

Experimental and Theoretical Investigation of Dynamic Response on a Cantilever Beam Submerged In Fluid

Ch. Ratnam¹, G. Nageswara Rao², BSN Murthy³ and M. Rajashekhar⁴

¹Professor, Dept. of Mechanical Engineering, Andhra University, Visakhapatnam, India.

²Assistant Professor, Dept. of Mechanical Engineering, Vignan's Lara Inst.Of Tech. & Science, Guntur, India.

³Associate Professor, Dept. of Mechanical Engineering, GITAM University, Visakhapatnam, India

⁴Assistant Professor, Dept. of Mechanical Engineering, GITAM University, Visakhapatnam, India

ABSTRACT:- Vibration of submerged structures requires a deterministic knowledge of vibration analysis and structures coupling. The vibrational characteristics such as the natural frequencies of systems are depending on the fluid in which the beam is submerged. In general the natural frequencies are decrease in reduction in stiffness due to damage. But the natural frequencies of structure would also change as a result of changes in mass. In this paper the influence of pressure force can be considered as added mass which is derived analytically from the hydrostatic pressure force for a submerged cantilever beam at different depths are studied. The results are obtained theoretical by using finite element method (FEM) with considering added mass from analytical method to the structural mass. It is observed that the natural frequencies decrease with increasing added mass from finite element method. Experimentally measured frequency response is also decreases with increasing depth of immersion of cantilever beam in water by using Laser Doppler vibrometer(LDV). The results are presented in the form of tables and graphs for both cases.

Keywords:- natural frequencies, hydrostatic pressure force, cantilever beam, LDV,FEM.

I. INTRODUCTION

In practice there exist several fluid interaction structures like ships, turbines, Dams, Gates and Tanks are subjected to hydrostatic pressure forces. In the design of these structures it is therefore necessary to compute the magnitude of hydrostatic force and to locate their points of application on the structures. In this paper, the effect of hydrostatic force on the submerged structure is represented as added mass which influence the natural frequencies of the structures.

The dynamic response is used directly in conventional damage detection methods which are measured from experiment and/or numerical techniques. An overview of the various damage detection methods using system's modal parameters was given by Doebling et al. to estimate the crack size and location [1]. Most of the authors utilized frequency changes which are obtained from both analytical and experimental modal tests. Srinivas, et al. [2-9] employed modal data to identify damage location and extent in residual force concept. The damage estimation methods are predominately based on the change in natural frequencies. Liang.etal [10] evaluate the added mass effect of water on Francis turbine runner by using numerical modal analysis with FEM method considering the surrounding fluid domain by comparing the frequencies in air and in water. The natural frequencies are considerably reduced by the presence of water. Rodriguezet al [11] studied experimentally the influence of still water on the modal characteristics of a reduced scale model of a Francis turbine. The authors noticed there is change in modal properties. Anita Kaczor [12] addresses the free vibration analysis of floating plates. The action of the liquid on the vibrating plate was described by the boundary integral equation. The liquid mass matrix is calculated which is added to the plate mass matrix. Estimation of natural frequency of the bearing system was explained by Pol et al[13] on principle of hydrodynamic mass of fluid. The hydrodynamic concept was described by Lamb, Stokes, Batton & Brikhooff and other investigators. They concluded the natural frequency is related to inverse of density.Fushun Liu et al[14] used frequency response of structures to estimate the added mass matrix by conducting experiment also. The frequency response of the beams depending on the fluid in which immersed Erdal Uzunlar et al[15] reported frequency response of microcantilevers immersed in gaseous, liquid and supercritical carbon dioxide.AwladHossain[16] address the prediction of density and viscosity of fluid media by using experimentally measured frequency response of partially submerged mini-cantilever beams.Hence estimation of dynamic response of fluid interacted systems plays avital role in prediction of damaged structures in fluid

II. MATHEMATICAL BACKGROUND

The governing equation of motion of a beam having n degrees of freedom for forced vibration can be written as

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = F(t) \quad (1)$$

Where $[M]$, $[C]$ and $[K]$ are mass, damping and stiffness matrices of the system, $\{x(t)\}$ is displacement vector, and $F(t)$ is applied load vector. Neglecting damping and force in Eq.(1) as a free undamped vibration problem. The required characteristic equation can be derived from Eq. (1) as follows

$$[K - \lambda_i M]\phi_i = 0 \quad (2)$$

Where $[K]$ and $[M]$ are the global stiffness and mass matrix, λ_i and ϕ_i are the i^{th} mode eigenvalue and eigenvector respectively.

2.1 Finite Element Formulation

The cantilever beam is modeled for Finite Element Method. The elemental stiffness and mass matrix for beam are formulated in the following section. The elemental stiffness and mass matrix are given as

$$k_e = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ 6l_e & 4l_e^2 & -6l_e & 2l_e^2 \\ -12 & -6l_e & 12 & -6l_e \\ 6l_e & 2l_e^2 & -6l_e & 4l_e^2 \end{bmatrix}$$

$$m_e = \frac{\rho A l}{420} \begin{bmatrix} 156 & 22l_e & 54 & -13l_e \\ 22l_e & 4l_e^2 & 13l_e & -3l_e^2 \\ 54 & 13l_e & 156 & -22l_e \\ -13l_e & -3l_e^2 & -22l_e & 4l_e^2 \end{bmatrix}$$

Where E = Modulus of elasticity, I = moment of inertia of the section, l_e = length of element, ρ = mass density of the beam material, A = cross-sectional area of the beam element.

2.2 Analytical method

The equation of motion of a multi degree of freedom system can be written as

$$\frac{\partial^4 y}{\partial x^4} + \frac{\rho A}{EI} \frac{\partial^2 y}{\partial t^2} = 0 \quad (1)$$

Where y is the displacement of the beam at distance x measured from the free end, considering that y depends on distance x and time t , and evolution in time is harmonically y can be written

$$Y = x(x)T(t) = x \sin \omega t \quad (2)$$

The solution of equation for cantilever beam after derivation and substitution in equation(1) by imposing boundary conditions is given as

$$1 + \cos \lambda \cdot \cosh \lambda = 0 \quad (3)$$

With $\lambda = \alpha l$ which permits to calculate the λ_i values for i vibrations modes of cantilever beam. The angular frequencies of the cantilever beam analytically given as

$$\omega_i = (\alpha l)^2 \sqrt{\frac{EI}{\rho A l^4}} \quad (4)$$

2.3. Determination of Added Mass

When a structure vibrates in a fluid which is subjected to a hydrostatic force. The pressure force acting on the submerged beam is given as

$$P = wAx \quad (5)$$

Where w = weight density of the fluid, A = cross sectional area of the submerged beam, x is the vertical distance from the surface of the liquid to the centroid of the beam. The natural frequencies of the beam with added mass given as

$$\omega_i = (\alpha l)^2 \sqrt{\frac{EI}{M l^3}} \quad (\text{rad/sec}) \quad (6)$$

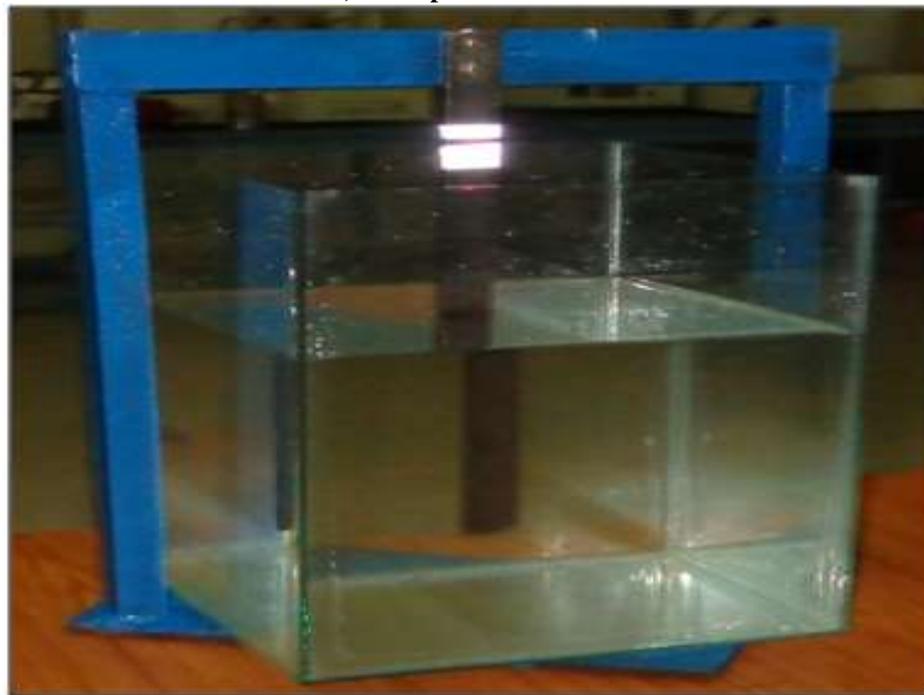
Where $M = m_s + m_a$, M is total mass and m_s is mass of structure, m_a is added mass

III. EXPERIMENTAL PROCEDURE

The frequency response of the cantilever beam is measured experimentally. An illustrative case of a cantilever beam frequency response is measured by using Laser Doppler vibrometer (LDV). Experimentation is conducted on cantilever beam in air and in water then repeated the procedure for partially submerged beam in water. The beam was excited with impact hammer and recorded the vibration signal in LDV. Later, the recorded signal was analyzed by signal processing in VibSoft to measure the frequency response. Data were recorded for both in air and partially submerged beam in water to assess the effect of water on dynamic characteristics of systems. The experimental setup for measuring natural frequency using LDV as shown in Fig.1



a) Setup with Vibraometer



b) Cantilever immersed in water
Fig. Experimental Setup

IV. RESULTS AND DISCUSSIONS

4.1. Finite element analysis

The methodology is illustrated with considering a cantilever beam, which is discretised into 10 elements. This leads to a finite element model containing 20 degrees of freedom (10 translations and 10 rotations) and the beam is depicted in Fig.2. The effect of added mass of beam structure on the modal properties such as natural frequencies is studied. The analyzed beam has the following geometrical dimensions. modulus of elasticity $E = 207 \text{ GPa}$, cross sectional area $A = 1.5 \times 10^{-4} \text{ m}^2$, moment of inertia $I = 4.5 \times 10^{-10} \text{ m}^4$, density $\rho = 7800 \text{ kg/m}^3$ and total length of the beam $l = 0.385 \text{ m}$. Two different situations for this case are considered as i) the beam is without added mass, ii) the beam is with added mass. Now finite element analysis is performed to

solve the eigenproblem of these conditions and the modal data are shown in Table 1 and Fig.3. It is observed from numerical technique that the natural frequency decreases with increase in added mass.

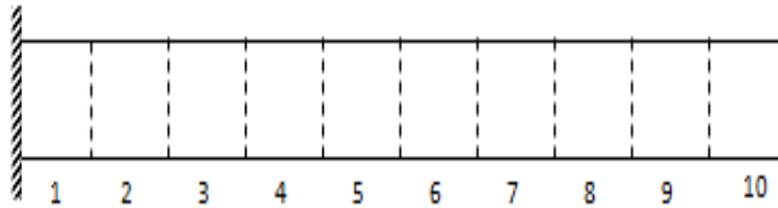


Fig. 2.Cantilever Beam

Table 1 The natural frequencies of the cantilever beam with added mass of 20%,30% and 40% to the structural mass

Sr.No	In air	% of mass added to the structural mass		
		20%	30%	40%
1	34.65	34.02	34.01	34.00
2	211.63	207.56	204.74	202.98
3	593.61	573.87	564.88	555.27
4	1172.69	1151.54	1141.63	1132.13
5	1945.87	1909.43	1892.01	1876.90
6	3231.63	3177.95	3154.64	3133.42

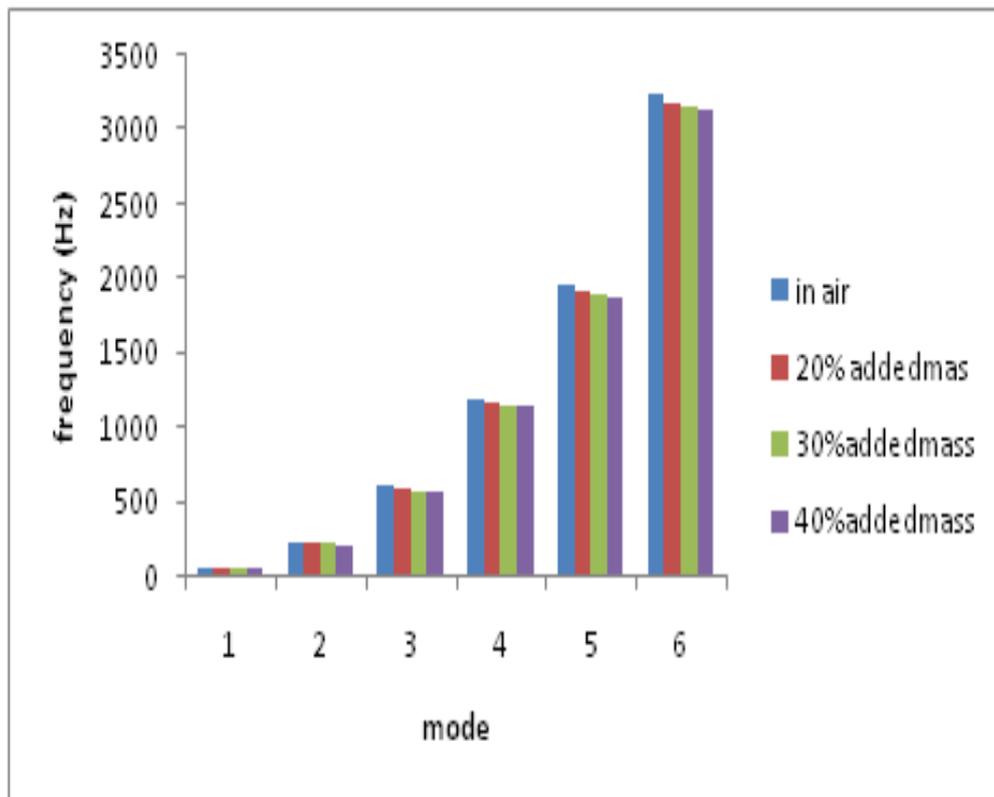
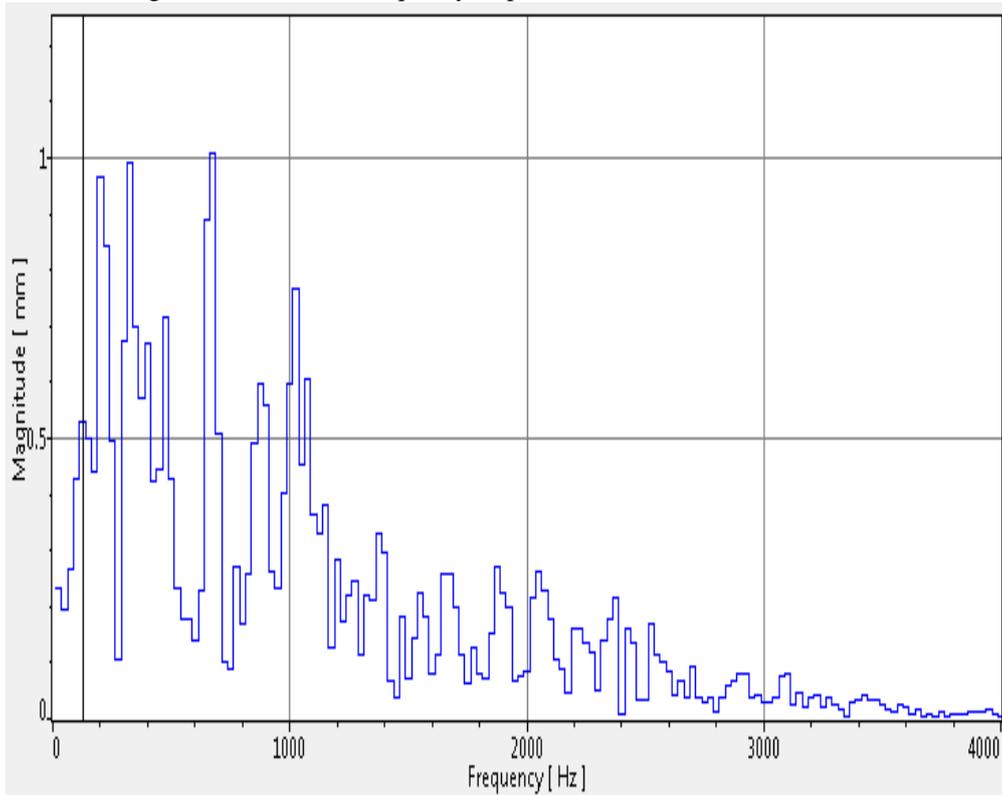


Fig.3. comparison of frequency response with and without added mass

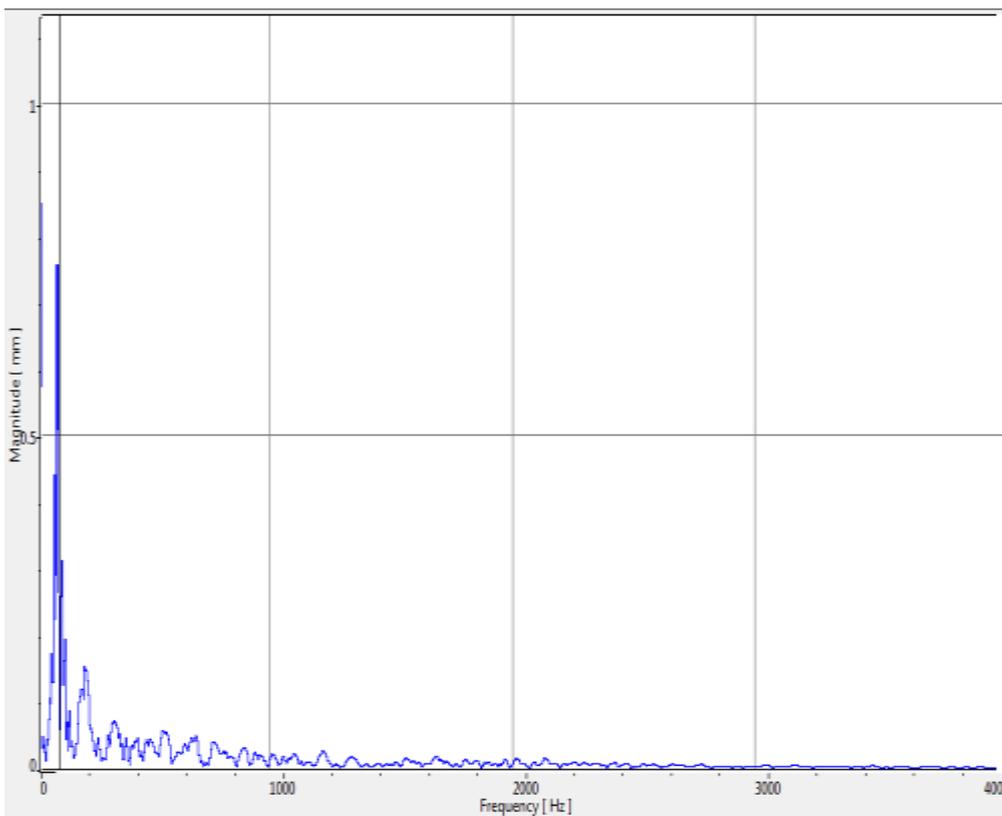
4.2. Experimental measurements

The natural frequencies of the beam in air and partially submerged in water were measured experimentally by using laser Doppler vibrometer (LDV). The frequency of each case was obtained from the

FFT spectrum analyzer. It is observed from experiment that the natural frequency decreases with increase in depth of immersion. Fig.4 is the measured frequency response in air and in water



a) Frequency response of the cantilever beam in air



b) Frequency response of the cantilever beam submerged in water

Fig.4 Measured frequency response

V. CONCLUSIONS

In this paper, a finite element formulation was adopted and then a computer code was developed in MATLAB for calculating and simulating for added mass to cantilever beam. If the added mass is considered on the cantilever beam, the frequencies of the beam are reduced. Experimentation was conducted on cantilever beam using Laser Doppler Vibrometer. The frequency of the partially submerged cantilever beam decreases with increase in the depth of immersion in water. The frequency of the cracked cantilever beam decreases with increase in the crack size for all modes of vibration. Experimental values are coinciding very close with the theoretical values.

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