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Free Vibration of Skew Laminated Composite Plates with Circular Cutout by Finite Element Method

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ABSTRACT

The free vibration of skew laminated composite plates with circular cutout is investigated by using finite element method based on a first order shear deformation theory with the help of ANSYS 14.5 – a commercial finite element program. The analysis was carried out by using eight noded isoparametric shell element. Simply supported and clamped boundary conditions are considered. The results are presented for 15º and 30º skew laminated composite plates with circular cutout. The effects of number of plies, boundary conditions, side to thickness ratios and skew angles on free vibration behavior of cross-ply and angle-ply plates are discussed.

Keywords: *ANSYS, Circular cutout, Finite element method, First order shear deformation theory, Free vibration, Skew laminated composite plates*

I. INTRODUCTION

Laminated composite plates are used commonly in aerospace structures (air craft/ space craft), civil engineering structures (liquid retaining structures), marine vehicles (ships) and nuclear structures (nuclear plants) as they are light-weight and have high specific strength and stiffness. In modern era of composites, skew laminated composite plates are also being increasingly used, for example in the wings of aircraft. Cutouts are necessitated mainly for the purposes of connecting or assembling and reducing weight. They are also required to meet the functional requirements such as ventilation, electrical or fuel lines, inspection during damage and so on. The dynamic behavior of structural members is an important problem to be studied by structural designers.

The free vibration of skew isotropic and laminated composite plates was extensively studied by many researchers over the past few decades. Barton [1] studied vibration of rectangular and skew cantilever plates for various conditions. Kaul and Cadambe [2] presented natural frequencies of thin skew plates by Rayleigh–Ritz method. Hasegawa [3] reported results for clamped parallelogram isotropic flat plates by using Rayleigh–Ritz method. Classen [4] modified the study of Barton [1] by following Fourier sine series solution technique which was applied to Rayleigh–Ritz method. Conway and Farnham [5] studied free vibration behavior of rhombic, triangular and parallelogram plates by using point matching method for various skew angles and boundary conditions.

Durvasula [6] presented the frequencies and mode shapes of clamped skew plates using Galerkin's method. Thangam Babu and Reddy [7] studied the free vibration behavior of orthotropic skew plates with two opposite edges simply supported and the other two edges free. Nair and Durvasula [8] presented the natural frequencies of isotropic and orthotropic skew plates for simply supported, clamped, free edge boundary conditions and for combinations of above conditions. Srinivasan and Ramachandran [9] used a numerical method to obtain the natural frequencies of orthotropic skew plates. Mizusuwa et al. [10,11] used the Rayleigh–Ritz method with B-spline functions and investigated the effects of skew angle on natural frequencies of isotropic skew plates. Liew and Lam [12] applied two dimensional orthogonal plate functions to flexural vibration of skew plates. Krishnan and Deshpande [13] used DKT finite element to determine the effects of fiber orientation angle, skew angle, aspect ratio and length-to-thickness ratio on the fundamental frequencies of single layer Graphite/Epoxy and Glass/Epoxy skew plates. McGee and Butalia [14] presented the results for the free vibration of thick and thin cantilever skew plates using C^0 continuous isoparametric quadrilateral element. Krishna Reddy and Palaninathan [15] used a high precision triangular plate bending element for study of free vibration characteristics of laminated composite skew plates by consistent mass matrix. Wang [16] studied free vibration of skew fibre reinforced composite plates for various skew angles and support conditions. Wang [17] studied free vibration of thin skew laminated plates. Garg et al. [18] have studied free vibration studies on isotropic, orthotropic, and layered anisotropic composites and sandwich skew plates using isoparametric finite element model. Srinivasa et al. [19] presented experimental and finite element studies on free vibration of skew laminated composite plates.

From the literature discussed, it can be noticed that researchers studied free vibration skew isotropic and laminated composite plates without cutouts. Presence of cutouts influences the dynamic behavior and hence worth to be studied. In this paper, free vibration analysis of skew laminated composite plates with circular cutouts by finite element method based on first order shear deformation theory (FSDT) with the help of ANSYS is presented. The analysis was performed for 15º and 30º skew plates with cross ply and angle ply fibre orientations. Thin and moderately thick plates with simply supported and clamped boundary conditions have been considered in the study.

II. CONVERGENCE AND VALIDATION

To show the computational efficiency of ANSYS, skew laminated composite plate was analyzed and compared with the results of the study by Wang [16]. 30º skew laminated cross-ply (90º/0º/90º/0º/90º), thin (a/h=1000) plate (a/b=1) with simply supported boundary condition *(* $u_o = v_o = w = 0$ *at x=0, a and y=0, b)* was considered. The material properties are

E1 = 400E9Gpa; E2, E3=10E9Gpa; ν12, ν23, ν31 = 0.25; G12, G31 = 6E9Gpa; G23=5E9Gpa. ρ = 2200 Kg/m^3. Where E = Young's Modulus, ν = Poisson's ratio, G = Shear Modulus and ρ = Density.

Eight noded isoparametric shell element (SHELL 281 in ANSYS) is used. Acceptable agreement of the present results (Table 1) with the results of the study by Wang [16] for 12x12 mesh size, prompted the use of 12 X 12 mesh size for the current study.

Table 1: Convergence study of non-dimensional frequencies ($\acute{\omega} = (\frac{\omega\, a^2}{\Pi^2 h}) *\sqrt{}\ (\rho/E2)$) for a cross- ply (90º/0º/90º/0º/90º), 30º skew laminated plate (a/b=1) with K_s (shear correction factor) = 5/6 for simply supported (SSSS) boundary condition.

a/h Ratio	Mesh Size	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
1000	4x4	3.3007	6.9132	10.7679	13.4851	18.0439	24.1527
	6x6	2.9965	5.6478	9.7735	10.2525	13.9895	17.5345
	8x8	2.9158	5.3291	8.8897	9.4821	12.9065	13.7933
	10x10	2.8825	5.2506	8.5803	9.3622	12.4860	12.5941
	12x12	2.8642	5.2222	8.5054	9.3185	12.2729	12.3042
	Wang [16]	2.8248	5.1891	8.4836	9.2574	12.1070	12.1301

III. FREE VIBRATION OF SKEW LAMINATED COMPOSITE PLATES WITH CIRCULAR CUTOUT BY FINITE ELEMENT METHOD

Free vibration analysis of skew plates (a/b=1) with circular cutout for skew angles (ψ) of 15 º and 30 º (Figure 1) by finite element method based on first-order shear deformation theory (FSDT) with the help of ANSYS was performed. Thin (a/h=1000) and thick (a/h=20) plates were analyzed. Both simply supported and clamped boundary conditions were considered. Other details are presented below.

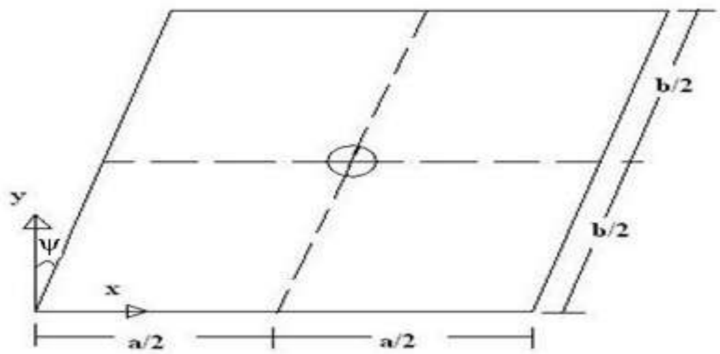

Figure 1: Skew laminated plate with circular cutout

3.1 Element Type

Eight-noded isoparametric shell element (SHELL 281 in ANSYS) was used for entire analysis (Figure 2). The element has six degrees of freedom at each node. The element can be used for analysis of both thin and moderately thick plates and is capable of analyzing layered members such as composite plates/shells.

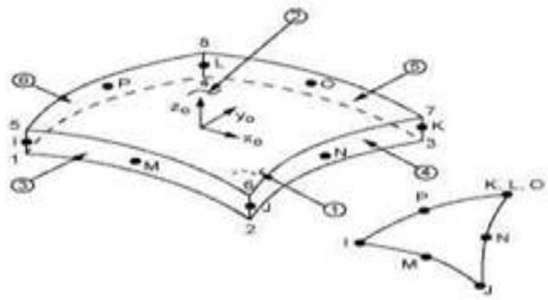

Figure 2: 8 – noded isoparametric shell element

3.2 Material Properties

The material is assumed to have the properties as specified below

E1 = 400E9Gpa; E2, E3=10E9Gpa; ν12, ν23, ν31 = 0.25;

G12, G31 = 6E9Gpa; G23=5E9Gpa. ρ = 2200 Kg/m^3

where E = Young's Modulus, ν = Poisson's ratio, G = Shear Modulus and ρ = Density

3.3 Fibre Orientation

Symmetric cross-ply (90°/0°/90°...) and symmetric angle-ply (-45°/45°/-45°…) were considered.

3.4 Boundary Conditions

(i) Simply supported (SSSS)

$u_o = v_o$ = w = 0 at x=0, a and y=0, b

(ii) Clamped (CCCC)

$u_o = v_o = w = 0;\ w_x = 0$ at x =0, a

$u_o = v_o = w = 0;\ w_y = 0$ at y=0, b

3.5 Solver Type

Block Lanczos method was used for obtaining natural frequencies and the method uses an assembled stiffness and mass matrix in addition to factoring matrices that are a combination of the mass and stiffness matrices computed at various shift points.

IV. RESULTS AND DISCUSSION

The non – dimensional frequencies obtained for 15° and 30° simply supported and clamped skew laminated plates with circular cutout of 50 mm radius for cross-ply (90°/0°/90°…) and angle-ply (-45°/45°/-45°) fibre orientations, were tabulated in the Tables 2 – 5. Frequencies corresponding to first six modes were tabulated for each type. The graphical representation of the frequencies tabulated in tables 2-5 were depicted in figures 3-10. From the analysis of tabulated data and graphs, the following inferences could be drawn.

a. Effect of number of layers: As the number of layers were increased from five to ten, the non-dimensional frequencies of higher magnitude were obtained.
b. Effect of a/h ratio: The magnitudes of non – dimensional frequencies were more for thin plates (a/h=1000) than the non-dimensional frequencies of moderately thick plates (a/h = 20).
c. Effect of Support Condition: Clamped boundary condition yielded higher magnitudes of non-dimensional frequencies rather than simply supported boundary condition.
d. Effect of fiber orientation: First, considering the 15° skew angle, from tables 2 and 3, the fundamental non-dimensional frequencies were found to be higher for angle ply (-45°/45°/-45°…) in most of the types except for ten layered a/h=1000 and a/h=20 clamped types, where cross ply (90°/0°/90°…) resulted in higher magnitudes.

Considering 30° skew angle, (Tables 4 and 5) the fundamental non-dimensional frequencies of cross ply (90°/0°/90°...) were higher for clamped plates irrespective of a/h ratio. In simply supported plates, except for five layered a/h=20 type, for which angle ply (-45°/45°/-45°) resulted in higher magnitude.

e. Effect of Skew angle: The non – dimensional frequencies increased with increase in skew angle from 15° to 30°.
f. Effect of Cutout size: From tables 6 and 7, irrespective of fibre orientation, for 15° skew angle, the non – dimensional fundamental frequencies increased with increase in cutout size – d/b ratio. For 30° skew angle, the non-dimensional fundamental frequencies decreased for d/b ratio of 0.2 and increased for 0.3 ratio.

V. CONCLUSION

The free vibration analysis of 15° and 30° skew laminated plates with circular cutout by finite element method based on first order shear deformation theory is carried out for various conditions with the help of ANSYS. Non – dimensional fundamental frequencies increased with increase in number of layers, a/h ratio and skew angle. Clamped boundary condition resulted in higher magnitudes of non – dimensional frequencies. In 15° skew laminated plates angle-ply resulted in higher magnitudes of non-dimensional fundamental frequencies except for ten layered clamped thin and moderately thick plates, for which cross-ply resulted in higher magnitudes. But for 30° skew laminated plates, cross-ply resulted in higher magnitudes of non-dimensional frequencies except for five layered simply supported moderately thick plate, for which angle-ply resulted in higher magnitudes. With increase in cutout size irrespective of the fibre orientation, for 15° skew angle, the non – dimensional fundamental frequencies increased with increase in cutout size – d/b ratio. For 30° skew angle, the non-dimensional fundamental frequencies decreased for d/b ratio of 0.2 and increased for 0.3 ratio.

Table 2: Non-Dimensional Frequencies ($ώ = (\frac{\omega\, a^2}{\Pi^2 h})*\sqrt{(\rho/E2)}$) of symmetric cross ply (90°/0°/90°…), 15° skew laminated Plate (a/b=1) with circular cut out of 50mm radius

Boundary Condition	a/h ratio	Number of layers	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
SSSS	1000	5	1.9604	4.5835	5.4035	7.6615	9.9676	11.0766
		10	2.3064	5.1001	6.1888	8.0061	11.8837	12.4170
	20	5	1.7217	3.7533	4.3673	5.9132	7.3605	8.0308
		10	1.8844	4.2972	5.0777	6.4773	8.6887	9.1140
CCCC	1000	5	3.7262	7.0640	8.1927	11.1533	13.6046	15.2621
		10	4.4892	8.3301	9.6251	12.4514	16.2678	17.5112
	20	5	2.9345	4.9620	5.5658	7.1843	8.3752	9.1008
		10	3.5772	5.8625	6.5615	8.1859	9.8013	10.4373

Table 3: Non-Dimensional Frequencies ($ώ = (\frac{\omega\, a^2}{\Pi^2 h})*\sqrt{(\rho/E2)}$) of symmetric angle ply (-45°/45°/-45°…), 15° skew laminated Plate (a/b=1) with circular cut out of 50mm radius

Boundary Condition	a/h ratio	Number of layers	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
SSSS	1000	5	2.4270	5.5364	6.3533	9.0703	11.1062	11.1952
		10	2.5453	5.8514	6.5564	9.3871	11.3113	11.5795
	20	5	1.8782	4.2426	4.7488	6.4325	7.7708	7.9547
		10	2.0416	4.5598	5.0356	6.8212	8.1214	8.3692
CCCC	1000	5	4.0210	7.9706	9.10167	12.0826	14.6121	14.9011
		10	4.0714	8.3104	9.3104	12.3205	14.8040	15.250
	20	5	3.0779	5.5262	6.0777	7.7165	8.9921	9.1085
		10	3.2095	5.8390	6.3913	8.0443	9.3558	9.5976

Table 4: Non-Dimensional Frequencies (ώ = $(\frac{\omega\, a^2}{\Pi^2 h})$*√ (ρ/E2)) of symmetric angle ply (90°/0°/90°…), 30° skew laminated Plate (a/b=1) with circular cut out of 50mm radius

Boundary Condition	a/h ratio	Number of layers	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
SSSS	1000	5	2.5399	5.3369	7.5863	8.1838	11.8837	12.5114
		10	2.5682	5.7373	7.8589	8.5412	12.8915	13.9596
	20	5	2.1097	4.1593	5.5692	5.5692	8.3776	8.5955
		10	2.2871	4.7203	6.1363	6.8081	9.3641	9.7905
CCCC	1000	5	4 .9953	8.5914	11.7102	12.3588	16.4535	17.8805
		10	5.4653	9.8628	12.4072	13.8557	13.8557	19.5542
	20	5	3.6257	5.5588	7.06311	7.4889	9.5439	9.7147
		10	4.0995	6.4743	7.7577	8.5937	10.9013	10.9843

Table 5: Non-Dimensional Frequencies (ώ = $(\frac{\omega\, a^2}{\Pi^2 h})$*√ (ρ/E2)) of symmetric angle ply (-45°/45°/-45°…), 30° skew laminated plate (a/b=1) with circular cut out of 50mm radius

Boundary Condition	a/h ratio	Number of Layers	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
SSSS	1000	5	2.6745	5.8081	8.2375	9.4030	12.4842	14.0605
		10	2.8929	6.1270	8.3991	10.1578	13.5771	14.7139
	20	5	2.0619	4.3585	5.7956	6.331	8.7299	9.07867
		10	2.2961	4.7250	6.1166	7.1957	9.47521	9.6615
CCCC	100	5	4.8983	8.5687	12.4469	12.5857	17.1839	18.0113
		10	4.9439	8.8610	12.0901	13.5222	17.9851	18.9301
	20	5	3.5871	5.7478	7.36590	7.36590	10.1662	10.3895
		10	3.7672	6.1078	7.56416	8.5328	10.7658	10.8912

Table 6: Variation of Non – dimensional Frequencies (ώ = $(\frac{\omega\, a^2}{\Pi^2 h})$ *√ (ρ/E2)) of symmetric cross ply (90°/0°/90°/0°/90°), skew laminated clamped plate (a/b=1; a/h=1000) with different circular cutout sizes (d/b = 0.1, 0.2, 0.3)

Skew angle	d/b ratio	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
15°	0.1	3.7262	7.0640	8.1927	11.1533	13.6046	15.2621
	0.2	3.7896	6.9625	8.0778	11.0100	13.3594	15.4093
	0.3	3.9681	3.9681	7.5191	11.2250	13.5792	15.7485
30°	0.1	4.9953	8.5914	11.7102	12.3588	16.4535	17.8805
	0.2	4.9616	7.8105	10.6980	12.4765	15.6957	16.6025
	0.3	5.4220	7.7290	10.6072	12.9903	16.3610	17.0450

Table 7: Variation of Non – dimensional Frequencies (ώ = $(\frac{\omega\, a^2}{\Pi^2 h})$*√ (ρ/E2)) of symmetric angle ply (-45°/45°/-45°/45°/-45°) skew laminated clamped plate (a/b=1; a/h=1000) with different circular cutout sizes (d/b = 0.1, 0.2, 0.3)

Skew angle	d/b ratio	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
15°	0.1	4.0210	7.9706	9.10167	12.0826	14.6121	14.9011
	0.2	4.1413	7.8204	8.6240	11.9378	14.4284	15.7130
	0.3	4.3808	7.3958	8.7633	12.3367	13.8240	16.8695
30°	0.1	4.9953	8.5914	11.7102	12.3588	16.4535	17.8805
	0.2	4.8657	8.6167	11.3077	12.8831	16.8089	17.8462
	0.3	5.6561	8.5899	11.4773	13.8425	17.5069	18.3894

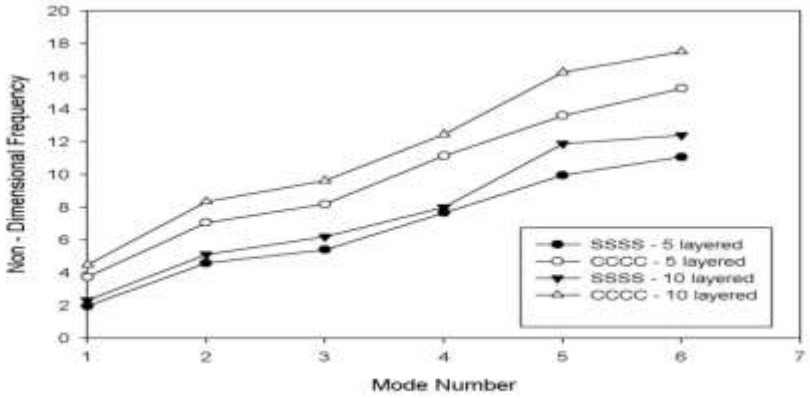

Figure 3: First six natural frequencies ($\acute{\omega} = (\frac{\omega\, a^2}{\Pi^2 h})*\sqrt{}\ (\rho/E2)$) of symmetric five and ten layered cross ply (90°/0°/90°…), 15° skew laminated thin (a/h=1000) plate (a/b=1) with circular cutout for simply supported and clamped boundary conditions (K_s = 5/6).

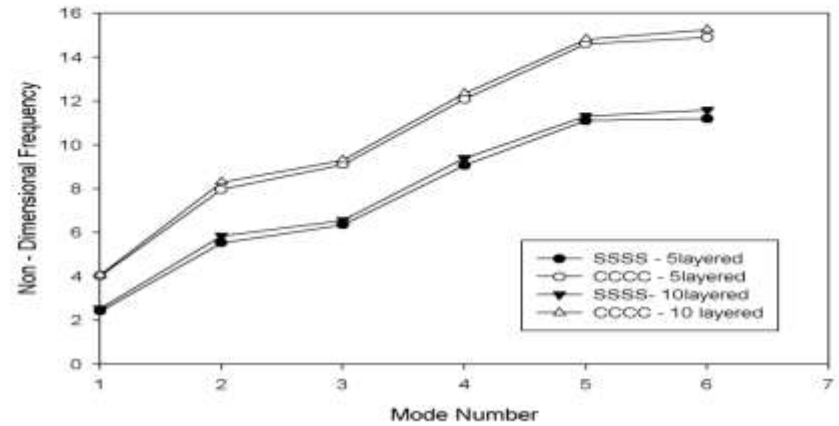

Figure 4: First six natural frequencies ($\acute{\omega} = (\frac{\omega\, a^2}{\Pi^2 h})*\sqrt{}\ (\rho/E2)$) of symmetric five and ten layered angle-ply (-45°/45°/-45°…), 15° skew laminated thin (a/h=1000) plate (a/b=1) with circular cutout for simply supported and clamped boundary conditions (K_s = 5/6).

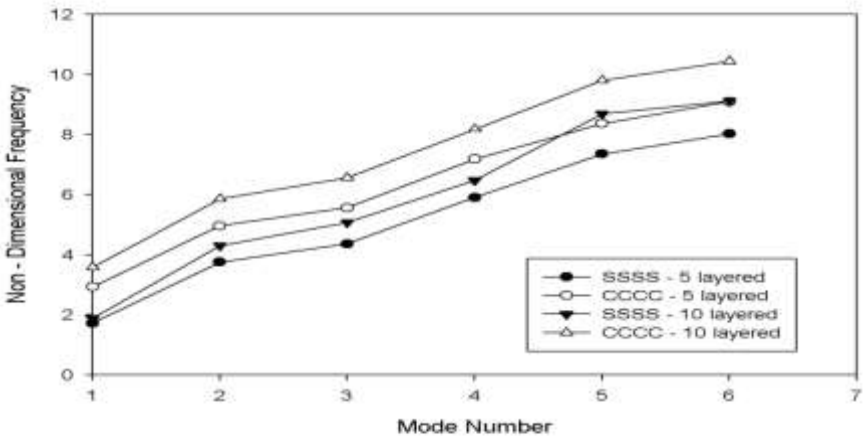

Figure 5: First six natural frequencies ($\acute{\omega} = (\frac{\omega\, a^2}{\Pi^2 h})*\sqrt{}\ (\rho/E2)$) of symmetric five and ten layered cross ply (90°/0°/90°…), 15° skew laminated moderately thick (a/h=20) plate (a/b=1) with circular cutout for simply supported and clamped boundary conditions (K_s =
5/6).

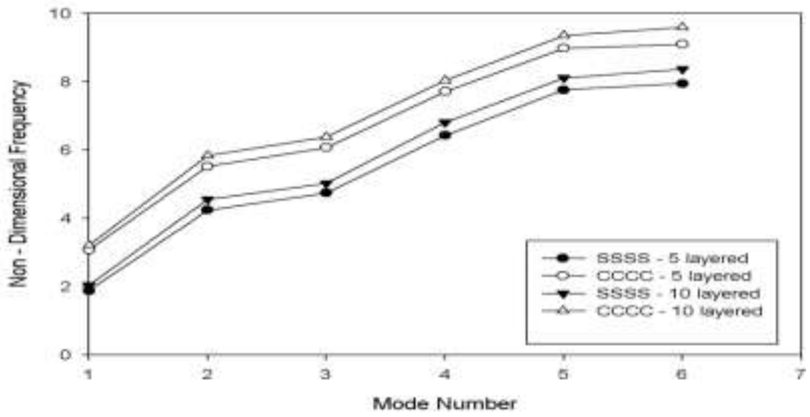

Figure 6: First six natural frequencies ($\acute{\omega} = (\frac{\omega\, a^2}{\Pi^2 h})*\sqrt{(\rho/E2)}$) of symmetric five and ten layered angle-ply (-45°/45°/-45°…), 15° skew laminated moderately thick (a/h=20) plate (a/b=1) with circular cutout for simply supported and clamped boundary conditions
(K_s = 5/6).

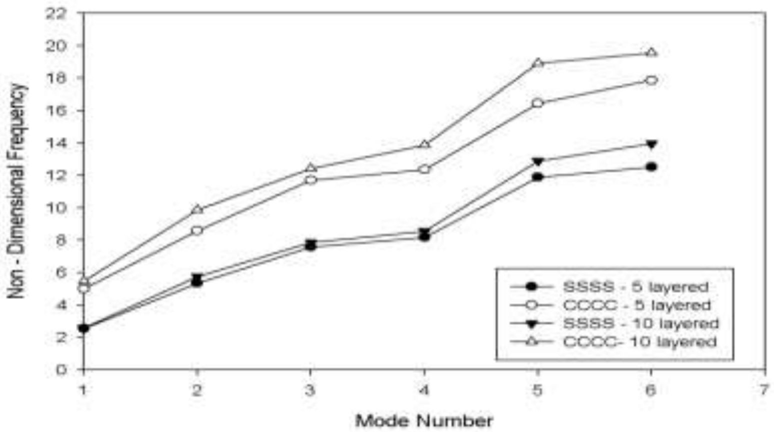

Figure 7: First six natural frequencies ($\acute{\omega} = (\frac{\omega\, a^2}{\Pi^2 h})*\sqrt{(\rho/E2)}$) of symmetric five layered and ten layered cross ply (90°/0°/90°…), 30° skew laminated thin (a/h=1000) plate (a/b=1) with circular cutout for simply supported and clamped boundary conditions (K_s = 5/6).

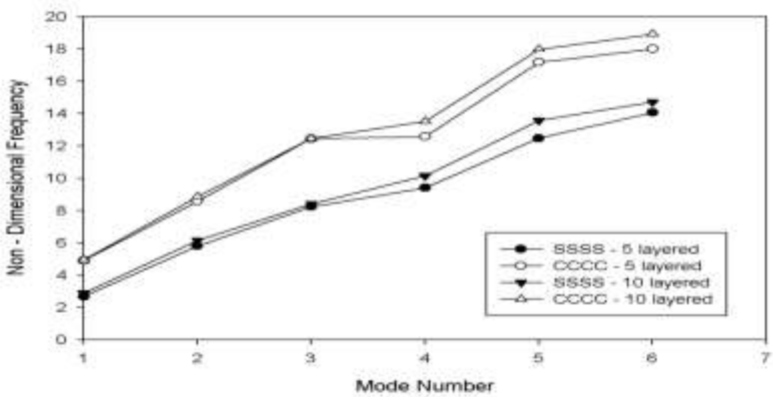

Figure 8: First six natural frequencies ($\acute{\omega} = (\frac{\omega\, a^2}{\Pi^2 h})*\sqrt{(\rho/E2)}$) of symmetric five layered and ten layered angle-ply (-45°/45°/-45°…), 30° skew laminated thin (a/h=1000) plate (a/b=1) with circular cutout for simply supported and clamped boundary conditions (K_s = 5/6).

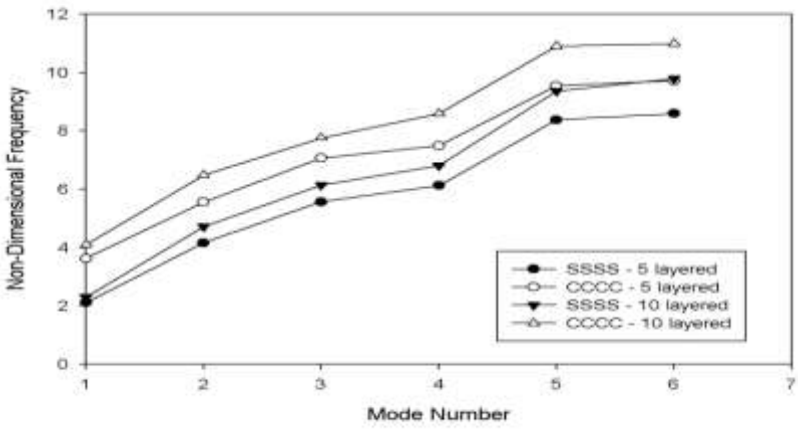

Figure 9: First six natural frequencies (ώ = ($\frac{\omega\, a^2}{\Pi^2 h}$)*√ (ρ/E2)) of symmetric five layered and ten layered angle-ply (90°/0°/90°…), 30° skew laminated thick (a/h=20) plate (a/b=1) with circular cutout for simply supported and clamped boundary conditions (K_s = 5/6).

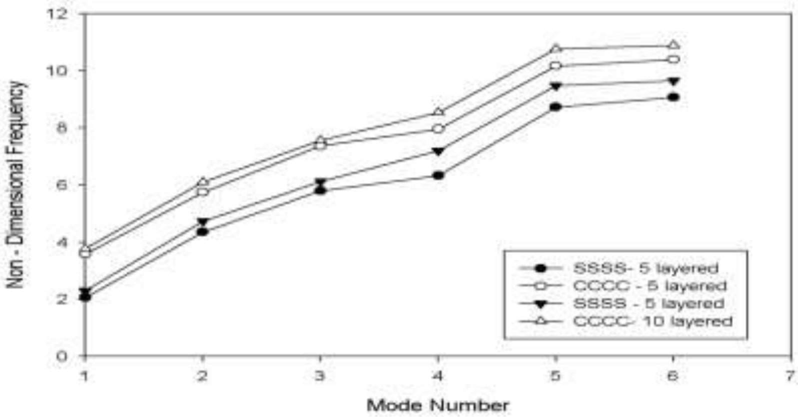

Figure 10: First six natural frequencies (ώ = ($\frac{\omega\, a^2}{\Pi^2 h}$)*√ (ρ/E2)) of symmetric five layered and ten layered angle-ply (-45°/45°/-45°…), 30° skew laminated thick (a/h=20) plate (a/b=1) with circular cutout for simply supported and clamped boundary conditions (K_s = 5/6).

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