} - \Psi_{md})$$

$$i_{dr} = \frac{1}{X_{lr}} (\Psi_{dr} - \Psi_{md})$$

Based on the above equations, the Electromagnetic torque and rotor speed can be determined as follows,

$$T_e = \left(\frac{3}{2}\right) \left(\frac{P}{2}\right) \left(\frac{1}{\omega_b}\right) (\Psi_{ds} i_{qs} - \Psi_{qs} i_{ds})$$

$$\omega_b = \int \left(\frac{P}{2J}\right) (T_e - T_l)$$

With P = no of poles of Machine.

$T_e$  = Electromagnetic torque developed

#### IV. MATLAB/SIMULINK RESULTS

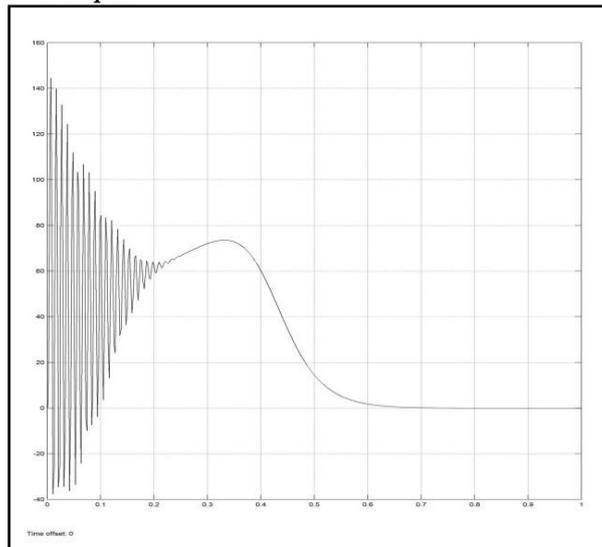
The induction motor of 3 hp was tested in this simulated model. The results of the simulation are given for the first induction motor with the following specifications:

Input Parameters:[5]

1	Hp Rating	3
2	$V_{line}$	220 Volts
3	f	50
4	$R_s$ & $R_r$	0.435 $\Omega$
5	$X_{ls}$	0.816 $\Omega$
6	J	0.089
7	$X_m$	26.13 $\Omega$
8	RPM	1500

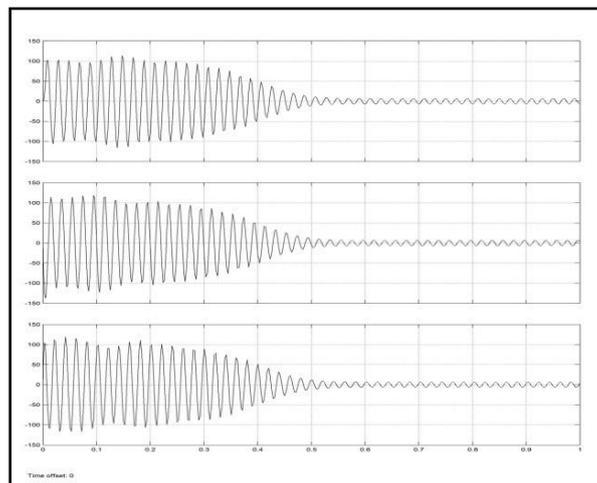
The various performance parameters of an Induction motor are the electromagnetic torque developed, rotor speed, and stator and rotor currents in each of three phase windings.

**A. Electromagnetic Torque Developed**



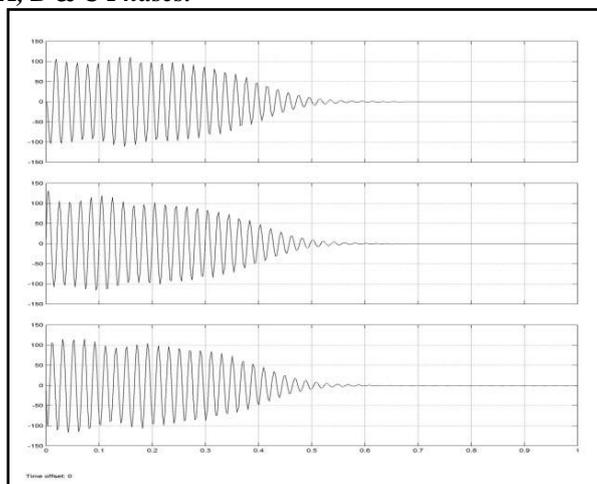
*Fig. 3 Electromagnetic Torque Developed*

**B. Stator Currents In A, B & C Phases**



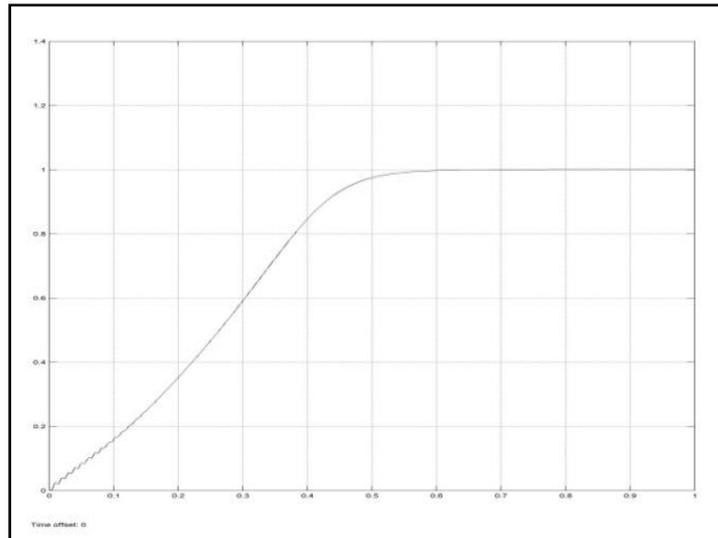
*Fig. 4 Stator currents in A, B & C phases*

**C. Rotor Current in Each A, B & C Phases.**



*Fig. 5 Rotor currents in A, B & C phases*

**D. Speed of Rotor in Per Unit with Base Speed as 1500 Rpm.**



*Fig. 5 Speed of Rotor*

**V. Conclusion**

In this paper, an implementation and dynamic modelling of a three-phase induction motor using Matlab/Simulink is presented in a step-by-step manner. The model was tested by 3 hp induction motor.[5,6] The simulated machine has given a satisfactory response in terms of the torque and speed characteristics. This concludes that the Matlab/Simulink is a reliable and sophisticated way to analyse and predict the behaviour of induction motors using the theory of reference frames.

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