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Hybrid Methods of Some Evolutionary Computations AndKalman Filter on Option Pricing

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ABSTRACT: *The search for a better option price continues within the financial institution. In pricing a put option, holders of the underlying stock always want to make the best decision by maximizing profit. We present an optimal hybrid model among the following combinations: Kalman Filter-Genetic Programming(KF-GP), Kalman Filter-Evolutionary Strategy(KF-ES) and Evolutionary Strategy -Genetic Programming(ES- GP). Our results indicate that the hybrid method involving Kalman Filter-Evolutionary Strategy(KF-ES) is the best model for any investor. Sensitivity analysis was conducted on the model parameters to ascertain the rigidity of the model.*
Keywords:*Evolutionary Computation, Kalman Filter, Option Pricing.*

I. INTRODUCTION

Optimization is a mathematical technique through which best possible results are obtained under the given set of conditions. Some of the methods used in optimization to address option problems are the traditional and Evolutionary Computation methods. Option is one of the important research areas in the financial world. Investors are provided with a world of opportunity through options, since they are given the power to adjust market situations. These investors can choose to protect a particular asset by hedging to minimize risk. A lot of people including investors, traders, and financial institutions are still searching for an optimal way to make maximum profit. Many financial institutions recently use options because of its higher returns. Xu and Zang(1975), used Kalman Filter to compute the stock prices and concluded that the method was fast and easy. Kumar and Mansukhani(2011), also used the same algorithm in a situation where the estimate was linear and discrete. The method removed the uncertainty while still maintaining vital information. Kalman Filter gave very good predictions for the prices of stocks.Zheng(2015), used genetic programming in a situation where some genetic operators were changing within the algorithm. Crossover and mutation were varied during the runs in an attempt to solve a real world option pricing problem. The varying genetic programming method improved performance of calculated option price. Chidambaran et al.(1998), developed a procedure upon which genetic programming was applied to compute for option price. A given set of data was used to test for efficiency which proved be good. Their model showed that the approximated true solution is better than the Black-Scholes model when stock prices followed a jump-diffusion process. It was proved that the program out-performed other models in many different settings due to it robustness and efficiency.

Investors are therefore confronted with the challenge of finding the optimal option price which may be trapped in a local optimum (Ackora-Prah et al., 2014). In this paper we investigate three hybrid methods on option price for a European put.

II. THEORETICAL CONCEPT

European Put Option. This provides the person the right to buy an underlying asset at a maturity time at a strike price K, but not obliged to do so. ItsPayoff $P_T$is:

$$P_T = \begin{cases} (K - S_T)^+, K > S_T & \text{if exercise} \\ 0 \text{ if not exercise} \end{cases}$$

The seller of European put option will always expect the underlying asset priceto fall below the strike price at the expiry date. The investor exercises atmaturity with an intrinsic value P_t, which will help in calculating the optionprice as:

$$e^{-rT}P_T \;\Rightarrow\; e^{-rT}(K-S_T)^+.$$

Stochastic process. This is given by $\{S_t\}_{0\le t\le T}$ and deals with a collection of random variable on $(\Omega,\mathcal{F})$, where each $\gamma\epsilon\Omega$, $S_t(\gamma)$ is a sample path of S_t that is associated with γ, $\Omega=\gamma_1,\gamma_2,\gamma_3,\cdots$ is a sample space and $\mathcal{F}$ is a $\sigma-$ algebra, $\mathcal{F}$ is the set of all observable event for a single trial which have the following properties;

i) $\emptyset,\Omega\epsilon\mathcal{F}$, where $\emptyset$ is an empty set.
ii) If B $\epsilon\mathcal{F}$, then $\Omega\backslash B\epsilon\mathcal{F}$.
iii) If $B_i\epsilon\mathcal{F}$, then $\cup_{i=1}^{\infty}B_i\epsilon\mathcal{F}$,

The probability space $(\Omega,\mathcal{F},\mathbb{P})$ is equipped with a filtration or a collection of σ-algebra $\{\mathcal{F}_t\}_{0\le t\le T}$, $\mathbb{P}$ is the probability measure on $\mathcal{F}$, where $\mathbb{P}(\gamma)\epsilon[0,1]$ $\forall\gamma\epsilon\Omega$, such that $\mathbb{P}(\Omega)=1$ and $\mathcal{F}_0\subseteq\mathcal{F}_s\subseteq\mathcal{F}_t\subseteq\mathcal{F}_T$ for all $0\le s\le t\le T$

Standard Brownian Motion. It is a one-dimensional stochastic process $\{W_t\}_{t>0}$ which is indexed with a non-negative real number t, with the following properties (Mcwilliams,2005):

i) $W_0=0$.
ii) Given a probability 1, the function $t\to W_t$ implies that W_t is continuous on the interval $[0,\infty)$, W_t has a stationary independent increment and has a normal distribution with a zero mean and variance of 1.

Model of the Underlying Asset. If S_t is the price of the underlying asset, μ is the drift and σ the volatility, S_t is said to follows a Geometric Brownian Motion model if it satisfies

$$dS_t \;=\; \mu S_t dt + S_t\sigma dW_t.$$

Using Itô lemma on $d\ln S_t$ we have:

$$\begin{aligned} d\ln S_t &= \frac{1}{S_t}dS_t - \frac{1}{2S_t^2}dS_t^2 \\ &= \frac{1}{S_t}S_t[\mu dt+\sigma dW_t] - \frac{1}{2S_t^2}S_t^2(\mu dt+\sigma dW_t)^2 \end{aligned}$$

$$d\ln S_t = \frac{1}{S_t}S_t[\mu dt+\sigma dW_t] - \frac{1}{2S_t^2}S_t^2(\mu^2dt^2+2\mu dt\sigma dW_t+\sigma^2 dW_t^2)$$

$$\text{as}\quad (dt)^2, dW_t dt\to 0, (dW_t)^2\to dt$$

$$\begin{aligned} &= \frac{1}{S_t}S_t[\mu dt+\sigma dW_t] - \frac{1}{2S_t^2}S_t^2\sigma^2 dW_t^2 \\ &= \mu dt+\sigma dW_t-\frac{1}{2}\sigma^2 dt. \end{aligned}$$

Applying the fundamental theorem of calculus we have,

$$\begin{aligned} \int_0^t d\ln S_t &= \int_0^t\left(\mu-\frac{1}{2}\sigma^2\right)dt+\sigma dW_t \\ \ln S_t-\ln S_0 &= \left(\mu-\frac{\sigma^2}{2}\right)t+\sigma W_t-\left(\mu-\frac{\sigma^2}{2}\right)0+\sigma W_0 \end{aligned}$$

$$\ln S_t - \ln S_0 = \left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t$$
$$S_t = S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t}.$$

Given the values for our parameters μ and σ we can compute the solution for a Geometric Brownian Motion throughout the given interval.

Mean and Variance of the Underlying Asset. The mean of the underlying asset is;

$$E[S_t] = E\left[S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right) t}\right] E[e^{\sigma W_t}],$$

$$\begin{aligned} E[S_t] &= S_0 e^{\left(\mu t - \frac{\sigma^2}{2}\right) t} e^{\frac{t\sigma^2}{2}} \\ &= S_0 e^{\mu t - \frac{t\sigma^2}{2} + \frac{t\sigma^2}{2}} \\ &= S_0 e^{\mu t}. \end{aligned}$$

The variance of the underlining asset is as shown below;

$$\begin{aligned} Var[S_t] &= E[S_t^2] - (E[S_t])^2 \\ &= S_0^2 e^{(2\mu + \sigma^2)t} - (S_0 e^{\mu t})^2 \\ &= S_0^2 e^{2\mu t + \sigma^2 t} - S_0^2 e^{2\mu t} \\ &= S_0^2 e^{2\mu t}(e^{\sigma^2 t} - 1). \end{aligned}$$

State space formulation. We use Geometric Brownian Motion to simulate the underlying asset price as follows:

$$dS_t = S_t \mu dt + S_t \sigma dW_t$$

$$dX_t = \mu dt + \sigma dW_t$$

where $X_t = \ln S_t$

$$\Delta X_t = \mu \Delta t + \sigma \varepsilon \sqrt{\Delta t}$$
$$X_t - X_{t-1} = \mu \Delta t + \sigma \varepsilon \sqrt{\Delta t}.$$

Since $\omega \sim N(0, \sigma^2 \Delta t)$ then,

(1) $$X_t = X_{t-1} + \mu(t_i - t_{i-1}) + \omega_t,$$

Kalman Filter. The invention of Kalman Filter has seen an extensive re-search area and has become appealing due to its simple and robust nature.This method is a recursive mathematical process that fuses model output withobserved data to estimate the hidden variable. It is an optimal estimator sinceit minimizes the mean square error of the estimated parameter, which is knownto be a variance minimizing algorithm that always update the state estimatewhen measurement becomes available by minimizing the trace of the error co-variance. If the state space is linear then the Kalman scheme can be used forstate estimation thus:

(2) $$X_t = F_t X_{t-1} + f_t + \omega_t$$

with F as a transition matrix that relates the state at various time step X_t, f_t applies its effect on the state and ω_t is a Gaussian with a zero mean and a given variance Q_t.

The measurement equation is of the form,

$$Y_t = H_t X_t + \nu_t \tag{3}$$

Y_t is the measurement parameter or the observation, H_t is a measurement operator which projects the state onto the measurement equation and ν_t is the measurement noise with R_t as its variance.

Kalman Filter equations are:
Prior estimate is given as;

$$X_t^- = E[X_t|Y_{t-1}] = F_t X_{t-1}^+ + f_t. \tag{4}$$

Prior error estimate covariance is:

$$P_t^- = E[e_t^-(e_t^-)^T] = F_t P_{t-1}^+ (F_t)^T + Q_t. \tag{5}$$

Predicted measurement equation is shown below;

$$Y_t^- = E[Y_t|Y_{t-1}] = H_t X_t^-. \tag{6}$$

Posterior estimate is denoted by

$$X_t^+ = X_t^- + K_t(Y_t - Y_t^-) \tag{7}$$

Posterior error estimate covariance uses the equations:

$$\begin{aligned} P_t^+ &= E[e_t^+(e_t^+)^T] \\ &= E[((I - K_t H_t)(e_t^-) - K_t\nu_t)((I - k_t H_t)(e_t^-) - K_t\nu_t)^T] \\ &= (I - K_t H_t)P_t^-(I - K_t H_t)^T + K_t R_t K_t^T. \end{aligned} \tag{8}$$

We equate the minimum trace of P_t^+ to zero.

$$\begin{aligned} \frac{\partial tr P_t^+}{\partial K_t} &= -P_t^- H_t^T + K_t H_t P_t^- H H_t^T + K_t R_t = 0 \\ K_t &= P_t^- H_t^T (H_t P_t^- H_t^T + R_t)^{-1} \end{aligned} \tag{9}$$

From equation(8) and (9), we have the posterior error covariance estimate as:

$$\begin{aligned} P_t^+ &= P_t^- - P_t^- K_t^T H_t^T - K_t H_t P_t^- + K_t(H_t P_t^- H_t^T + R_t)K_t^T \\ &= (I - K_t H_t)P_t^- \end{aligned} \tag{10}$$

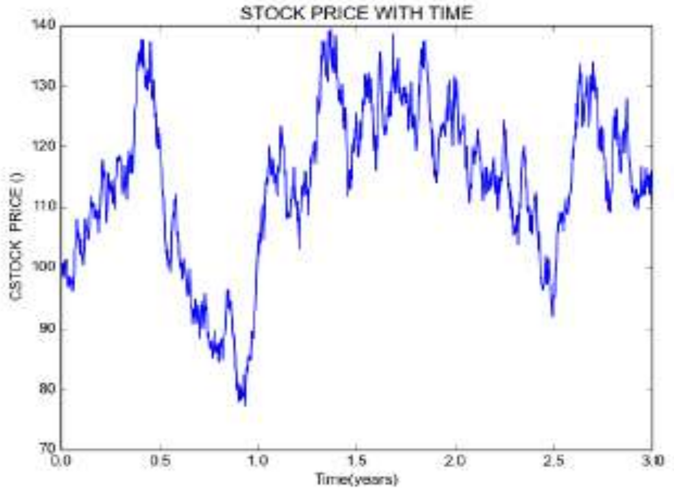
Evolutionary Strategy. This method involves the random change of experimental setup. This experimental strategy led to good result when it was tested.The major quality of Evolutionary Strategy is its ability to incorporate majorparameters of the strategy such as standard deviation and the correlation coefficient (covariance) of a normally distributed mutation.

Genetic programming. Genetic programming (GP) begins with a lot of randomly created computer programs. This population of programs is progressively evolved over a series of generations. The evolutionary search uses theDarwinian principle of natural selection (survival of the fittest) and analogs ofvarious naturally occurring operations, including crossover (sexual recombination), mutation, gene duplication, gene deletion. Genetic programming worksin an automated environment for creating a working computer program from
a high-level problem statement.

Graphical Simulation of the Underlying asset. The values for the simulation are as follows: underlying asset price S0 = 100, interest rate = 0:15,volatility = 0:35 and the maturity time T = 3. The graph below shows thesimulation of the underlying asset price over the given time [0; 3]: From thegraph it can be seen that the price of the asset changes with time. The priceof the asset assume positive values as shown in the graph.

III. Methodology

3.1. Kalman Filter and Genetic Programming Algorithm. Numerical values are assigned to the given parameters to estimate the option price. The price of the underlying asset follows a Geometric Brownian motion model. The initial price of the underlying asset is $S_0 = 100$, the strike price $K = 140$ with volatility $\sigma = 0.35$ with an interest rate of $\mu = 0.15$ and a maturity time $T = 3$



years. Kalman filter is used to generate random stock price(S_T). These random prices serve as our population of individuals, with a given fitness function of $e^{-rt}(K - S_T) < S_0$. Roulette wheel is used in our selection. We then used the fitness value of each individual to calculate the proportion of each individual on the Roulette wheel. Selection is applied randomly for the next stage of crossover and mutation, which are two point crossover and sub-tree mutation. After the stopping criteria has been met, the option price was obtained. The summary of the use of the hybrid algorithm using python 2.7 is:

i) Random population of stock price at maturity by using the Kalman filter.
ii) Genetic programming in a tree form(identical ramped grow).
iii) Fitness value evaluation, only positive values from our option pricing were selected.
iv) Selection; roulette wheel selection.
v) Recombination; two point recombination.
vi) Mutation; sub-tree mutation.
vii) Stopping criteria.

3.2. Kalman Filter and Evolutionary Strategy Algorithm. We generate random stock price using Kalman Filter. These random numbers served as our population of individuals, with a given fitness function of $e^{-rt}(K - S_T) < S_0$. During the stage of selection (μ, λ)-selection is used. Sexual intermediate recombination was applied on the parent population and the strategy parameters. Mutation was applied on the new generation. After stopping criteria has been met we obtained the value of option price, which is from the fittest offspring. We present the summary of steps used in the algorithm.

i) Random population of stock price at maturity by using the Kalman Filter.
ii) Evolutionary strategy was then applied on the option pricing equation.
iii) We evaluate the fitness value.

vi) Mutation to generate the fittest offspring.
vii) Stopping criteria.

3.3. Genetic Programming and Evolutionary Strategy Algorithm. Thestock price are from the log form of stock price equation using Geometric Brownian Motion model. The first part of this Hybrid solution was from GeneticProgramming for selection and crossover until the new population was generated. Evolutionary Strategy is then applied during mutation. The new mutatedpopulation is used to compute the option price of a European put option. Thesolution to the hybrid method is obtained after the stopping criteria has beenmet. Simulations was done using python 2.7 below;

4. Conclusion

Hybrid, Kalman Filter and GP gave GH ¢36:57, Kalman Filter and Evolutionary Strategy gave GH ¢27:03 and Genetic Programming and EvolutionaryStrategy gave ¢34:85 as the value of the option price. Hybrids gave us threesolutions from which we obtained our global minimum option price, which wasKalman Filter and Evolutionary Strategy. It is therefore advisable for investorsto resort to our method in the option market. This solution will help investorsto position themselves in the option market by reducing the loses that sometimes occur.

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